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SCHOLAR
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FOR FEDERAL
& KPK BOARD

11

A Conceptual Approach to

PHYSICS

SUBJECTIVE

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S E R I E S

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Approach to

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Preface

ALLAH Almighty be thanked a million times for showering his special blessing a work of such magnitude. Those who have under taken such a Herculean task can fully understand the obstacles, barriers and problems which need to be overcome.

This book is unique in its kind and texture as it caters for the need of the students and teachers alike. It covers a wide range of textual styles on one hand and on the other tends to develop the knowledge, skills and appetite for attaining practical knowledge for future fears.

We also owe our deepest sense of gratitude to Scholar Publications for inculcating in us a self belief which proved out to be our greatest source of strength.

We medge that any suggestions from our worthy teaching fraternity shall be highly appreciated for the improvement and a mention about any mistake.

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Every Chapter gives you

- ◆ Learning Objects
- ◆ Questions Answers Type
- ◆ Topic Wise MCQ's
- ◆ Important Formulae
- ◆ Exercise Answers with Explanations
- ◆ Numerical Problems
- ◆ Solved Examples
- ◆ Past Boards MCQ's - Short Questions
Long Questions
- ◆ Tit - Bits, Do You Know
- ◆ Self - Assessment Tests
- ◆ Model Ppaers
- ◆ For Your Information, Intrusting Information
Answers with Explanations.

CHAPTER

1

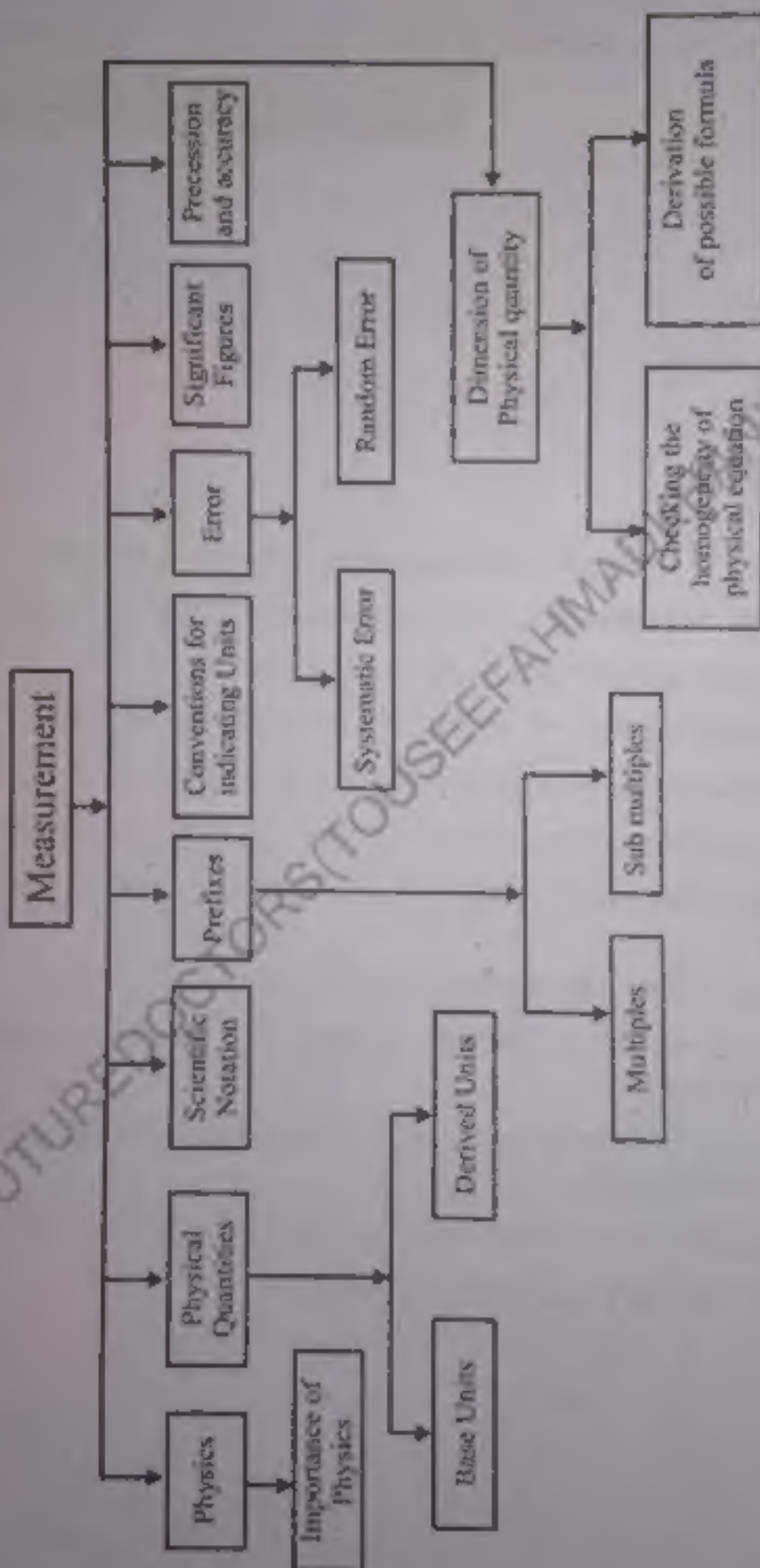
MEASUREMENTS

Learning Objectives

- ❖ Describe the scope of Physics in science, technology and society.
- ❖ State SI base units, derived units, and supplementary units for various measurements.
- ❖ Express derived units as products or quotients of the base units.
- ❖ State the conventions for indicating units as set out in the SI units.
- ❖ Explain why all measurements contain some uncertainty.
- ❖ DISTINGUISH between systematic errors (including zero errors) and random errors.
- ❖ Identify that least count or resolution of a measuring instrument is the smallest increment measurable by it.
- ❖ Differentiate between precision and accuracy.
- ❖ Assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties.
- ❖ Quote answers with correct scientific notation, number of significant figures and units in all numerical and practical work.
- ❖ Check the homogeneity of physical equations by using dimensionality and base units.
- ❖ Derive formulae in simple cases using dimensions.

Chapter No. 1

CONCEPT MAP



Q.1 Define Physics? Explain the scope and importance of physics in science, technology and society?

Ans: PHYSICS AND ITS SCOPE

"Physics is the branch of science in which we study about properties of matter and energy; and their mutual relationship."

OR

"Physics is the branch of science that involves the study of the physical world in specific and physical universe in general: energy, matter, and how they are related."

Scope and Importance of Physics

- In physics modeling of the natural world is made with theory, and is usually expressed quantitatively with mathematical description.
- Even if we do not study PHYSICS as a subject, we depend on it for nearly everything.
- From walking to driving a car, from cooking to using a gadget, from cutting a tree to building a new house everything involves physics.
- Even as we read this sentence, physics is at work. Physicists investigate the motions of electrons and rockets, the energy in sound waves and electric circuits, the structure of the proton and that of the universe.
- Physics is also called 'the fundamental science' because the subject of study of all branches of natural science like chemistry, astronomy, geology, and biology are constrained by laws of physics. We can say 'All other natural sciences stem from physics. Chemistry is essentially applied physics and biology is applied chemistry'.
- For technologies to develop physics is essential.
- Physics generates fundamental knowledge needed for the future technological advances that will continue to drive the economic engines of the world.
- So many pivotal discoveries of the 20th century – including the laser, television, radio, computer technology plus internet, DNA and nuclear weapons are all credited to advancement in physics.
- Physics contributes to the technological infrastructure and provides trained personnel needed to take advantage of scientific advances and discoveries. It is important for improvement in health, telecommunication, transport and design of our future.
- Nearly all consumer goods we use at home have been developed from research in physics.

Q.2 What is system of units? In SI what is meant by base, derived and supplementary units?

Ans: SYSTEM INTERNATIONAL (SI)

A complete set of units for all physical quantities is called system of units.

However, to form a system we do not need to define every quantity. We take only a few quantities (called base quantities) and base units to agree on accessible and invariable standards for measurement such that all other quantities and units are expressed in terms of those quantities.

The International System of Units (abbreviated SI from *système internationale*, the French version of the name) is a scientific method of expressing the magnitudes or quantities of important natural phenomena.

Science require that quantities must be defined and measured. Things that cannot be measured like beauty, love, hate, are all not science.

On the other hand quantities like length, time, density, temperature, electric fluxes can be measured therefore they are called physical quantities.

"The quantities which can be measured are called physical quantities."

Different systems of quantities:

In earlier times scientists around the world were using different systems of units for their liking. Three such systems, the MKS, the CGS and the FPS (or British) system were in use extensively till recently.

In 1960 an international committee agreed on a single system for whole world, the system's official name is the Systems International, or SI, meaning International System.

We can use other systems and its units (fahrenheit, pounds, and miles) for our convenience but in science we must always use SI.

Base Units: In SI, SEVEN physical quantities chosen arbitrarily as base quantities and their corresponding units are called base units.

Table 1.1: SI BASE QUANTITIES AND BASE UNITS			
Base Quantity		SI Base Unit	
Name	Symbol	Name	Symbol
Length	l, x, r etc.	meter	m
Mass	m	kilogram	Kg
Time, duration	t	second	s
Electric current	I	ampere	A
Thermodynamic temperature	T	kelvin	K
Amount of substance	n	mole	mol
Luminous intensity	I_v	candela	cd

Derived Units: A quantity and its unit obtained and developed from base quantities and their respective units without giving any consideration to the directional properties are called derived quantities and its units.

Some of the derived quantities are given by:

TABLE 1.2: SI DERIVED QUANTITIES AND DERIVED UNITS			
Derived Quantity		SI Coherent Derived Unit	
Name	Symbol	Name	Symbol
Area	A	Square meter	m^2
Volume	V	Cubic meter	m^3
Speed, velocity	V, \vec{V}	Meter per second	ms^{-1}
Acceleration	a	Meter per second squared	ms^{-2}

Supplementary Units: These are such units which can neither be placed as base units nor in derive units.

Pure geometrical units (radian and the steradian) were classified by the System International (SI) as supplementary units.

(But this designation was abrogated in 20th CGPM (french words Conférence générale des poids et mesures abbreviated from General Conference on Weights and Measures) in 1995 and the units were grouped as derived units).

There are two supplementary units i.e. radian and steradian.

Q.3 Differentiate between radian and steradian. Discuss in detail

Ans: **Radian:** radian is the unit of plane angle.

"One radian (1 rad) is the angle subtended at the center of a circle by an arc with a length equal to the radius of the circle."

Formula:

Mathematically

$$\text{Number of radians in } \frac{4\pi \times \text{length}}{\text{Radius of circle}} = \frac{s}{r}$$

Relation radian measurement and degree measurement.

$$\text{Number of degrees in one revolution} = 360^\circ \quad \text{--- (1)}$$

$$\text{Where as the number of radians in one revolution} = \frac{\text{Circumference of Circle}}{\text{Radius of same circle}}$$

$$\text{Number of radians in one revolution} = \frac{2\pi r}{r} = 2\pi \text{ radians}$$

Comparing Eq. (1) & (2)

As number of degrees in one revolution = Number of radians in one revolution

$$\text{Therefore, } 2\pi \text{ rad} = 360^\circ$$

$$\text{OR } 1 \text{ rad} = \frac{360}{2\pi} = \frac{360^\circ}{2 \times 3.14} = 57.3^\circ$$

An angle of approximate 57° corresponds to 1 radian.

In one rotation,

$$\theta = 2\pi \text{ radians} = 2 \times 3.14 \text{ radians} = 6.28 \text{ radians}$$

$$\text{And } 360 = 2\pi \text{ rad}$$

$$\therefore 1^\circ = \frac{2\pi}{360} \text{ rad} = \frac{2 \times 3.14}{360} \text{ rad} = 0.01745 \text{ rad}$$

Steradian Steradian is the unit for solid angle

Steradian is defined as the solid angle subtended at the centre of a sphere by a surface equal to the square of radius of that sphere

Formula:

Mathematically,

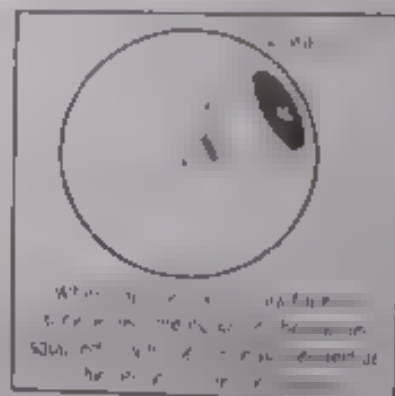
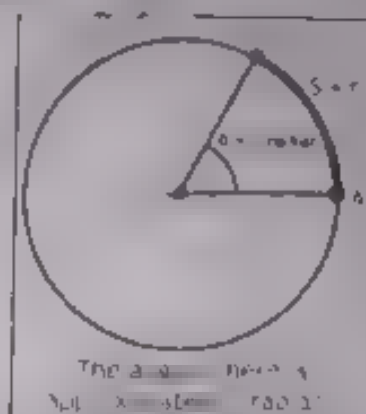
$$\text{Number of steradians in sphere} = \frac{\text{Area of sphere}}{r^2}$$

Since

Surface area of closed sphere of radius r is $4\pi r^2$

herefore

$$\text{Number of steradians in sphere} = \frac{4\pi r^2}{r^2}$$

Sphere (or any closed surface) subtends 4π (12.56 sr)**Assignment 1.1:**

A pulley of radius 0.9 m is used to lift a bucket from the well. If I took 3.6 rotations for the pulley to take water out of the well, how deep is water in the well?

Solution

$$r = 0.9 \text{ m}$$

$$n = 3.6 \text{ rotations} = 3.6 \times 2\pi \text{ radian} = 3.6 \times 2 \times 3.14 \text{ radian} = 22.6 \text{ radians}$$

$$\text{Depth of well, i.e. distance covered by pulley in 3.6 rotations} = s = r\theta$$

Since

$$s = r\theta$$

$$\therefore s = 0.9 \times 22.6$$

$$s = 20.3 \text{ m} \approx 20 \text{ m}$$

MCQ's

1. Silicon is obtained from

A. Water

B. Metals

C. Wood

D. Sand

2. Mass is a _____ quantity
(A) Derived (B) Base (C) Both derived and base (D) None of these
3. The SI unit of solid angle is.
(A) steradian (B) Degree (C) Revolution (D) Radian
4. Solid angle subtended at the centre of sphere of radius 'r' in steradian is
(A) 2π (B) 4π (C) 8π (D) 16π
5. An example of derived unit is
(A) candela (B) ampere (C) coulomb (D) mole
6. Which is not a base unit in SI units?
(A) kilogram (B) joule (C) ampere (D) kelvin
7. The unit of work in base units is _____
(A) kg m s^{-1} (B) kg m s^{-2} (C) kg m s^{-3} (D) kg m s^{-4}
8. The angle made by Ice Cone at its edge is a
(A) Plane angle (B) Solid angle (C) Critical angle (D) Abtuse angle
9. Candela is the SI unit of
(A) Charge (B) Luminous intensity (C) Power (D) Refractive index
10. The S.I unit of power in terms of base unit are
(A) kg m s^{-2} (B) kg m s^{-3} (C) kg m s^{-4} (D) kg m s^{-5}
11. Which of the following is a derived quantity?
(A) Force (B) Mass (C) Length (D) Time
12. Which is derived unit?
(A) candela (B) ampere (C) kelvin (D) newton
13. Unit used for the factor $\sqrt{\frac{1}{g}}$ may be
(A) meter (B) second (C) kilogram (D) radian

Answers Key

1 D	2 B	3 A	4 B	5 C	6 B	7 C	8 B	9 B	10 D	11 A	12 D
13 B											

Q 4 What is scientific notation?**Ans:** Scientific Notation

"Numbers are expressed in standard form called scientific notation, which employs power of ten."

OR

"Writing a number in powers of 10 or standard form $M \times 10^p$ is called scientific notation."

It is the product of a number greater than 1 and less than 10 (called the mantissa) and a power (or exponent) of 10

$$\text{number} = \text{mantissa} \times 10^{\text{power}}$$

- For example if someone writes that human body contains approximately 7,000 000 000,000,000 000 000 000 atoms = 7×10^{27} , where 7 is the mantissa and 27 is the exponent
- Another advantage of scientific notation is that it makes it easy to add, subtract, multiply and divide large numbers. For example, to multiply two numbers in scientific notation, we multiply their mantissas and then add their exponents
- If we wanted to estimate, as how many atoms are contained in the bodies of all the people on Earth we could do this calculation easily. The population of earth is approximately 7 billion (or 7×10^9). To find our answer we have to multiply 7×10^{27} by 7×10^9 . We do this by multiplying the two mantissas and adding their exponents

$$(7 \times 10^{27}) \times (7 \times 10^9) = (7 \times 7) \times 10^{27+9} = 49 \times 10^{36} = 4.9 \times 10^{37}$$

POINT TO POWER

Why is it important to have a standard system of units that is used by all scientists and engineers?

In December 1966, the NASA launched the Mars Viking orbiter on a scientific mission to the Red Planet from Mars. Nine months later on September 23, 1970, the orbiter descended to the surface of the planet Mars. The investigation showed that the orbital conditions were not as fast as an error in the orbital determination between two teams working on the computer. They had used English and metric systems and positions where the two groups disagree. The result of the error might well have indicated a failure.



Q 5 What are prefixes? Draw their table

Ans: Prefixes:

These are letters or symbols written before unit of the measurement to represent certain specific powers of ten. Some prefixes have powers of ten positive (multiplier) and some have negative power of ten (Sub multiplier). OR

A mechanical through which a term is selected with a symbol unit is given a proper name to its power of ten is called prefix to the power of ten.

For example,

- The length of a house is 5×10^3 m equivalent to 5 kilometres (km).
- The distance of a satellite 8.25×10^4 m from Earth can be expressed as 8.25×10^4 kilometres (km).

PREFIXES		
Prefix	Decimal Multiplier	Symbol
Yotta	10^{24}	Y
Zetta	10^{21}	Z
Exa	10^{18}	E
Peta	10^{15}	P
Tera	10^{12}	T
Giga	10^9	G
Mega	10^6	M
Kilo	10^3	k
Hecto	10^2	h
Deca	10^1	da
Deci	10^{-1}	d
Centi	10^{-2}	c
milli	10^{-3}	m

micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p
femto	10^{-15}	f
atto	10^{-18}	a
zepto	10^{-21}	z
yocto	10^{-24}	y

Q 6 Define the following units of lengths which are mostly used in measurements
Light Year, Angstrom, Micron

ANS Prefixes

i. Light Year:

A Light year (ly) is a non SI unit of length.

It is defined as the distance that light travels in a vacuum in one year.

$$1 \text{ Light year} = 9.461 \times 10^{15} \text{ m}$$

For example

- Spiral galaxies like our own Milky Way measure approximately 2×10^5 light years in diameter.
- Our nearest neighbor galaxy is the small spiral galaxy Andromeda, which has been determined to be approximately 2.5 million ly away.

ii. Angstrom: The angstrom (symbol \AA) is a non SI length.

It is unit of length equal to 10^{-10} meter or 0.1 nanometer (nm).

For example

- A hydrogen atom has a size of about 5×10^{-11} m.
- Nucleus of helium is only 1 femtometer (10^{-15} meters) in diameter.

iii. Micron: A micron (symbol μ) is an obsolete name of micrometer.

It is equal to 1×10^{-6} meters.

For examples

- Red Blood Cells (RBCs) are approximately 10 microns in diameter.
- Human hair is between 1 to 100 microns in diameter.

MCQ's

- One Peta is equal to _____
(A) 10^9 (B) 10^{14} (C) 10^{16} (D) 10^{18}
- The ratio of 1 nanometre to 1 attometre is _____.
(A) 10^9 (B) 10^6 (C) 10^9 (D) 10^{12}
- The prefix pico is equal to _____
(A) 10^{-12} (B) 10^{-15} (C) 10^{-18} (D) 10^{-21}
- One tera is equal to _____
(A) 10^{12} (B) 10^9 (C) 10^6 (D) 10^3
- Which of the following is least multiple _____
(A) piko (B) femto (C) nano (D) atto
- The prefix atto stands for _____
(A) 10^9 (B) 10^{18} (C) 10^{-9} (D) 10^{-18}
- How many nanometers are in a meter?
(A) 10^{-9} (B) 10^9 (C) 10^9 (D) 10^9
- One femto is equal to _____
(A) 10^{-15} (B) 10^{-12} (C) 10^{-18} (D) 10^{-21}
- One giga is equal to _____
(A) 10^9 (B) 10^{18} (C) 10^{-9} (D) 10^{-18}

Answers Key

1 B

2 A

3 A

4 C

5 D

6 D

7 C

8 D

9 A

DO YOU KNOW

here are some non-SI related units for instances extensively used around the world. Some of these units are discussed below.

A. Light Year A light year (ly) is a non-SI unit of length, is defined as the distance the light travels in a vacuum in one year, which is 9.46×10^{15} m. For example, a star nearest like our own Milky Way measured approximately 2×10^5 light years is distance. Our nearest neighbor galaxy is the great spiral galaxy Andromeda, which is thought determined to be approximately 2.5 million ly away.

B. Angstrom The angstrom or angstrom (Å) is also a non-SI and internationally recognized unit of length equal to 10^{-10} m or 0.1 nanometre. For example, a helium atom has a size of about 1 Angstrom, while its nucleus is only 10^{-14} m or 10^{-5} Å in diameter.

C. Micron A micron is an obsolete name of a millimetre, which is a thousandth part of the metre, about 10^{-6} metres. For example, Red Blood cells are about approximately 10 microns in diameter. Human hair is between 50 and 100 microns in diameter.

Q.7 Write down the conventions for indicating the units. What are scientific notation?

Ans: Conventions for Indicating the Units (WRITING UNIT SYMBOLS AND NAMES)

Unit symbols

- Unit symbols are printed in roman, upper case. For example m for metre, s for second and Pa for pascal.
- e.g. when writing pico meter, p m is wrong, correct is pm.
- Compound prefixes are never used. For example, micro nano meter or pm is not µm (micro micro meter).
- To multiply or divide the two units, the normal rules of algebraic (multiplication or division) apply.
 - ⇒ Multiplication must be indicated by a space or a half-high (centred) dot (·).
 - ⇒ Division is indicated by a horizontal line, by a slash (oblique stroke, /) or by negative exponents.
- For example, N m or N·m for a newton metre and m/s or $m s^{-1}$ for metre per second.
- It is not allowed to use abbreviations for and symbols for fractions, such as second per second sq mm (for either m^2 or square millimetre), cc (for either cm^3 or cubic centimetre).
- When multiple of unit is raised to the power, the power applies to the whole multiple not just the unit.

Unit names

- Full name of the unit does not begin with a capital letter even if named after scientist, e.g. newton.
- The symbol of units named after a scientist has initial capital such as N for newton.
- A multiple or sub multiple prefix is part of the unit and is written before the unit symbol without any space.

Q.8 Define error. What are its different types?

Ans: ERRORS:

"The difference between measured value and acceptable value in any measurement is called an error."

OR

"Error is the doubt that exists about the result of any measurement. For every measurement, even the most careful, there is a way of marking a doubt which is called Error."

Types of Errors

The errors in measurement can be broadly classified as (A) systematic errors and (B) random errors.

- A. Systematic Errors** The systematic errors are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are
- (a) **Instrumental Errors** This error arises due to faulty behaviour of an instrument.

- As there is uncertainty to some extent in every measurement, therefore every measurement need to be written in the form

$$\text{measurement} = \text{best estimate} \pm \text{uncertainty}$$

Example

For example a measurement $(5.07 \pm 0.02) \text{ g}$ means that the experimenter is confident that the actual value for the quantity being measured lies between 5.05 g i.e. $(5.07 - 0.02) \text{ g}$ and 5.09 g i.e. $(5.07 + 0.02) \text{ g}$.

Types of Uncertainties

Absolute Uncertainty or Uncertainty = least count of measuring instrument
It is denoted by the symbol Δ and has the same units as the quantity.

i. **Fractional Uncertainty or Relative Uncertainty** = $\frac{\text{Absolute Uncertainty}}{\text{Measurement}}$

It is denoted by the symbol $\frac{\Delta}{x}$ and has no units.

ii. **Percentage Uncertainty** = Fractional Uncertainty $\times 100\%$

$$\frac{\text{Absolute Uncertainty}}{\text{Measurement}} \times 100\%$$

For example let's find a cylinder measured with Vernier calipers. It has a length of 2.35 cm so

Measurement of length = 2.35 cm

Uncertainty = L.C. of Vernier calipers = $\pm 0.01 \text{ cm}$

So its length will be written as

$$l = 2.35 \pm 0.01 \text{ cm}$$

Indicating Uncertainty in Calculation

A numeric measure of confidence in a measurement is referred to as uncertainty. A lower uncertainty indicates greater confidence. Uncertainties are usually expressed by using statistical methods.

A Sum or difference: Absolute uncertainties are added

- Suppose two physical quantities A and B have measured values

$$A \pm \Delta A \text{ and } B \pm \Delta B \text{ respectively}$$

where ΔA and ΔB are their absolute uncertainties. The following steps are followed for the result $Z \pm \Delta Z$ in the sum and difference.

Sum: Let $Z = A + B$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. We have by addition $Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$ $Z \pm \Delta Z = (A + B) \pm (\Delta A + \Delta B)$ The maximum possible uncertainty is $Z \pm \Delta Z = (A + B) \pm (\Delta A + \Delta B)$	Difference: Let $Z = A - B$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. We have $Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$ $Z \pm \Delta Z = (A - B) \pm (\Delta A + \Delta B)$ The maximum value of the uncertainty ΔZ is $\Delta Z = \Delta A + \Delta B$
--	---

B Product or quotient: Fractional uncertainties are converted into percentage uncertainties which are added

- Suppose two physical quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute uncertainties and $\Delta A\%$ and $\Delta B\%$ are their percentage uncertainties. The following steps are followed for the result $Z \pm \Delta Z$ in their product and quotient.

Product: Suppose $Z = AB$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then $Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$ Convert fractional uncertainty to percentage uncertainty $Z \pm \Delta Z = (A \pm \Delta A\%)(B \pm \Delta B\%)$	Quotient: Suppose $Z = \frac{A}{B}$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then $Z \pm \Delta Z = \frac{(A \pm \Delta A)}{(B \pm \Delta B)}$ Convert fractional uncertainty to percentage uncertainty $Z \pm \Delta Z = \frac{(A \pm \Delta A\%)}{(B \pm \Delta B\%)}$
---	---

Multiply the product and add percentage uncertainties

$$Z \pm \Delta Z = AB \pm (\Delta A\% + \Delta B\%)$$

$$Z \pm \Delta Z = AB \pm (\Delta A + \Delta B\%)$$

Convert back to absolute fractional uncertainty

$$Z \pm \Delta Z = AB \pm \Delta Z$$

Divide the ratio and add percentage uncertainties

$$Z \pm \Delta Z = A/B \pm (\Delta A\% + \Delta B\%)$$

$$Z \pm \Delta Z = A/B \pm (\Delta A + \Delta B\%)$$

Convert back to absolute fractional uncertainty

$$Z \pm \Delta Z = A/B \pm \Delta Z$$

C Power P (in Watts) is multiplied by power

thereby converted back to fractional uncertainty and then converted back to absolute

$$P \pm \Delta P = V \pm \Delta V$$

and the measured values of V are $V \pm \Delta V$

$$P \pm \Delta P = (V \pm \Delta V)^2$$

Convert the uncertainty to percentage uncertainty

$$\Delta P = \Delta V \times 2$$

$$P \pm \Delta P = V^2 \pm 2\Delta V$$

$$Z \pm \Delta Z = V^2 \pm 2\Delta V$$

Convert back to absolute fractional uncertainty

$$Z \pm \Delta Z = V^2 \pm \Delta Z$$

Assignment 1.2:

A physicist calculated the wall width of half brick thickness. The brick is laid in a flat position, lengthwise called 'stretcher position' as (13.6 ± 0.1) cm. And one brick thickness (the brick is placed in flat position lengthwise orthogonal to wall called 'header position') as (23.6 ± 0.1) cm. Calculate the difference in width of walls with uncertainty in it.

Solution

Given Data

To Find

$$W = W_1 - W_2$$

$$W = 13.6 \pm 0.1 - 23.6 \pm 0.1$$

$$W = -10 \pm 0.2$$

$$W = 10 \pm 0.2$$

Assignment 1.3:

The voltage $V \pm \Delta V$ is measured as $7.3 \text{ V} \pm 0.1 \text{ V}$ and current $I \pm \Delta I$ is measured as $2.3 \text{ A} \pm 0.1 \text{ A}$. Calculate the resistance R by using Ohm's Law as $R = V/I$.

Solution

Given Data

$$V = 7.3 \pm 0.1$$

$$I = 2.3 \pm 0.1$$

$$R = V/I$$

$$R = 7.3/2.3$$

$$R = 3.17$$

$$R = 3.17 \pm \Delta R$$

$$R = 3.17 \pm \Delta R$$

$$R = 3.17 \pm \Delta R$$

$$R = 3.17 \pm \Delta R$$

$$R = 3.17 \pm \Delta R$$

DO YOU KNOW

Technical specification data

Variable uncertainty

Variable uncertainty

Variable uncertainty

Variable uncertainty

Variable uncertainty

Variable uncertainty

Variable uncertainty

Variable uncertainty

Variable uncertainty

Variable uncertainty

Variable uncertainty

Variable uncertainty

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Variable uncertainty

$$\text{Percentage uncertainty in } I = \frac{\Delta I}{I} \times 100\% = \frac{0.05}{1.8} \times 100\% = 2.8\% \approx 3\%$$

$$\text{Thus total uncertainty in } R = 1\% + 2\% = 3\%$$

$$\text{Absolute uncertainty in } R = 1.8 \times 10^{-2} \times 3\% = 0.00054$$

$$\text{So, } R = (1.8 \pm 0.00054) \text{ m}$$

Assignment 1.4:

The radius of sphere 'r' is measured with vernier callipers as $(r \pm \Delta r) = (2.25 \pm 0.01) \text{ cm}$. Calculate the volume of sphere $(47.7 \pm 0.5) \text{ cm}^3$.

Solution

Given Data Radius of sphere $r = 2.25 \text{ cm}$
Absolute uncertainty in radius $\Delta r = 0.01 \text{ cm}$

To Prove $V = 47.7 \pm 0.5 \text{ cm}^3$

$$V = \frac{4}{3} \pi r^3$$

$$\Delta V = 3 \pi r^2 \Delta r$$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{\Delta V}{V} = \frac{3 \pi r^2 \Delta r}{\frac{4}{3} \pi r^3}$$

$$\frac{\Delta V}{V} = \frac{9 \Delta r}{4 r}$$

$$\text{Absolute uncertainty in } V = \frac{9 \Delta r}{4 r} \times V = \frac{9 \times 0.01}{4 \times 2.25} \times 47.7 \text{ cm}^3$$

$$\text{Percentage uncertainty in } V = \frac{1}{4} \times \frac{\Delta r}{r} \times 100\% = 0.9\%$$

$$\text{Total percentage uncertainty in } V = 0.9\% + 2\% = 2.9\% \approx 3\%$$

$$\text{Absolute uncertainty in } V = \frac{3}{100} \times 47.7 \text{ cm}^3 = 1.431 \text{ cm}^3$$

$$\text{So, } V = (47.7 \pm 1.431) \text{ cm}^3$$

MCQ's

- The percentage uncertainty in mass and velocity of an object are 2% and 3% respectively. Which of the following is the maximum uncertainty in the measurement of its kinetic energy?
A. 11% B. 10% C. 14% D. 16%
- If $s = 10.5 \pm 0.1 \text{ cm}$ and $\pi = 3.14 \pm 0.01$, A is given by
A. $16.1 \pm 0.1 \text{ cm}$ B. $16.3 \pm 0.1 \text{ cm}$ C. $16.1 \pm 0.0 \text{ cm}$ D. $16.3 \pm 0.4 \text{ cm}$
- The percentage uncertainty in radius of a sphere is 2%. Which of the following is the total percentage uncertainty in the volume of a sphere?
A. 4% B. 6% C. 8% D. 10%

Answers Key

1. B

2. B

3. C

Q 10 What are significant figures? How can we estimate the number of significant figure in the physical measurement?

Ans: Significant Figures

In any measurement, the accurately known digits and the first doubtful digit are called significant figures.

OR

In other words, a significant figure is the one which is known to be reasonably reliable importance.

The concept of significant figures is very important and useful. It shows the extent or limit to which readings are reliable.

Rules for Finding Significant Figures

1. NON ZERO digits are always significant. That is all the digits from 1 to 9 are significant, e.g. the number of significant figures in 47872 is 5.
 2. ZERO in between two significant digits is always significant, e.g. the number of significant figures in 1015006 is 7.
 3. ZEROS to the left of significant figures are not significant, e.g. the number of significant figures in 0.000538 is 3.
 4. ZEROs to the right of the significant figure may or may not be significant.
 - (i) In decimal fractions zero to the right of a decimal fraction are significant, e.g. in 5.200 there are 4 significant figures.
 - (ii) However, if the number is an integer, number of significant figures depends upon the least count of the measuring instrument.
- Example** consider the measurement 8000 kg.
 If the least count is 1 kg then there are four significant figures and the measured value will be expressed as 8.000×10^3 kg.
 Similarly if the least count is 10 kg and the correct measurement in appropriate significant figures is 8.00×10^3 kg, then there are three significant figures.
5. In scientific notation or standard form, the figures other than power of ten are significant, e.g. in number 2.1000×10^6 there are 5 significant figures.

Significant Figures in Calculation:**(a) Addition and Subtraction****Rule:**

When two or more quantities are added or subtracted, the result is as precise as the least precise of the quantities. After adding or subtracting, round the result by keeping only as many decimal places as are in the figure containing least decimal places of the quantities that were added or subtracted.

For example, $44.56005 + 0.0698 + 1103.2 = 1147.82985$.

We do not want to write all of those digits in the answer. Answer should be written upto one decimal place because data has least one decimal place.
 the sum is written = 1147.8

(b) Multiplication and Division**Rule:**

When quantities are multiplied or divided, the result has the same number of significant figures as the quantity with the smallest number of significant figures.

For example: A calculator gives $45.26 \times 2.41 = 109.0766$.

Since the answer should have only three significant figures, we round the answer to

$$45.26 \times 2.41 = 109$$

In scientific notation, we write 1.09×10^2 .

Tip

In a series of calculations, rounding to the correct number of significant figures should be done only at the end, not at each step. Rounding at each step would increase the error. That roundoff error could snowball and have an adverse effect on the accuracy of the final answer. It's a good idea to keep at least two extra significant figures in calculations, then round at the end.

Assignment 1.5:

Calculate the answers to the appropriate number of significant figures

(a) $0.31 + 0.1$

(b) $658.0 + 23.5478 + 1345.29$

(c) 8×7

(d) $0.9935 \times 10.48 = 13.4$

(e) $5.5 / 1.1$

(f) $73.2 + 18.72 = 91.9$

Solution:

(a) $0.31 + 0.1 = 0.41 \approx 0.4$ (data has least 1 decimal place)

(b) $658.0 + 23.5478 + 1345.29 = 2026.8378 \approx 2026.8$ (data has least 1 decimal place)

- (c) $8 \times 7 = 56 = 60$ (data has least one significant figure)
 (d) $0.9934 \times 10.48 \times 1.14 = 1.1951272 = 1.395$ (0.9934 = 10 = 1.40 = 0) (data has least 3 significant figure)
 (e) $5.5 - 1.1 = 4 = 5.0$ (data has 2 significant figures)
 (f) $\frac{7 \times 2 + 18.77}{1.4} = 55.152931176 = 55$ (data has 2 significant figures)

MCQ's

- How many Significant figures are in 0.008767
 (A) 3 (B) 4 (C) 5 (D) 6
- What is the sum of three numbers 2.7543, 4.10, 1.273 up to correct decimal place?
 (A) 8.12 (B) 8.13 (C) 8.1273 (D) 8.127
- Which is correct for the diameter of wire when measured by a screw gauge of least count 0.001 cm?
 (A) 2.3cm (B) 2.31cm (C) 2.32cm (D) 2.3124cm
- In scientific notation the number 0.0001 may be written as _____
 (A) 10^{-4} (B) 10^{-1} (C) 10^{-10} (D) 10×10^4
- A student added three figures 72.1, 3.32 and 0.003. What is the correct answer regarding the rules of the addition of the significant figures?
 (A) 75.423 (B) 75.42 (C) 75.4 (D) 75.5
- Number of significant figures in 01.030 mm are _____
 (A) 2 (B) 3 (C) 4 (D) 5
- 275.00 has the significant digits _____
 (A) 2 (B) 3 (C) 4 (D) 5
- Significant figures in 0.0010 are _____
 (A) 1 (B) 2 (C) 3 (D) 4
- Which one of the following is the correct record for the diameter of a ball when measured with a vernier calipers of least count of 0.01 cm
 (A) 5.3 cm (B) 5.31 cm (C) 5.312 cm (D) 5.3124 cm
- The significant figures in 0.04080 are _____
 (A) 2 (B) 4 (C) 5 (D) 6

Answers Key

1. A	2. B	3. C	4. C	5. C	6. C	7. D	8. B	9. B	10. B
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Q 11 Differentiate between precision and accuracy in measurement.**Ans: PRECISION AND ACCURACY**

When a value is measured, two parameters precision and accuracy affect the quality of the measurement. Therefore it is important to clearly distinguish between them.

Precision

- In measurements the term precision describes the degree of exactness with which a measurement is made and stated (that is, the position of the last significant digit, i.e. last digit of least count).
 For example, precision of the measurement 293 (000) km is 1000 km. (The position of the last significant digit is in the thousands place).
 Similarly the precision of the measurement 0.0210 s is 0.0001 s. (The position of the last significant digit is in the ten thousandths place).
- A precise measurement is the one which has less absolute uncertainty.
- The precision of a measurement is associated with the least count of the measuring instrument.
- The precision of a measurement is determined by the instrument or device being used.
- Smaller the least count of the instrument, measurement will be more precise and vice versa.
- Precision means how close the measured values are to each other.

Accuracy:

- In measurement the accuracy describes the closeness of a measured value to the actual value of the measured quantity.
- The accuracy of a measurement is the difference between your measurement and the accepted correct answer. The bigger the difference, the less accurate your measurement.
- An accurate measurement is one which has less fractional or percentage error (i.e. per cent uncertainty). The accuracy of a measurement depends on the fractional or relative error (uncertainty, or percentage uncertainty) in that measurement.
- Accuracy is related with the measurement.
- Smaller the fractional or percentage uncertainty, measurement will be more accurate and vice versa.
- The accuracy of a measurement depends upon the number of significant digits. The greater the number of significant digits given in a measurement, the better is the accuracy, and vice versa. For example, the accuracy of the measurement 0.025 cm is indicated by two significant digits.
- Accuracy means how close a measured value (result) is to the actual (true) value.

TABLE PRECISION AND ACCURACY

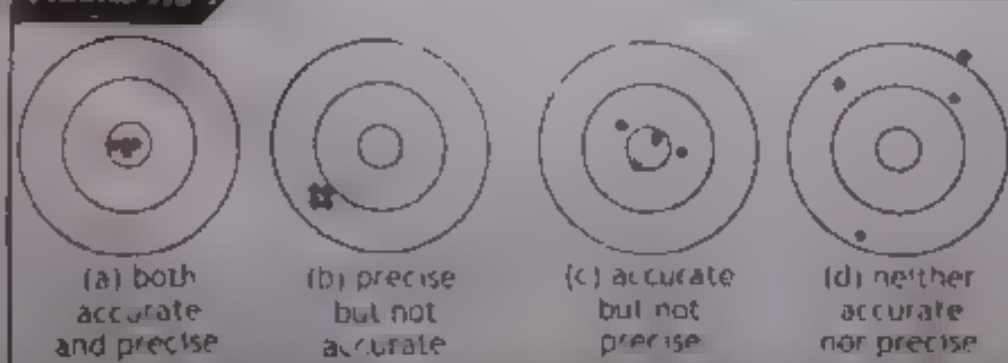
Measurement	Precision	Accuracy (significant digits)
1.4 m	1 m	2
2.50 m	10 cm	3
34.000 km	1000 m	5
20.005 kg	0.01 kg	5
0.000005 kg	0.000001 kg	6
78 N	1 N	2
4.750 s	0.01 s	4
100.000 km	0.001 km	6

Accuracy shows how well the results of a measured value agree with the actual value (that is the accepted value as measured by competent experimenters). As it is difficult to know the actual true value, it is only predicted theoretically and then is accepted based on the results of repeated experiments. For example, the accepted value of acceleration due to gravity is 9.80 m/s^2 .

Understanding Precision and Accuracy Using Game of Dots

Consider a dart game with bull's eye at the centre as shown in the Figure.

- If the darts are close to the bull's eye and close to each other, there is both accuracy and precision as in Figure (a).
- If all of the darts are very close together, but far from the bull's eye, there is precision but not accuracy as in Figure (b).
- If the darts are spread around the bull's eye, there is mathematical accuracy because the average of the darts is in the bull's eye as in Figure (c).
- If the darts are neither close to the bull's eye nor close to each other, there is neither accuracy nor precision as in Figure (d).

FIGURE 1.3

$$\begin{aligned} [R.H.S.] &= [1 T^{-1}] + [1 T^{-1}] \\ &= 2[1 T^{-1}] \end{aligned}$$

As 2 has no dimension being a number so

$$[R.H.S.] = [1 T^{-1}]$$

Thus $[R.H.S.] = [L.H.S.]$

Hence the equation is dimensionally correct

(b) $[L.H.S.] = [S] = [L]$

$$\begin{aligned} [R.H.S.] &= \left\{ (V_0) \left[\frac{1}{t} \right] + \frac{1}{2} a \left[\frac{1}{t^2} \right] \right\} \\ &= [L T^{-1}] [T] + [L T^{-2}] [T^2] \\ &= [L T^{-1+1}] + [L T^{-2+2}] \\ &= [L T^0] + [L T^0] \\ &= [L] + [L] = 2[L] \\ &= [L] \end{aligned}$$

$$\Rightarrow [L.H.S.] = [R.H.S.]$$

So, equation is dimensionally consistent

Assignment 1.7:

Find an expression for the time period 'T' of a simple pendulum. The time period 'T' may depend upon (i) mass 'm' of the bob of the pendulum, (ii) length 'l' of pendulum, (iii) acceleration due to gravity 'g' at the place where the pendulum is suspended.

Solution:

To Find:

Relation for the time period of simple pendulum - $T = ?$

Calculations:

The relation for the time period 'T' will be of the form

$$T \propto m^a \times l^b \times g^c \times \theta^d$$

$$\text{or } T = \text{constant } m^a l^b g^c \theta^d \quad (1)$$

Taking dimensions on both sides, we get

$$\text{As } S = r\theta$$

$$\text{Or } \theta = \frac{S}{r}$$

$$\text{Thus } \theta = [L L^{-1}]$$

$$[T] = \text{constant } [M]^a [L]^b [L L^{-1} T^{-2}]^c [L L^{-1}]^d$$

Comparing the dimensions on both sides

$$[T]^1 = [T]^{-2d}$$

$$[M]^0 = [M]^a$$

$$[L]^0 = [L]^{b+c+d}$$

Equating powers on both sides, we get

$$2d = 1 \quad \text{or} \quad d = -\frac{1}{2}$$

$$a = 0$$

$$\text{and } b + d = 0$$

$$\text{Or } b = -d \quad \text{or} \quad b = \frac{1}{2}$$



$$\text{and } \theta = [L]^{-1} T = [L]^{-1} T$$

Substituting the values of a, b, c and d in equ (1)

$$T = \text{constant} \times m^0 \times l^{\frac{1}{2}} \times 1 \times g^{-\frac{1}{2}}$$

$$\text{or } T = \text{constant} \times l^{\frac{1}{2}} \times g^{-\frac{1}{2}}$$

$$\text{or } T = \text{constant} \sqrt{\frac{l}{g}}$$

DIMENSIONAL FORMULA (DIMENSIONS) OF PHYSICAL QUANTITIES

Quantity	Formula	Dimensions	SI Unit	In terms of base unit
1. Displacement	Distance in a particular direction	[L]	m	m
2. Area	(Length) ²	[L ²]	m ²	m ²
3. Volume	(length) ³	[L ³]	m ³	m ³
4. Density	Mass/volume	M/L ³	kg/m ³	kg/m ³
5. Speed	Distance/time	[L/T]	m/s	m/s
6. Acceleration	a = Δv/Δt	[L/T ²]	m/s ²	m/s ²
7. Force	F = ma	[ML/T ²]	newton (N)	kg m/s ²
8. Momentum	P = mv	[ML/T]	N s	kg m/s
9. Work	W = Fd	[ML ² /T ²] = [ML ² /T ²]	joule (J)	kg m ² /s ²
10. Kinetic energy	K.E. = 1/2 mv ²	[ML ² /T ²]	joule (J)	kg m ² /s ²
11. Torque	τ = Force × moment arm	[ML ² /T ²] [ML ² /T ²]	N m	kg m ² /s ²
12. Power	P = $\frac{\text{Work}}{\text{Time}}$	[ML ² /T ³] = [ML ² /T ³]	watt (W)	kg m ² /s ³
13. Pressure	P = $\frac{\text{Force}}{\text{Area}}$	[ML/T ²] [L] = [ML/T ²]	N/m ² and Pascal (Pa)	kg/m s ²
14. Stress	= Force/area	[ML/T ²]	N/m ²	kg/m s ²
15. Modulus of elasticity	= $\frac{\text{Stress}}{\text{Strain}}$	[ML/T ²]	N/m ²	kg/m s ²
16. Gravitational constant G	= $\frac{\text{Force} \times (\text{distance})^2}{\text{Mass}}$	[M ⁻¹ L ³ T ⁻²]	N m ² /kg	kg m ³ /s ²

17 Co-efficient of viscosity	$\eta = \frac{\text{Velocity}}{\text{Velocity gradient}}$	$\left[\frac{ML^{-1}}{T} \right]$	Nsm	$kg\ m\ s^{-1}$
18 Time period	$T = \frac{2\pi}{\omega}$	$[T]$	s	s
19 Frequency	$\omega = \frac{\text{Angle}}{\text{Time}}$	$[T^{-1}]$	Hz	
20 Force constant (or Spring constant)	$k = \frac{\text{Force}}{\text{Extension}}$	$\left[\frac{ML^{-1}}{T^2} \right]$	N/m	$kg\ s^{-2}$
21 Angle	$\theta = \frac{\text{Arc length}}{\text{Radius}}$	$[1]$	rad or (rad)	
22 Angular velocity	$\omega = \frac{\text{Angle}}{\text{Time}}$	$[T^{-1}]$	rad/s	
23 Angular acceleration	$\alpha = \frac{\text{Angular velocity}}{\text{Time}}$	$[T^{-2}]$	rad/s ²	
24 Angular momentum	$L = mvr$	$[ML^2 T^{-1}]$	$kg\ m^2\ s^{-1}$	$kg\ m^2\ s^{-1}$

QUANTITIES HAVING SAME DIMENSIONS

Quantities	DIMENSIONS
1 Length, distance, displacement, wavelength, radius, diameter, thickness, depth, height, year, wave length, half period, etc.	$[L]$
2 Momentum, impulse	$[MLT^{-1}]$
3 Angular momentum, Planck's constant	$[ML^2 T^{-1}]$
4 Work, Energy, Moment of force, Torque	$[ML^2 T^{-2}]$
5 Pressure, Stress, Elastic constant, Energy density	$[ML^{-1} T^{-2}]$
6 Frequency, Angular velocity, Angular frequency	$[T^{-1}]$
7 Force, weight, tension, spring	$[MLT^{-2}]$

MCQ's

- Which pair has the same dimension?
(A) Work and Power (B) Force and Torque (C) Torque and Power (D) None
- Which of the following are the dimensions of work?
(A) $[MLT^{-1}]$ (B) $[ML^2]$ (C) $[MLT^{-2}]$ (D) $[MLT]$
- Which of the following are the dimensions of light year?
(A) $[L]$ (B) $[T]$ (C) $[MLT^{-1}]$ (D) $[L]$
- Which of the following pair has the same dimensions?
(A) Work and Power (B) Momentum and Energy (C) Power and Pressure (D) Work and Torque

5. The dimensions $[ML^{-1}T^{-2}]$ belongs to
 A. Pressure (D) Momentum (C) Force (D) Heat energy
6. Which of the following are the dimensions of density?
 A. ML^{-3} (D) ML^{-2} (C) ML^{-1} (D) $ML^{-2}T^{-1}$
7. $[M^1L^1T^{-1}]$ refers to.
 A. velocity (D) time period (C) frequency (D) force
8. Which of the following are the dimension of torque?
 A. $[ML^2T^{-2}]$ (D) $[ML^2T^{-1}]$ (C) $[ML^2T^{-2}]$ (D) $[ML^2T^{-1}]$
9. Which of the following are the dimension of acceleration due to gravity?
 A. $[ML^{-1}T^{-2}]$ (D) $[ML^{-1}T^{-1}]$ (C) $[ML^{-1}T^{-2}]$ (D) $[ML^{-1}T^{-1}]$
10. Which of the following are the dimension of angular velocity?
 A. $[T^{-1}]$ (D) $[T]$ (C) $[T^{-1}]$ (D) $[T]$
11. Which of the following pair have same dimension?
 A. Work and power (D) Mass, time and velocity (C) Force and torque (D) Torque and power
12. The dimension of $\frac{1}{S}$ is same as that of
 A. Time (D) Energy (C) Velocity (D) Force
13. The dimensions of the relation $\frac{F}{m}$ are equal to the dimensions of
 A. Force (D) Acceleration (C) Energy (D) Force
14. Which of the following are the dimension of Power?
 A. $[ML^2T^{-3}]$ (D) $[ML^2T^{-2}]$ (C) $[ML^2T^{-3}]$ (D) $[ML^2T^{-2}]$
15. Which of the following are the dimension of coefficient of viscosity?
 A. $[ML^{-1}T^{-1}]$ (D) $[ML^{-1}T^{-1}]$ (C) $[ML^{-1}T^{-1}]$ (D) $[ML^{-1}T^{-1}]$

Answers Key

1 D	2 C	3 D	4 D	5 D	6 C	7 C	8 C	9 C	10 D	11 B	12 A
13 D	14 C	15 A									

General Information

1. Which of the following is equal to the numerical value of one light year?
 A. $9.46 \times 10^{15} \text{ m}$ (D) $9.46 \times 10^{16} \text{ m}$ (C) $9.46 \times 10^{14} \text{ m}$ (D) $9.46 \times 10^{15} \text{ km}$
2. How many number of colors used in the process of color printing to produce the entire range of colors?
 A. 4 (D) 5 (C) 6 (D) 7
3. Light year is the unit of
 A. length (D) Mass (C) Time (D) Speed

Answers Key

1 C	2 A	3 A
-----	-----	-----

Key Points

- Physics: The study of the physical world in specific and physical universe in general.
- System International (SI) System of Units is adopted specifically by the science community for measurement of physical quantities. The SI units consists of seven fundamental units from which all the units for other physical quantities developed called derived units.
- Least count or resolution: The smallest increment measurable by measuring instrument.
- Error: The doubt that exists about the result of any measurement.
- Uncertainty: The quantification or magnitude of error or doubt in measurement.
- Precision: The degree of exactness with which a measurement is made and stated.
- Accuracy: The closeness of a measured value to the actual value of the measured quantity.
- Significant figures: In any measurement the number of accurately known digits and first doubtful digit.
- Dimension: Expressing a physical quantity in terms of base physical quantities by using special symbols. Dimensions of a physical quantity help to understand its relation with base physical quantities.

Solved Examples

Example 1.2

Two connected gears are rotating. The smaller gear has a radius of 0.4 m and the larger gear's radius is 0.7 m. What is the angle through which the larger gear has rotated when the smaller gear has made one complete rotation?

Given Data: Larger gear's radius $r_1 = 0.7 \text{ m}$
 Smaller gear's radius $r_2 = 0.4 \text{ m}$
Required: Angle of rotation for larger gear ' θ_1 ' = ?
Solution

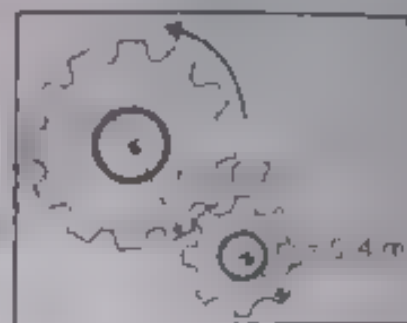
The smaller gear performs one complete rotation i.e. $\theta_2 = 2\pi$, hence path length travelled by it is $s = r_2 \theta_2$

Putting values $s = 0.4 \text{ m} \times 2\pi$
 $s = 0.8\pi \text{ m}$

So, an arc of length s on the larger circle would form an angle θ_1 as

$$s = r_1 \theta_1$$

Putting values $\theta_1 = \frac{s}{r_1} = \frac{0.8\pi}{0.7}$
 $\theta_1 = 3.6 \text{ radians}$ Ans



Extension Exercise

What is the angle in degrees through which the larger gear has rotated?

$$\theta_1 = 3.6 \text{ radians} = 3.6 \times 57.3^\circ = 206.3^\circ$$

Example 1.3

If $d \pm \Delta d = (101.41 \pm 0.05) \text{ mm}$ represents the internal diameter of the metal pipe and $d \pm \Delta d = (102.79 \pm 0.05) \text{ mm}$ represents the external diameter of the metal pipe then find the thickness of the metal part of the pipe including uncertainty in it

Given Data: Internal diameter $d \pm \Delta d = (101.41 \pm 0.05) \text{ mm}$
 External diameter $d \pm \Delta d = (102.79 \pm 0.05) \text{ mm}$

Required: Thickness of Pipe ' t ' = ?

Solution.

To find the thickness of the pipe we would subtract the internal diameter from the external diameter however we will add the fractional uncertainties.

$$d = d_2 - d_1 \pm (\Delta d_1 + \Delta d_2)$$

Putting values $d = 102.79 - 101.41 \pm (0.05 + 0.05) \text{ mm}$
 $d = 1.38 \pm 0.10 \text{ mm}$

We are not to subtract uncertainties when subtracting measurements. Always gets worse as more measurements are combined.

Since the difference in the radii is required for the thickness t that difference is d so the uncertainty must be divided by 2 as the percentage uncertainty remains the same. Hence

$$t = \frac{d}{2} = \left(\frac{1.38}{2} \pm \frac{0.10}{2} \right) \text{ mm}$$

$$t = (0.69 \pm 0.05) \text{ mm}$$

Example 1.4

The length and width of a rectangular room are measured to be $l = (l \pm \Delta l) = (3.955 \pm 0.005) \text{ m}$ and $w = (w \pm \Delta w) = (3.050 \pm 0.005) \text{ m}$. Calculate the area $A = (A \pm \Delta A)$ of the room and its uncertainty.

Given length $l = (3.955 \pm 0.005) \text{ m}$
 width $w = (3.050 \pm 0.005) \text{ m}$

Required Area $A = (A \pm \Delta A) = ?$

Solution.

For the product percentage uncertainties are added. The length l and width w in percentage uncertainties are

$$= 4.050 \text{ m} \pm \frac{0.05 \text{ m}}{4.050 \text{ m}} \times 100\% = 4.050 \text{ m} \pm 0.12\%$$

and $w = 2.955 \text{ m} \pm \frac{0.005 \text{ m}}{2.955 \text{ m}} \times 100\% = 2.955 \text{ m} \pm 0.17\%$

Since the area of a rectangle is the product of length and width

$$A = l \times w \quad \text{or} \quad A = 4.050 \text{ m} \pm 0.12\% \times 2.955 \text{ m} \pm 0.17\%$$

In multiplication the percentage uncertainties are added

$$A = 4.050 \text{ m} \times 2.955 \text{ m} \pm (0.12\% + 0.17\%) \quad \text{or} \quad A = 12.20 \text{ m}^2 \pm 0.29\%$$

To convert it back to fractional uncertainty we have

$$A = 12.20 \pm \frac{0.29}{100} = 12.20 \text{ m}^2$$

Or $A = 12.20 \pm 0.35 \text{ m}^2$

Example 1.10 A ball drops from rest from an unknown height 'h'. The time 't' it takes for the ball to hit the ground is measured to be $(t \pm \Delta t) = (1.3 \pm 0.2) \text{ s}$. The height is related to this time by the equation $h = \frac{1}{2}gt^2$ (where $g = 9.81 \text{ m/s}^2$). Assume that the value for 'g' carries no uncertainty and calculate the height 'h' including its uncertainty.

Given time $t \pm \Delta t = (1.3 \pm 0.2) \text{ s}$

acceleration due to gravity ' g ' = 9.81 m/s^2

Required Solution height $h = h \pm \Delta h = ?$

For the power percentage uncertainties is multiplied with power. The percentage uncertainty in time t is

$$t = 1.3 \pm \frac{0.2}{1.3} \times 100\% = 1.3 \pm 15.4\%$$

Since the Height h is given by $h = \frac{1}{2}gt^2$

Putting values $h = \frac{1}{2} \times 9.81 \text{ m/s}^2 \times (1.3 \text{ s})^2 \pm 15.4\%$

For the power percentage uncertainties is multiplied with power. therefore

$$h = \frac{1}{2} \times 9.81 \text{ m/s}^2 \times 1.69 \text{ s}^2 \pm 2 \times 15.4\%$$

Or $h = 8.30 \text{ m} \pm 30.8\%$

To convert it back to absolute uncertainty we have

$$h = 8.30 \pm \frac{30.8}{100} \times 8.30 \text{ m}$$

Or $h = 8.3 \pm 2.6 \text{ m}$

Example 1.11 Calculate the answers to the appropriate number of significant figures.

(a) $0.35 - 0.1$

(b) $32.567 + 135.0 + 1.4567$

(c) $420.03 + 299.270 + 99.068$

(d) 14×8

(e) $(2400)(3.45)(16.21)$

(f) $\frac{32.09 + 1.2 - 17.035}{19.8}$

Solution.

(a) Calculating

$$0.35$$

$$\pm 0.1$$

$$0.25$$

Not worrying about significant figures, the result of $0.35 - 0.1 = 0.25$

But, according to the rules of significant figures, the result should have the same number of decimal places as the input with the fewest number of decimal places. The result of our calculation should be rounded to the tenths place. so $0.35 + 0.1 = 0.2$

Or $0.35 + 0.1 = 0.2$ Answers

(b) Calculating

$$\begin{array}{r} 32.567 \\ 135.0 \\ + \quad 1.4567 \\ \hline 169.0237 \end{array}$$

But, since according to the rules of significant figures, the result should be rounded to the lowest number of decimal places as in the input given numbers. The result of our calculation should be rounded to one decimal place.

So

Or $32.567 + 135.0 + 1.4567 = 169.0$ Answers.

(c) Calculating:

$$\begin{array}{r} 420.03 \\ 299.270 \\ + \quad 94.068 \\ \hline 818.368 \end{array}$$

But, since according to the rules of significant figures, the result should be rounded to the lowest number of decimal places as in the input given numbers. The result of our calculation should be rounded to two decimal place. Therefore

Or $420.03 + 299.270 + 94.068 = 818.37$ Answers.

(d) Calculating

$$\begin{array}{r} 14 \\ \times \quad 8 \\ \hline 112 \end{array}$$

However, according to the rules of significant figures, the result should have the same number of significant figures as the quantity with the smallest number of significant figures. In this case the number 14 has two significant digits and number 8 has one significant digit. Therefore, the result of our calculation should be rounded to only one significant digit. Therefore

Or $14 \times 8 = 100 = 1 \times 10^2$ Answers.

(e) Calculating:

$$\begin{array}{r} 2400 \\ 3.45 \\ \times \quad 16.21 \\ \hline 134,218.8 \end{array}$$

However, according to the rules of significant figures for multiplication, the result should have the same number of significant figures as the quantity with the smallest number of significant figures. In this case the number 2400 has two significant digits, number 3.45 has three significant digits and number 16.21 has four significant digits. Therefore, the result of our calculation should be rounded to two significant digits. Hence

Or $(2400)(3.45)(16.21) = 30,000$ Answers.

(f) Calculating

$$\begin{array}{r} 32.09 \\ + \quad 1.2 \\ \hline 33.29 \end{array}$$

Although the answer should have one decimal place but we will keep both and proceed

$$\begin{aligned} 3 \times 29 \\ = 87 \\ 16255 \end{aligned}$$

Again the answer should have one decimal place but we will keep all digits

$$\begin{aligned} 16255 \\ \div 8 = 2031.875 \end{aligned}$$

If we go according to the rules of significant figures for division, the result should have the same number of significant figures as the quantity with the smallest number of significant figures. In this case the number 6.2 has two significant digits and number 268 has three significant figures. Therefore the result of our calculation should be rounded to two significant digits. Hence

$$\text{or } \frac{16255}{8} = 2031.875 \approx 0.831 \text{ Answers}$$

When a solid sphere moves through a liquid the liquid opposes the motion with a force F . The magnitude of F depends on the coefficient of viscosity η (having dimensions $[M^{-1} L^{-1} T]$) of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.

Given

Force F has dimension $[M L T^{-2}]$

Dimension of radius r is $[L]$

Dimension of velocity v is $[L T^{-1}]$

Dimension of viscosity η is $[M^{-1} L^{-1} T]$

Required
Solution

Let the formula be $F = k \eta^a r^b v^c$

Let the dimensions of force F be equal to the dimensions of $k \eta^a r^b v^c$ by the following equation

$$[M L T^{-2}] = [M^{-1} L^{-1} T]^a [L]^b [L T^{-1}]^c$$

Where k is a constant of proportionality and a, b, c are unknown constants.

$$M^1 L^1 T^{-2} = M^{-a} L^{-a+b+c} T^{-a-c}$$

$$M^1 L^1 T^{-2} = M^{-a} L^{-a+b+c} T^{-a-c}$$

Comparing the powers of M, L and T on both sides

For M	For T	For L
$1 = -a$	$-2 = -a - c$	$1 = -a + b + c$
$a = -1$	$a + c = 2$	$a + b + c = 1$
	$-1 + c = 2$	$-1 + b + c = 1$
	$c = 3$	$b = -1$
	$c = 3$	$b = -1$
	$c = 3$	$b = -1$
	$c = 3$	$b = -1$
	$c = 3$	$b = -1$

Putting values in the above equation we get

$$F = k \eta^{-1} r^{-1} v^3$$

Dimensional analysis does not give information about the value of constant, however from experiment we know that $k = 6\pi$

$$\text{or } F = 6\pi \eta r v$$

Answer



Text Book Exercises

Q.1 Select the correct answer of the following questions

- What is the radian measure between the arms of watch at 5:00 p.m?
A 1 radian B 2 radian C 3 radian D 4 radian
- $1^\circ =$ _____
A 0.1745 radian B 1 radian C 1.745 radian D π radian
- The metric prefix for 0.000001 is
A hecto B micro C nano D nano
- Which of the following is the CORRECT way of writing units?
A 71 Newton B 12 gms C 8 kg D 1.45×10^3
- A student measures a distance several times. The readings lie between 49.8 cm and 50.2 cm. This measurement is best recorded as,
A (49.8 ± 0.2) cm B (49.8 ± 0.4) cm C (50.0 ± 0.2) cm D (50.0 ± 0.4) cm
- The percent uncertainty in the measurement of (3.76 ± 0.25) m is:
A 4% B 6.6% C 25% D 33%
- The temperatures of two bodies measured by a thermometer are $t_1 = (20 \pm 0.5)^\circ\text{C}$ and $t_2 = (50 \pm 0.5)^\circ\text{C}$. The temperature difference and the error therein is,
A $(30 \pm 0.5)^\circ\text{C}$ B $(30 \pm 1.5)^\circ\text{C}$ C $(30 \pm 1)^\circ\text{C}$ D $(30 \pm 1.5)^\circ\text{C}$
- $(5.0\text{m} \pm 4.0\%) \times (3.0\text{s} \pm 3.3\%) =$
A $15.0\text{ms} \pm 13.2\%$ B $15.0\text{ms} \pm 7.3\%$ C $15.0\text{ms} \pm 0.7\%$ D $15.0\text{ms} \pm 15.3\%$
- $(2.0\text{m} \pm 1.0\%) =$
A $8.0\text{m} \pm 1.0\%$ B 8.0m C $8.0\text{m} \pm 1.0\%$ D $8.0\text{m} \pm 6.0\%$
- The number of significant figures in measurement of 0.003500cm are
A 5 B 4 C 3 D 9
- How many significant figures does $1.362 - 25.2$ have?
A 2 B 3 C 4 D 8
- Compute the result to correct number of significant figures $4.56 \times 10^3 + 2 \times 10^3$
A 29 m B 28.5 m C 28.5 m D 28.8 m
- If 7.635 and 4.81 are two significant numbers. The result of their addition in significant digits is
A 36.72435 B 46.445 C 46.44 D 36.7
- The precision of the measurement 355.000 km is
A 10 km B 100 km C 1.00 km D 1.000000 km
- $[\text{ML}^2\text{T}^{-2}]$ are dimension of
A strain B refractive index C capacitance D all of these
- $[\text{M}^2\text{L}^2\text{T}^{-2}]$ are dimension of
A strain B refractive index C capacitance D all of these
- The dimensions of torque are
A ML^2T^{-2} B ML^2T^{-1} C ML^2T^{-1} D ML^2T^{-2}

No.	Option	ANSWER	EXPLANATION
1	C	3 rad	<p>Angle covered in 5 hours</p> <p>If a wheel is divided into 6 parts, angle covered when hour's hand covers one hour is</p> $\theta = \frac{360^\circ}{6} = 60^\circ$ <p>We know angle covered in 5 hours is $60^\circ \times 5 = 300^\circ$</p> <p>$\therefore \theta = \frac{300^\circ}{100} \times \frac{\pi}{180} = \frac{5\pi}{3} \text{ rad}$</p>

			$\Rightarrow \theta = 150 \times \frac{\pi}{180} \text{ rad}$ $\Rightarrow \theta = 2.6 \text{ rad} \approx 2.7 \text{ rad}$
2	A	11.7 rad	$360^\circ = 2\pi \text{ rad} \Rightarrow 1^\circ = \frac{2\pi}{360} \text{ rad} = \frac{2 \times 3.14}{360} \text{ rad}$ $= 0.01745 \text{ rad}$
3		10 m	10×10^6
4		1 km	
5		100 m	If we convert it to km then it will be 0.1 km
6		10 m	$\% \Delta = \frac{0.5}{3.76} \times 100\% = 13.3\%$
7		10 m	$t_2 - t_1 = (50 - 20) \pm (0.5 + 0.5) = 30 \pm 1^\circ \text{C}$
8		10 m	$10 \text{ m} + 1 \text{ m} + 1 \text{ m} = 12 \text{ m}$
9		10 m	$10 \text{ m} + 1 \text{ m} + 1 \text{ m} = 12 \text{ m}$
10		10 m	If we convert it to km then it will be 0.1 km
11	13	3	$6.562 = 26.6$ As answer should be up to least decimal places in the given data, 25.2 has one decimal place so answer will be 26.6 after rounding off which is correct.
12	3	28.8 m	$10 \text{ m} + 1 \text{ m} + 1 \text{ m} = 12 \text{ m}$ As answer should be up to least decimal places in the given data, 25.2 has one decimal place so answer will be 26.6 after rounding off which is correct.
13	1	36.7	If we convert it to km then it will be 0.1 km
14		100 km	Since least count of measurement is 100 km as predicted by measurement.
15		10 m	If we convert it to km then it will be 0.1 km
16		10 m	If we convert it to km then it will be 0.1 km



Short Answers of the Exercise

- Q 2 Write short answers of the following questions
- Q 1 For an answer to be complete the units need to be specified. Why?
- Ans:** Explanation
It is meaningless for the number without a unit and unit does not mean anything without a number.
For example:
Let us consider drawing a line using scale.
The measurement given is 5 [meaningless]
When the given is a number, it will lead to a confusion whether to draw a line of 5 cm or m.
Hence the above mentioned statement is meaningless.
Since the measurement is 5 cm. This is making complete sense.



Q.2 What are the advantages of using International System of Units (SI)?

Ans: **Advantage System of Units (SI)**

- It consists of only seven base quantities, most of the derived quantities are obtained by multiplying and dividing the base units.
- Each quantity has only one standard unit, so we don't need to specify a quantity with different units.
- There are no numerical definitions or constants for students to memorize. For example, the quantity power is defined as energy per unit time ($\text{Power} = \frac{\text{work}}{\text{time}}$). Therefore the SI unit of power (watt), is defined as the unit of energy per the unit of time i.e. joule/second.
- SI units are coherently derived as the simple algebraic division or multiplication of a few base units, using the same equation as the quantity being measured.
- SI uses decimals exclusively, eliminating difficult fractions and mixed numbers.

Q.3 How many radians account for circumference of a circle? How many steradians account for circumference of a sphere?

Ans: **Radians in One Circle**

Mathematically,

$$\text{Number of radians } (\theta) = \frac{\text{Arc Length}}{\text{Radius of same circle}} = \frac{s}{r}$$

$$\text{So, the number of radians in one revolution} = \frac{\text{circumference of circle}}{\text{Radius of same circle}}$$

$$\text{Number of radians in one revolution} = \frac{2\pi r}{r} = 2\pi \text{ radians}$$

Steradians in One Sphere

Mathematically

$$\text{Number of steradians in sphere} = \frac{\text{Area of Sphere}}{r^2}$$

Since

$$\text{Surface area of closed sphere of radius } r \text{ is } 4\pi r^2$$

Therefore

$$\text{Number of steradians in sphere} = \frac{4\pi r^2}{r^2} = 4\pi \text{ sr}$$

Sphere subtends $4\pi \text{ sr}$ (12.56 sr)

Q.4 What is least count error? How can least count error be reduced?

Ans: **Least Count Error:**

The smallest value that can be measured by the measuring instrument is called its least count.

- All the readings or measured values are good only up to this value.
- For example, a vernier calipers has the least count as 0.01 cm, a spherometer may have a least count of 0.001 mm.
- Least count error belongs to the category of random errors but within a limited size, it occurs with both systematic and random errors.

Reduction

We can reduce the least count error by using instruments with higher resolution, improving experimental techniques etc.

Q.5 Why including more digits in answers, does not make it more accurate?

Ans: Including more digits in answers, does not make it more accurate but it can make answer more precise.

Explanation: Accurate answer is the one which is closer to actual or acceptable value. For example, actual length of a wire is 10.6 cm. If student A measures its value as 10.5 cm and student B measures it as 10.15 cm. We see that Student A measurement is closer to actual value, so his measurement is more accurate than measurement of student B, although student B reading has more digits.

Q 6 What determines the precision of a measurement?

Ans: Precision means how close the measured values are to each other. Smaller the least count of the instrument, measurement will be more precise and vice versa.

Example: Consider length of a wire is measured with Vernier Calipers.

$$L = 10.5 \pm 0.1 \text{ mm}$$

This means that its true length lies between 10.4 mm to 10.6 mm.

Then the length of the same wire is measured by screw gauge which is

$$L = 10.50 \pm 0.01 \text{ mm}$$

This means that its length lies between 10.49 mm to 10.51 mm.

We see that measurement taken by the screw gauge is more precise than that of vernier calipers, because they are closer to each other.

Q 7 If two quantities have different dimensions, is it possible to multiply and/or divide. Can we add and/or subtract them?

Ans: YES, we can multiply and divide quantities with different dimensions and addition and subtraction they will give us dimensions of any derived quantity.

For Example: As $\text{force} = \text{mass} \times \text{acceleration}$

$$\text{Dimension of force} = \text{mass} \times \text{dimension of acceleration}$$

$$F = M \cdot L T^{-2}$$

$$[F] = [MLT^{-2}]$$

Remember on a scale, quantities can be added or subtracted only if they have the same dimensions.

Q 8 The human pulse and the swing of a pendulum are possible time units. Why are they not often used?

Ans: A quantity can be taken as time standard if it is some fixed and invariable quantity.

Human Pulse:

It can not taken as time standard because pulse rate varies as a person's condition changes.

Swinging Pendulum:

As the time period of the simple pendulum can be expressed as,

$$T = 2\pi \sqrt{\frac{l}{g}}$$

But the time period can not serve as reasonable time standard due to the following reasons.

Reason:

- (i) Length of pendulum may change due to change in temperature. & the change in length will change the time period.
- (ii) Time period of the simple pendulum varies with g and g varies with latitude.
- (iii) Frictional force of air and support may affect the time period.

Q 9 If an equation is dimensionally correct, is that equation a right equation?

Ans: If an equation is dimensionally correct, then that equation may or may not be a right equation.

For Example;

Consider the equation of Newton's 2nd law $F = ma$

We know that this equation is correct equation in any sense.

But if we write $F = 2\pi ma$ where 2π is dimensional constant, so by applying principle of dimensional consistency, dimensions on L.H.S. and R.H.S. will be same therefore, it will be dimensionally correct equation.



Comprehensive Questions

Q3 Give a short response to the following questions

1. Define Physics? Explain the scope and Importance of physics in science, technology and society?

Ans See Ex 1

2. What is system of units? In SI what is meant by base, derived and supplementary units?

Ans See Ex 2

3. What conventions are used in SI to indicate units?

Ans See Ex 3

4. What are errors? Differentiate between systematic and random errors?

Ans See Ex 4

5. What is uncertainty in measurement? explain the propagation of uncertainty in addition, subtraction, multiplication and division?

Ans See Ex 5

6. What are significant figures? What are the rules for determining significant figures in the final result after addition, subtraction, multiplication and division?

Ans See Ex 6

7. Differentiate between precision and accuracy in the measurement.

Ans See Ex 7

8. What is meant by dimensions of physical quantities? What are the uses and applications of dimensions and units?

Ans See Ex 8



1. A circular pizza is cut into 3 equal parts. One piece of pizza is taken out. Estimate the degree measure of the angle subtended by one piece of pizza and convert the measure into radians. What is the radian measure of the angle of the remaining part of pizza?

Solution

Given Data

Number of pieces of a circular pizza = 3

To Find, Angle subtended by one piece = ?

Angle subtended by remaining part pizza = ?

Solution: Angle subtended by circular pizza = $\theta = 2\pi$ radian

Angle subtended by one of the three pieces is

$$\theta = \frac{2\pi}{3} \text{ radian}$$

$$\theta = \frac{(2 \times 3.14)}{3} \text{ radian} = 2.09 \text{ rad}$$

Angle subtended by the remaining two pieces of pizza is

$$\theta = 2 \times 2.09 \text{ rad} = 4.18 \text{ rad}$$



2. The length of a pendulum is (1.5 ± 0.01) m and the acceleration due to gravity is taken into account as $(9.8 \pm 0.1) \text{ m s}^{-2}$. Calculate the time period of the pendulum with uncertainty in it.

Given Data: Length $= L = 1.5 \pm 0.01 \text{ cm}$
 Acceleration $= g = 9.8 \pm 0.1 \text{ ms}^{-2}$

To Find:

%age uncertainty in $T = \% \Delta T = ?$

Solution:

Calculation of Time Period:

\therefore Formula of time period is: $T = 2\pi \sqrt{\frac{L}{g}}$
 $\Rightarrow T = 2 \times 3.14 \sqrt{\frac{1.5}{9.8}} = 2.5 \text{ s}$

%age uncertainty in $L = \frac{\Delta L}{L} \times 100\%$
 $= \frac{0.01}{1.5} \times 100\% = 0.67\%$

%age uncertainty in $g = \frac{\Delta g}{g} \times 100\%$
 $= \frac{0.1}{9.8} \times 100\% = 1.020\%$

\therefore Formula of time period is: $T = 2\pi \sqrt{\frac{L}{g}}$
 $T = 2\pi \left(\frac{L}{g}\right)^{\frac{1}{2}}$
 %age uncertainty in $T = \frac{1}{2} \times (\% \text{age uncertainty in } L + \% \text{age uncertainty in } g)$
 $= \frac{1}{2} \times (0.67\% + 1.020\%)$
 $= \frac{1}{2} \times (1.69\%) = 0.84\% = 0.8\%$

$\Rightarrow \% \Delta T = \pm 0.8\%$

$\Rightarrow T = 2.5 \pm 0.8\%$

3. Determine the area of a rectangular sheet with length $(l \pm \Delta l) = (1.50 \pm 0.02) \text{ m}$ and width $(w \pm \Delta w) = (0.20 \pm 0.01) \text{ m}$. Calculate the area $(A \pm \Delta A)$.

Given Data: $l \pm \Delta l = 1.50 \pm 0.02 \text{ cm}$
 $w \pm \Delta w = 0.20 \pm 0.01 \text{ cm}$

To Prove: $A \pm \Delta A = ?$

Solution: Calculation of Area
 $A = L \times W$
 $A = 1.50 \times 0.20$
 $A = 0.30 \text{ cm}^2$

%age Uncertainty in $L = \frac{\Delta L}{L} \times 100\%$
 $= \frac{0.02}{1.50} \times 100\% = 1.33\%$

%age uncertainty in $W = \frac{\Delta W}{W} \times 100\%$
 $= \frac{0.01}{0.20} \times 100\% = 5\%$

\Rightarrow Total %age uncertainty in $A = \% \text{age uncertainty in } L + \% \text{age uncertainty in } W$
 $= 1.33\% + 5\%$

$$= 6.33\%$$

Absolute uncertainty in Area (ΔA) is calculated as

$$\Delta A = \pm \frac{6.33}{100} \times 0.30 = 0.018 \text{ cm}^2 = 0.02 \text{ cm}^2$$

$$\Rightarrow \text{Area of rectangular sheet} = A \pm \Delta A = 0.30 \pm 0.02 \text{ cm}^2$$

4. Calculate the answer up to appropriate numbers of significant digits

(a) $246.24 + 238.278 + 98.3$

(b) $1.4 = 2.638 + 117.25$

(c) $(2.66 \times 10^4) - (1.03 \times 10^3)$

(d) $(112 \times 0.456) / (3.2 \times 120)$

(e) 168.99×9

(f) $1023 \div 8.5489$

Solution:

(a) $246.24 + 238.278 + 98.3$

$$= 582.818$$

$$= 582.8$$

(As data has least 1 decimal place,

(b) $1.4 + 2.639 + 117.25$

$$= 3.6946 + 117.25$$

$$= 120.9446$$

$$= 120.9$$

(As data has least 1 decimal place)

(c) $(2.66 \times 10^4) - (1.03 \times 10^3)$

$$= 26600 - 1030$$

$$= 25570$$

$$= 2.5570 \times 10^4$$

$$= 2.56 \times 10^4 \quad (\text{As data has at least 3 significant figures})$$

(d) $(112 \times 0.456) / (3.2 \times 120)$

$$\frac{51.072}{384} = 0.133 = 0.13 \quad (\text{As data has at least 2 significant figures})$$

(e) 168.99×9

$$= 1520.91$$

$$= 2000$$

(As data has least one significant digit so 1520.9, is rounded off to 2000 which contains one significant digit)

(f) $1023 \div 8.5489$

$$= 103.5489$$

$$= 1032$$

(As in given data 1023 has no decimal place, so answer w. ll also don't have any decimal place)

5. Calculate the answer up to appropriate numbers of significant digits

(a) The ratio of mass of proton ' m_p ' to the mass of electron ' m_e '

$$\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27} \text{ kg}}{9.1096 \times 10^{-31} \text{ kg}}$$

(b) The ratio of charge on electron ' q_e ' to mass of electron ' m_e '

$$\frac{q_e}{m_e} = \frac{1.67 \times 10^{-19} \text{ C}}{9.1096 \times 10^{-31} \text{ kg}}$$

Solution

(a) The ratio of mass of proton ' m_p ' to the mass of electron ' m_e '

$$\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27} \text{ kg}}{9.1096 \times 10^{-31} \text{ kg}}$$

$$= 0.18332 \times 10^4$$

$$= 1.8332 \times 10^3$$

$$= 1.83 \times 10^3$$

(Answer should be up to 3 significant figures as these are least significant digits in the given data.)

(b) The ratio of charge on electron ' q_e ' to mass of electron ' m_e '

$$\begin{aligned} \frac{q_e}{m_e} &= \frac{1.6 \times 10^{-19}}{9.1096 \times 10^{-31}} \\ &= (1.75838864 \times 10^{12}) \text{ C/kg} \\ &= (1.76 \times 10^{12}) \text{ C/kg} \\ &= 1.8 \times 10^{12} \text{ C/kg} \end{aligned}$$

(Answer should be up to 2 significant figures as these are least significant digits in the given data.)

6. Find the dimensions of

(a) Planck's constant ' h ' from formula $E = hf$

Where E is the energy and f is frequency.

(b) gravitational constant ' G ' from the formula $F = G \frac{m_1 m_2}{r^2}$

Where ' F ' is force, ' m_1 ' and ' m_2 ' are masses of objects and ' r ' is the distance between centers of objects

Solution. (a) Dimension of Planck's constant

From equation $E = hf$

$$[h] = \frac{[E]}{[f]} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T]$$

(b) Formula for gravitational force $F = G \frac{m_1 m_2}{r^2}$

$$\text{Or } G = \frac{F}{m_1 m_2 / r^2}$$

$$\begin{aligned} \text{Dimension of } G &= \frac{[ML^2T^{-2}][L]^2}{[M][M]} \\ &= \frac{ML^4T^{-2}}{M^2} \\ &= [M^{-1}L^4T^{-2}] \end{aligned}$$

7. Show that (a) $K.E. = \frac{1}{2}mv^2$ and (b) $P.E. = mgh$ are dimensionally correct

Solution. (a) $K.E. = \frac{1}{2}mv^2$

$$\begin{aligned} [LHS] &= [K.E.] = [W] = [F]d \\ &= [MLT^{-2}][L] = [ML^2T^{-2}] = [ML^2T^{-2}] \\ [RHS] &= \frac{1}{2}[m][v^2] = [M][LT^{-1}]^2 = [ML^2T^{-2}] \end{aligned}$$

Since dimensions of $[LHS] = [RHS]$, so given equation is dimensionally correct

(b) $P.E. = mgh$

$$\begin{aligned} [LHS] &= [P.E.] = [W] = [F]d \\ &= [MLT^{-2}][L] = [ML^2T^{-2}] = [ML^2T^{-2}] \\ [RHS] &= [m][g][h] = [M][LT^{-2}][L] = [ML^2T^{-2}] \end{aligned}$$

Since dimensions of $[LHS] = [RHS]$, so given equation is dimensionally correct



Additional Conceptual Short Questions With Answers

1. What are the uses of dimensions?

Ans: The three main uses are

- Checking the correctness of the possible formula (equation)
- Deriving a relationship between different physical quantities in a physical problem.
- Converting one system of units into another

2. What are the limitations of dimensional analysis?

Ans: The limitations are

- The numerical values of constant of proportionality in an equation cannot be determined with the help of dimensions. It can be determined experimentally or theoretically.
- Physical relations involving logarithmic, exponential or trigonometric functions (all dimensionless), cannot be derived and verified.
- It does not tell whether the quantity is scalar or vector.

3. Name the physical quantities which have the same dimensions but different units?

Ans: The following quantities have the same dimensions but different units

- Work and torque have same dimensions $[ML^2T^{-1}]$
- Velocity & work is $[ML^2T^{-1}]$ but torque is $[Nm]$

4. Prove that followings equations are homogeneous w.r.t. dimensions

$$a_c = \frac{v^2}{r}$$

Ans: For centripetal acceleration

$$a_c = \frac{v^2}{r}$$

$$[LHS] = a_c = [LT^{-2}]$$

$$[RHS] = \left[\frac{v^2}{r} = \frac{[L^2T^{-2}]}{[L]} = [LT^{-2}] \right]$$

Since dimension of $[LHS] = [RHS]$, so given equation is dimensionally correct.

Unit of $a_c = m/s^2$

$$S: [a_c] = L/T^2$$

$$U: [a_c] = m/s^2$$

$$SI: [v] = [LT^{-1}]$$

5. What do you understand by "Assignable errors" and "Unassignable errors"?

Ans: Assignable Errors: The errors to which we can assign a cause are called assignable errors. For example zero error is an example of assignable error. The error arising in measurement due to poor calibration of the instrument is also an assignable error.

Unassignable Error: The errors in measurement to which we cannot assign a cause. These are due to unknown reasons, hence are called random errors.

6. Show that equations of motions

$$V_f^2 = 2as + V_i^2$$

are dimensionally consistent?

Ans: The given equation is

$$V_f^2 = 2as + V_i^2$$

$$[LHS] = [V_f^2] = [L^2T^{-2}] = [L^2T^{-2}]$$

$$[RHS] = 2a[s] + [V_i^2]$$

$$= 2[L^2T^{-2}] + [L^2T^{-2}]$$

$$= 2[L^2T^{-2}] + [L^2T^{-2}]$$

$$= 4[L^2T^{-2}]$$

$$= [L^2T^{-2}]$$

Since dimension of $[LHS] = [RHS]$, so given equations are dimensionally correct.



MCQ's From Past CBSE Papers (FEDERAL BOARD)

1. Which of the following is not a unit of pressure?
 A kg m^{-2} B $\text{kg m}^{-1} \text{s}^{-2}$ C N m^{-2} D pascal
2. In colour printing entire range of colours can be obtained by mixing _____ colours.
 A Seven B Six C Five D Four
3. Which of the following pair have same dimensions?
 A Power, Speed B Force, Momentum C Work, Torque D Velocity, Acceleration
4. Numbers of significant figures in 8.70×10^4 are _____
 A 1 B 2 C 4 D 5
5. SI unit of time is _____
 A second B Cesium second C Krypton second D day
6. Light year is unit of _____
 A Distance B time C length D speed
7. Solid angle of sphere is _____
 A 12.57 sr B 6.28 sr C $4 \pi \text{ sr}$ D $2 \pi \text{ sr}$
8. Base unit of linear momentum are _____
 A N s^2 B kg m/s C kg m D N s
9. Which of the following is dimensionless?
 A Stress B Pressure C Surface tension D Strain
10. Which of the following is not unit of work?
 A Joule B Nm C $\text{kg m}^2 \text{s}^{-2}$ D W s
11. The dimensions of Power are _____
 A $[\text{M L T}^2]$ B $[\text{M L T}^3]$ C $[\text{M L}^2 \text{T}^{-3}]$ D $[\text{M L}^2 \text{T}^{-2}]$
12. The significant figures in 34.878 are _____
 A 4 B 3 C 5 D 6
13. Which of the following pairs of units are both derived units?
 A Kilogram, Angstrom B Ampere, Degree C Newton, Joule D Joule, Watt
14. The Prefix one peta is _____
 A 10^3 B 10^6 C 1 D 10^9
15. One year is equal to _____
 A $3.15 \times 10^7 \text{ s}$ B $3.15 \times 10^8 \text{ s}$ C $3.15 \times 10^6 \text{ s}$ D $3.15 \times 10^5 \text{ s}$
16. Which of the following may be used as valid formula to calculate speed of ocean waves? (v = speed, g = acceleration due to gravity, λ = wavelength, ρ = density, h = depth)
 A $gh\lambda$ B $\sqrt{\lambda g}$ C $\frac{g}{\rho h}$ D ρgh
17. In a cricket match 500 spectators are counted one by one. How many significant figures will be there in the final result?
 A 1 B 2 C 3 D 4

Answers Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
D	A	C	D	B	C	C	B	C	D	C	B	C	D	A	B	C



SELF-ASSESSMENT PAPER

Question No 1

Choose the correct answer from the given options

Total Mark 40

(1 x 5 = 5)

SECTION - A

1. Solid angle subtended at the centre of sphere of radius r in steradian is
(A) 2π (B) 4π (C) 8π (D) 16π
2. The SI unit of power in terms of base unit are
(A) kg m s^{-2} (B) $\text{kg m}^2 \text{s}^{-2}$ (C) kg m s^{-1} (D) $\text{kg m}^2 \text{s}^{-1}$
3. How many nanometers are in a meter?
(A) 10^9 (B) 10^6 (C) 10^3 (D) 10^2
4. The percentage uncertainty in mass and velocity of an object are 2% and 3% respectively. Which of the following is the maximum uncertainty in the measurement of its kinetic energy?
(A) 1.5% (B) 6% (C) 8% (D) 1%
5. Significant figures in 0.0010 are _____
(A) 1 (B) 2 (C) 3 (D) 4
6. Which of the following pair have same dimensions?
(A) Momentum and impulse (B) Work and torque
(C) Force and torque (D) Torque and angular momentum

Question No 2

Give short answers of followings

3 x 7 = 21

SECTION - B

- (i) How many radians account for circumference of a circle? How many steradians account for circumference of a sphere?
- (ii) What determines the precision of a measurement?
- (iii) The human pulse and the swing of a pendulum are periodic time units. Why are they not standard?
- (iv) Show that $a \propto \frac{1}{r^2}$ and $PE \propto r$ are dimensionally correct.
- (v) Two connected gears are rotating. The smaller gear has a radius of 4 cm and the larger gear's radius is 1 m . What is the angle through which the larger gear has rotated when the smaller gear has made one complete rotation?
- (vi) Differentiate between the terms 'precision' and 'accuracy'.
- (vii) Write down any three significant conventions for writing the SI units and their uses.

Question No 3

Extensive Questions

13

SECTION - C

- (a) Find an expression for the time period T of a simple pendulum. The time period T may depend on mass m of the bob of the pendulum, its length l of pendulum, its acceleration due to gravity g at the place where the pendulum is suspended. (05)
- (b) The length of a pendulum is $(1.5 \pm 0.1)\text{ m}$ and the acceleration due to gravity is taken into account as $(9.8 \pm 0.1)\text{ m/s}^2$. Calculate the time period of the pendulum with uncertainty in it. (05)
- (c) Write down any three advantages and three limitations of dimensional analysis. (04)

/// The End ///



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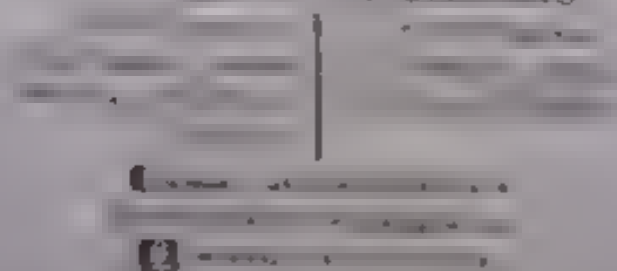
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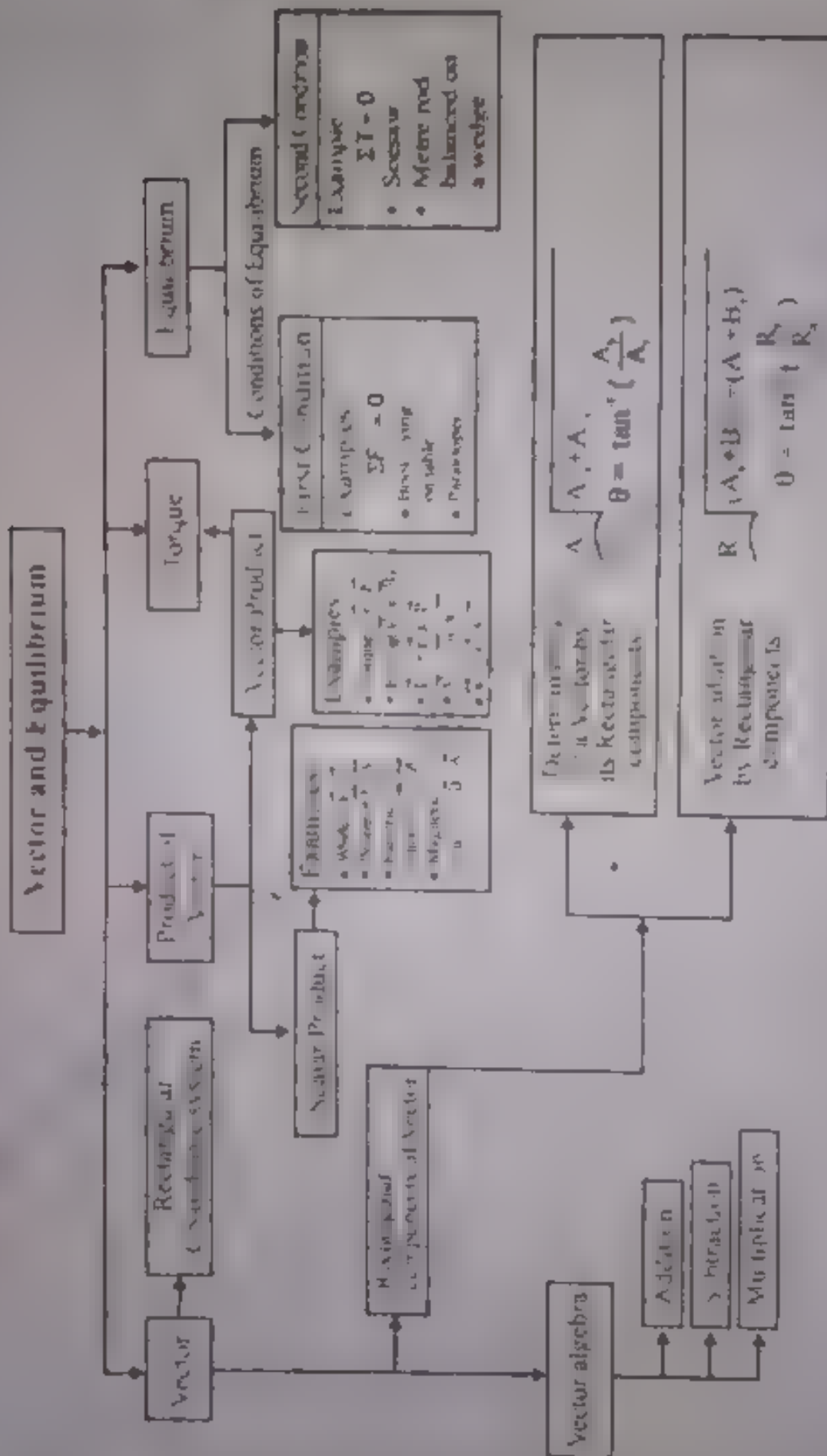
2

VECTORS & EQUILIBRIUM

Learning Objectives

- Define the addition of vectors.
- Define the subtraction of vectors.
- Explain the triangle rule for addition of vectors.
- Explain the parallelogram rule for addition of vectors.
- Use the triangle rule to find the resultant of two vectors.
- Use the parallelogram rule to find the resultant of two vectors.
- State the triangle rule for addition of vectors.
- Define the magnitude of a vector.
- Explain the direction of a vector.
- State the triangle rule for addition of vectors.
- State the parallelogram rule for addition of vectors.
- Give two examples of vector quantities.

CONCEPT MAP



Q 1 Define the following terms Magnitude, Scalars, Vectors

Ans **Magnitude** It is the numerical value of a physical quantity with suitable unit.
Scalars,

A physical quantity which can be completely described by **magnitude only** is called scalar quantity.

Examples Time, distance, mass, temperature, speed, energy, work, volume, area, electric charge etc.

Note that scalars can be added, subtracted, multiplied etc. like numbers using ordinary algebra.

Vectors,

A physical quantity which can be completely described by both **magnitude and direction** is called vector quantity.

Note that the vector quantities also obey the laws of vector addition i.e. they follow vector addition.

Examples, Force, velocity, displacement, torque, momentum, acceleration, weight, angular velocity, electric intensity etc.

Q 2 How is a vector represented?

Ans **Vector Representation**

A vector is represented in two ways

(i) Symbolic representation (ii) Graphical representation

Symbolic Representation

It is represented by **bold face letter** such as \mathbf{A} , \mathbf{d} , \mathbf{r} and a vector is also represented by a letter with an arrow placed above or below the letter such as \vec{A} or \vec{A} .

Graphical Representation

It is represented by a **straight line** with an **arrow head** at one end. The length of line represents magnitude of vector according to the scale chosen. Arrow head represents the direction of vector.

The starting point is called origin or tail point, and the end point is called head point.

Note Representation of magnitude of vector

The magnitude of vector is represented by **light face italic letter** such as A , d , r and V etc. In physics taking modulus of a vector is written as $|\mathbf{A}|$, $|\mathbf{V}|$ etc.



Fig. 2.1

Q 3 What are geometric vectors, Co-ordinate axes and algebraic vectors?

Ans I. **Geometric Vectors,** Geometric vectors are those vectors that are considered with reference to coordinate axes.

II. **Co-ordinate axes:** Any set of values that indicate the position of a point in a given reference system.

OR

Any system which helps us to locate position of a point w.r.t. to some reference point.

III. **Algebraic Vectors,** These are vectors drawn in a co-ordinate system (plane or space).

Q 4 What is Cartesian coordinate system or rectangular coordinate system?

Ans: Cartesian Co-ordinate System (Rectangular Coordinate System)

The set of two or three mutually perpendicular lines intersecting at a point is called cartesian or rectangular coordinate system.

- The lines are called *coordinate axes*
- One of these lines is called *x-axis* for *horizontal axis*,
- The other is called *y-axis* or *vertical axis*.
- The line perpendicular to both x and y -axis is called *z - axis*
- The point of intersection is called *origin*

Two dimensional coordinate system (Plane)

If the system consists of two perpendicular lines then it is called two dimensional coordinate system

The point where these lines intersect is called origin

- Each point in plane has two co-ordinates P (a b) Starting from the origin The point P is 'a' unit along x-axis and 'b' unit along y-axis

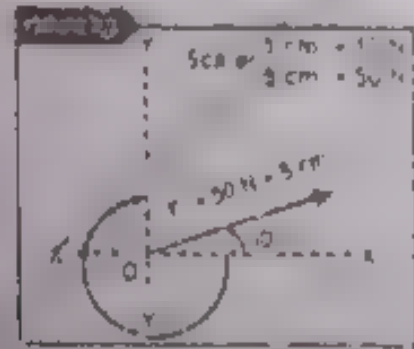
Direction of a vector in plane

It is represented by the angle which the vector makes with *positive x-axis* in *anti clock wise* direction

Example of A Vector in plane

Consider a force vector of 50 N making an angle of 30° with horizontal. is drawn as shown

The scale is 1 cm = 10 N, so its length 5 cm representing 50 N with 30° angle from x axis



Example of A Vector in space
Consider a force vector of 50 N making an angle of 30° with horizontal. is drawn as shown

Three dimensional co-ordinate systems (Space)

If the system consists of three mutually perpendicular lines, then it is called three dimensional co-ordinate system.

Each point in space has three co-ordinates P (a b, c)

Direction of a vector in space

It is represented by *three* angles which the vector makes with x, y and z -axes



- Angle with X - axis is α
- Angle with Y - axis is β
- Angle with Z - axis is γ

Q 5 What are steps to represent a vector in cartesian coord nate system?

Ans: **STEPS TO REPRESENT A VECTOR IN CARTESIAN COORDINATE SYSTEM**

- The following method is used to represent a vector
1. Draw a cartesian coordinate system
 2. Select a suitable scale
 3. Draw a line in the specified direction
 4. Cut the line equal to the magnitude of the vector according to the selected scale
 5. Put an arrow in the direction of the vector

Q 6 How will you add vectors by graphical method?

Ans: **ADDITION OF VECTORS**

Vectors can be added by two different approaches namely graphical or geometrical method and analytical method

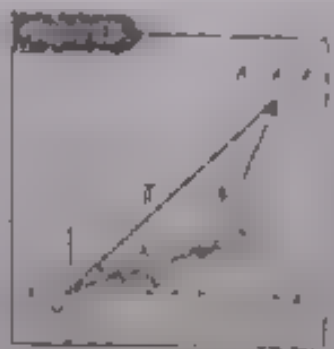
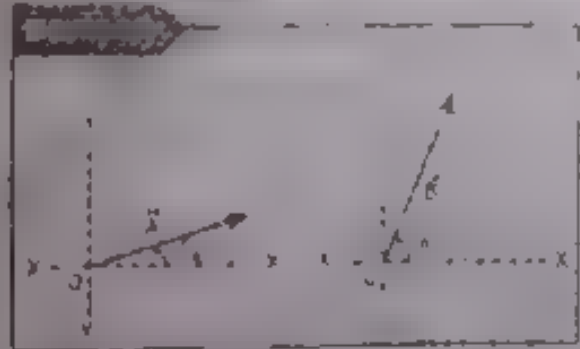
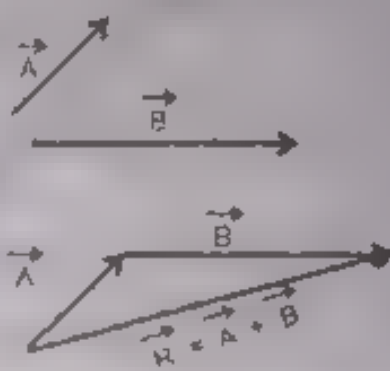
Graphical or Geometrical Method for Addition of Vectors

In this method vectors are added by sketching vector diagrams. For the graphical approach of addition of vectors we use Head-to-Tail Rule.

HEAD-TO-TAIL-RULE:

- Following steps must be followed to add vectors by head-to-tail rule
- i. Select a suitable scale for the representation of given vectors
 - ii. Sketch all the vectors according to the selected scale in a given direction
 - iii. Now join the tail of the second vector to the head of the first vector and draw its representative line according to the selected scale. The resultant vector is obtained, which is the vector to be added
 - iv. To find resultant vector, draw a line from the tail of the first vector to the head of the last vector and put an arrow on this line pointing towards the head of the last vector. This is the resultant vector.
 - v. To determine the magnitude of the resultant vector, measure the length of the resultant vector according to the selected scale
 - vi. To determine the direction of the resultant vector, draw a line from the tail of the first vector to the head of the last vector and put an arrow on this line pointing towards the head of the last vector. This is the resultant vector.

(Resultant Vector) is the vector which is the sum of the two vectors. It is the vector which is the sum of the two vectors.



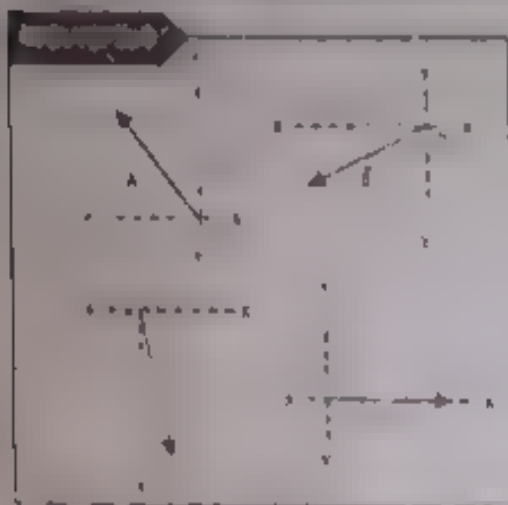
- Example:**
- Consider two vectors A and B shown in figure and taking scale as 1 cm = 1 unit and 1 unit = 1 m/s respectively as shown in the figure.
 - To add these vectors, we redraw them in a common origin.
 - Place them head to tail as in the steps mentioned above.
 - Such that the tail of vector B is on head of vector A.
 - Joining the tail of the first vector with the head of the last to get the resultant vector R as shown in Figure.

ADDITION OF MORE THAN TWO VECTORS

We can add any number of vectors using head to tail rule

Example

- Consider four vectors **A**, **B**, **C** and **D** in xy -plane as shown in the figure
- To add these vectors draw them to a common scale
- Place them head to tail. Such that the tail of each vector is on head of the previous vector
- Draw the resultant vector **R** from tail of first vector to head of last vector. Measure the length of vector **R** and convert it into magnitude of the quantity according to given scale
- To determine the direction measure the angle of resultant with x -axis



Q7 Does vector addition follow commutative property?

Let's Commutative Property

From figure it is clear that if either we add \vec{A} to \vec{B} or \vec{B} to \vec{A} the resultant will be the same. They will form a parallelogram.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

It means that when two vectors are added the resultant is the same, that is order of addition does not matter. Vector addition is commutative.



Example 2.4

A x -plane is moving at 120 m/s at an angle of 30° with x -axis, through a 30 m/s across wind blowing at angle of 260° with x -axis. Determine the resultant velocity of the x -plane.

Solution

Let $\vec{v}_1 = 120$ m/s making an angle of 30° with x -axis and

$\vec{v}_2 = 30$ m/s making an angle of 260° with x -axis.

Required

Resultant velocity of x -plane = \vec{v}

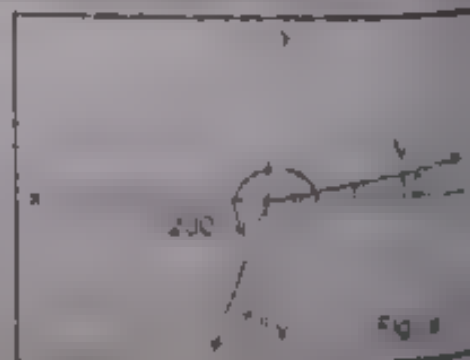
Calculation

The resultant velocity is given by

$$v = \sqrt{v_1^2 + v_2^2}$$

First we resolve \vec{v}_1 and \vec{v}_2 into its horizontal and vertical components. as it was a figure below we have,

$$v_{1x} = v_1 \cos 30^\circ = 120 \times \cos 30^\circ \text{ m/s}$$



$$\Rightarrow v_{1x} = 120 \times 0.985 \text{ m/sec}$$

$$\Rightarrow v_{1x} = 118.2 \text{ m/sec}$$

And $v_{1y} = v_1 \sin \theta = 120 \times \sin 10^\circ = 120 \times 0.174 \text{ m/sec}$

$$v_{1y} = 20.88 \text{ m/sec}$$

Similarly

$$v_{2x} = v_2 \cos \theta = 30 \times \cos 80^\circ$$

$$= 30 \times 0.174 \text{ m/sec}$$

$$\Rightarrow v_{2x} = 5.22 \text{ m/sec}$$

And $v_{2y} = v_2 \sin \theta = 30 \times \sin 80^\circ$

$$= 30 \times 0.985 \text{ m/sec}$$

$$\Rightarrow v_{2y} = 29.55 \text{ m/sec}$$

Now the x component of resultant velocity is given by

$$v_x = v_{1x} + v_{2x} = 118.2 + 5.22 = 123.42 \text{ m/sec}$$

$$\Rightarrow v_x = 123.42 \text{ m/sec}$$

Now the y component of resultant velocity is given by

$$v_y = v_{1y} + v_{2y} = 20.88 + 29.55 = 50.43 \text{ m/sec}$$

$$\Rightarrow v_y = 50.43 \text{ m/sec}$$

Putting the values of v_x and v_y in equation (1)

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(123.42)^2 + (50.43)^2}$$

$$\Rightarrow v = \sqrt{15234.5 + 2542} = \sqrt{17776.5}$$

$$\Rightarrow v = 133.37 \text{ m/sec}$$

The direction of v is given by

$$\theta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{50.43}{123.42}$$

$$\Rightarrow \theta = \tan^{-1}(0.408) = 24.2^\circ$$

Now the actual angle of v with positive x-axis is given by

$$\theta' = 360^\circ - \theta = 360^\circ - 24.2^\circ = 335.8^\circ$$

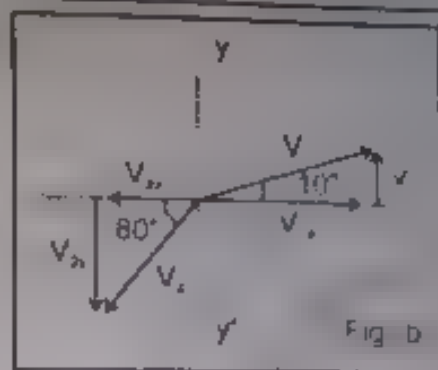


Fig. D

Point to Ponder

1. When two or more vectors are added or subtracted together, they must all have the same units and they all must be the same type of quantity.
2. In addition process, the vectors A, B which result R after addition are called components of vector R.
3. As we can add any number of vectors, so we can say that a vector may have infinite number of its components.

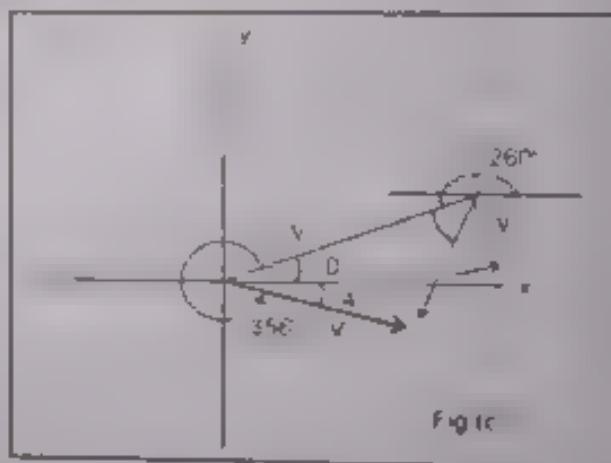


Fig. E

Q.8 How will you subtract two vectors? Explain in detail

Ans: Subtraction of Vectors

There is no direct method for determining the result of subtraction of two vectors. Even for subtraction we use the method of addition as

- i) draw the negative of the vector to be subtracted. If vector B is supposed to be subtracted from A, then first we find the negative of B which is $-B$.
- ii) Then join the tail of negative vector with the head of 1st vector according to head to tail rule i.e. we add this negative of B with A as shown.
- iii) To get their resultant draw a vector from tail of 1st vector A to head of negative vector $-B$ as shown by vector $(A - B)$.

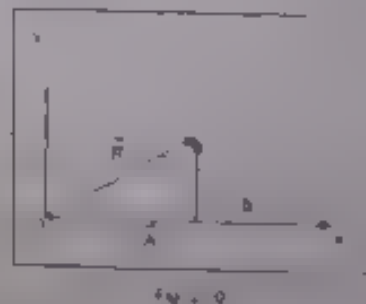
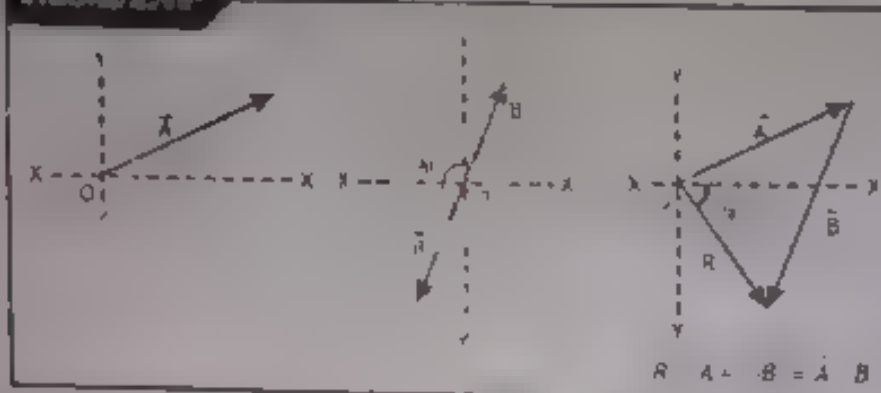


Fig. F

FIGURE 2.18



Q 9 Define and explain the following terms

(i) Resultant vector

(ii) Negative of vector

(iii) Unit vector

(iv) Null vector

(v) Equal vectors

(vi) Unequal vectors

Ans:

(i) **Resultant Vector**

'A vector having the same effect as the combined effect of all the vectors to be added is known as resultant vector.' Consider two vectors \vec{A} and \vec{B} . The sum vector $\vec{A} + \vec{B}$ is known as resultant vector and it can be expressed

$$\vec{R} = \vec{A} + \vec{B}$$



(ii) **Negative of a Vector**

A vector having the same magnitude as a given vector but opposite in direction is known as negative of vector.

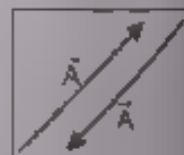
If \vec{A} is the given vector then the negative of \vec{A} is $-\vec{A}$ which has the same magnitude as \vec{A} but opposite in direction as shown in figure.

For Your Information

Resultant of two vectors will be maximum when they are along the same direction and will be minimum when they are opposite in direction.

(iii) **Unit vector**

'The dimensionless vector having magnitude one and representing the direction of a vector is called unit vector.'



A unit vector is represented by a letter with a small arrow on top and a dot on the letter.

Explanation:

A unit vector in the direction of \vec{A} is written as \hat{A} .

So
$$\hat{A} = \frac{\vec{A}}{A}$$

or
$$\hat{A} = \frac{\vec{A}}{A}$$

or
$$\hat{A} = \frac{\vec{A}}{A}$$



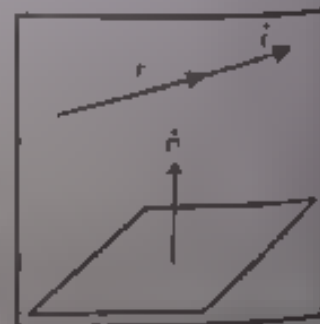
Special Unit Vectors:

(a) **Unit Orthogonal Vectors**

These are unit vectors along axes.

Generally, we take unit vectors $\hat{i}, \hat{j}, \hat{k}$ along x-axis, y-axis and z-axis respectively. The magnitude of each unit vectors \hat{i}, \hat{j} and \hat{k} is one i.e.

$$\hat{i} = \hat{j} = \hat{k} = 1$$



The three unit vectors \hat{i} , \hat{j} , \hat{k} do not change the magnitude or the dimensions of anything they indicate directions.

(b) Normal Unit Vector

The unit vector \hat{n} is used to represent the direction normal drawn on a certain surface e.g. \hat{n} is the unit vector normal to the surface of area S drawn perpendicular to surface.

(c) Unit Vector of Position Vector

is the unit vector in the direction of r .

(iv) Null Vector or Zero Vector

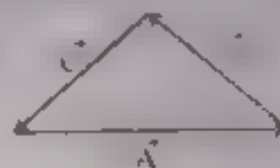
A vector having zero magnitude and arbitrary direction is called a null vector.

A null vector is denoted by $\vec{0}$.

- If a vector \vec{A} and a negative vector $-\vec{A}$ are to be added then their resultant vector can be expressed

$$\vec{A} + (-\vec{A}) = \vec{0}$$

- The resultant of number of vector is found by head to tail rule forming a closed polygon with n sides.



(v) Equal Vectors

"Two vectors are said to be equal vectors if they have the same magnitude and direction regardless of the position of their supports."

This means that two vectors having same magnitude and direction are equal to each other.

For Your Information
If \vec{A} and \vec{B} are equal magnitudes are equal to \vec{A} .

(vi) Unequal Vectors

Vectors having either different magnitude or direction or both are called unequal vectors.

Q 10 Explain the multiplication of a vector with a scalar

Ans Multiplication of a vector by a scalar

A vector can be multiplied by

1. a positive number
2. a negative number
3. zero
4. a scalar quantity with direction



1 Multiplication with positive number

When a vector \vec{A} is multiplied by a positive number n (i.e. $k > 0$). Then the product vector will have magnitude equal to kA and same direction as that of \vec{A} .

2 Multiplication with negative number

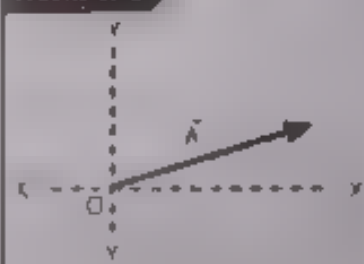
When a vector \vec{A} is multiplied by a negative number n , then the product vector will have magnitude equal to kA but direction opposite to that of \vec{A} .

3 Multiplication with Zero

When a vector is multiplied with zero we get null vector ($\vec{0}$).

$$\text{If } k = 0 \text{ then } k\vec{A} = \vec{0}$$

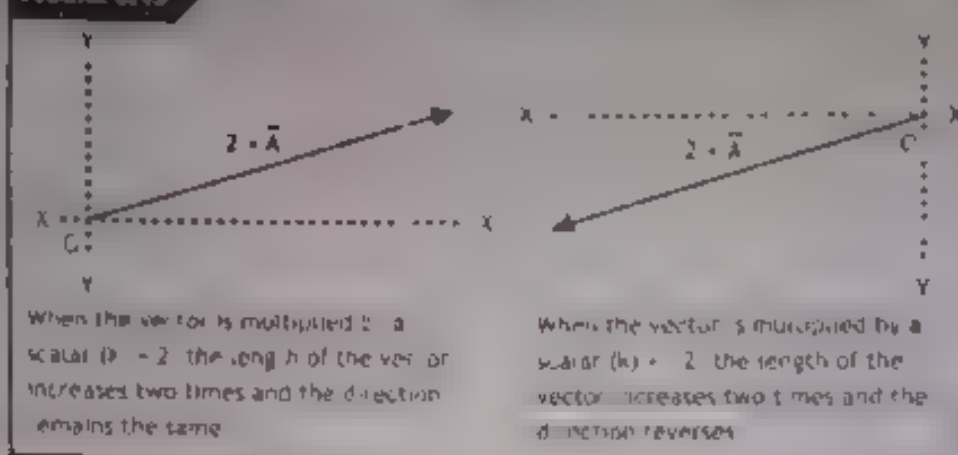
FIGURE 3.42



For Your Information

If vector \vec{A} is multiplied by a number n either positive or negative, the magnitude of new vector be $|n|A$.

FIGURE 2.13



4. Multiplication with scalar quantity

When a vector \vec{A} is multiplied by a scalar quantity k , then the product vector will be a new physical quantity whose dimensions are equal to the product of dimensions of k and \vec{A} .

Examples

- Product of mass m and velocity \vec{v} is momentum (kg m s^{-1}).
- Product of mass m and acceleration \vec{a} is force (kg m s^{-2}).
- Product of force F and time t impulse (kg m s^{-1}).

- Q.11** (a) Define the following vectors. Resolution of a vector, component of a vector and rectangular components of a vector.
 (b) How will you find rectangular components of a vector? Derive the formulae for magnitudes of rectangular components.
 (c) How will you find a vector from its rectangular components?

Ans:

(a) Resolution of a Vector

"The process of splitting up a vector into its components is called resolution of a vector."

Components of a vector

The effective value of a vector in a given direction is called component of a vector.

A vector may split up into two or more than two parts these parts are known as components of vector.

Rectangular Components of Vector

The components of a vector which are perpendicular to each other are called rectangular components.

(b) Resolution of A Vector

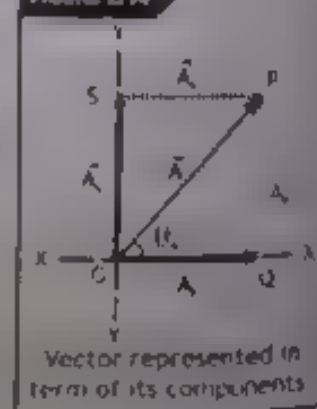
- Consider a vector A in the Cartesian coordinate system, represented by the line OP , making an angle θ as shown in the figure.
- Draw perpendiculars from point P on x -axis and y -axis which meets the axis at points Q and S respectively.
- Put arrow head from the direction of O towards Q and S such that they represent vectors as A_x (OQ) or $A_x \hat{i}$ and A_y (OS) or $A_y \hat{j}$ called the x and y rectangular components of vector A .

TIP

The effective value of a vector in a particular direction is called component of a vector.

Resolution of a vector is the reverse processes of addition of vectors.

FIGURE 2.14



Thus by head to tail rule

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

OR

$$A = A_x + A_y \quad (i)$$

As A_x and A_y are the components of vector A and A_x and A_y are mutually perpendicular, so these are called as rectangular components.

Magnitudes of Rectangular Components

From right angled triangle (OPQ)

From triangle (OPQ) we can write

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\text{Or } \cos\theta = \frac{A_x}{A}$$

$$A_x = A \cos\theta$$

This equation gives the magnitude of A_x in terms of A and θ .

Again From right angled triangle (OPQ)

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\text{Or } \sin\theta = \frac{A_y}{A}$$

$$\text{Or } A_y = A \sin\theta$$

These equations (i) and (ii) give magnitudes of rectangular components in terms of magnitude and direction of their vector.

Equation (i) can be written as

$$\vec{A} = (A \cos\theta) \hat{i} + (A \sin\theta) \hat{j}$$

(c) Determination of vector from its rectangular components

A vector can be determined by its rectangular components. A vector needs magnitude and direction angle for its complete description.

For magnitude:

The magnitude can find out by using Pythagoras theorem. In right triangle (OPQ)

$$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$$

$$A^2 = A_x^2 + A_y^2$$

$$A = \sqrt{A_x^2 + A_y^2}$$

The magnitude of vector can now be determined if the values of the magnitudes of components are known.

For direction:

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{A_y}{A_x}$$

$$\text{or } \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

This expression gives the direction of vector A in terms of angle θ with x axis.



MCQ's

- If R_x is negative and R_y is positive then the resultant vector lies in the _____ quadrant.
(A) 1st (B) 2nd (C) 3rd (D) 4th
- A force of 10 N is acting along x-axis. Which of the following is magnitude of its y-component?
(A) 10 N (B) (10)² (C) 1 N (D) 1 N
- A force of 10 N makes an angle of 30° with y-axis. Which of the following is magnitude of its x-component?
(A) 5 N (B) 8.66 N (C) 10 N (D) 2.66 N
- If R_x and R_y both are negative, then in which quadrant the resultant vector lies?
(A) 1st (B) 2nd (C) 3rd (D) 4th
- How many rectangular components, a vector have in space?
(A) 2 (B) 3 (C) 4 (D) 5
- At which angle with the x-axis, the magnitude of rectangular components of a vector are equal?
(A) 30° (B) 45° (C) 60° (D) 75°
- If R_x is positive and R_y is negative, then in which quadrant the resultant vector lies?
(A) 1st (B) 2nd (C) 3rd (D) 4th
- If a vector of magnitude 6 N is along y-axis then which of the following is its component along x-axis?
(A) 0 N (B) 5 N (C) 8.66 N (D) 10 N
- If a vector of magnitude 10 N is along y-axis then which of the following is its component along x-axis?
(A) 0 N (B) 5 N (C) 8.66 N (D) 10 N
- If vector A is along x-axis then which of the following is its component along y-axis?
(A) $A \sin \theta$ (B) $A \cos \theta$ (C) $A \tan \theta$ (D) Zero
- Which of the following is the resultant of two forces 30 N and 40 N acting parallel to each other?
(A) 30 N (B) 40 N (C) 5 N (D) 10 N
- Which of the following is the resultant of two forces 3 N and 4 N acting at right angle to each other?
(A) 5 N (B) 6 N (C) 7 N (D) 1 N
- Which of the following is the resultant of two forces 5 N and 12 N making an angle of 90° with each other?
(A) 17 N (B) 7 N (C) 13 N (D) 16 N

Answers Key

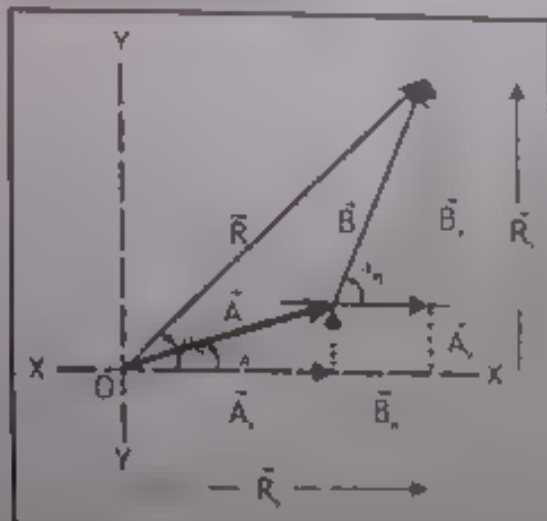
1 A	2 C	3 A	4 C	5 B	6 B	7 D	8 A	9 A	10 D	11 C	12 A
13 C											

Q 12 Describe the method of addition of vectors by rectangular components

Ans: Vector Addition by Rectangular Components

The analytical method for addition of vectors is called addition of vectors by rectangular components. This method is more mathematical in nature rather than geometrical. Therefore, it is regarded as more precise and accurate.

Consider two vector A and B. These vectors are added by head to tail rule. The resultant vector is given as $\vec{R} = \vec{A} + \vec{B}$ which is represented by vector OP.



To resolve the vectors A , B and R into rectangular components, we draw the perpendiculars from points P and M on x -axis shown in Fig. (c). OQ and OR are the magnitudes of x -components of vector A , B and R respectively. So

$$OR = OQ + QR$$

$$\text{OR} \quad OR = OQ + MN \quad QR = MN$$

$$R_x = A_x + B_x \quad \text{--- (1)}$$

$$\text{OR} \quad \text{In vector form}$$

$$\vec{R}_x = \vec{A}_x + \vec{B}_x$$

$$\text{OR} \quad R_x \hat{i} = A_x \hat{i} + B_x \hat{i} = (A_x + B_x) \hat{i}$$

- This shows that magnitudes of the sum of x -components of A and B is equal to the magnitude of x -component of resultant vector R .

Similarly, we can write

$$RP = RS + SP$$

$$RP = QM + SP \quad RS = QM$$

$$R_y = A_y + B_y \quad \text{--- (2)}$$

$$\text{OR} \quad \text{In vector form}$$

$$\vec{R}_y = \vec{A}_y + \vec{B}_y$$

$$\text{OR} \quad R_y \hat{j} = A_y \hat{j} + B_y \hat{j} = (A_y + B_y) \hat{j}$$

- This shows that the magnitudes of the sum of y -components of A and B is equal to the magnitude of y -component of the resultant vector R .
- Since $R_x \hat{i}$ and $R_y \hat{j}$ are the rectangular components of resultant vector R hence

$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

By putting values of magnitudes of R_x and R_y in eq. (1) and (2) we get

$$\vec{R} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

Magnitude:

The magnitude of resultant vector R is

$$R = \sqrt{R_x^2 + R_y^2}$$

or

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

Direction:

The direction of the resultant vector R is given by θ from

$$\tan \theta = \frac{R_y}{R_x}$$

or

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

$$\theta = \tan^{-1} \left(\frac{A_y + B_y}{A_x + B_x} \right)$$

Generalization:

For any number of coplanar vectors $\vec{A}, \vec{B}, \vec{C}, \dots$ we can write

$$\vec{R} = (A_x + B_x + C_x + \dots) \hat{i} + (A_y + B_y + C_y + \dots) \hat{j}$$

Its magnitude R can be written as

$$R = \sqrt{R_x^2 + R_y^2}$$

For Your Information

If A and B be two vectors having magnitude A and B then

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

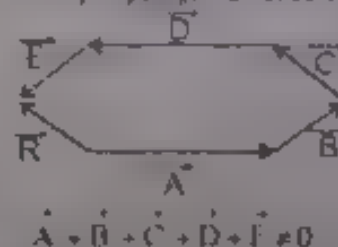
$$\theta = 0^\circ \quad R = \sqrt{A^2 + B^2 + 2AB} = A + B$$

$$\theta = 180^\circ \quad R = \sqrt{A^2 + B^2 - 2AB} = A - B$$

$$\theta = 90^\circ \quad R = \sqrt{A^2 + B^2}$$

Do You Know?

The sum of vector which forms the sides of open polygon is not zero



$$R = \sqrt{(A_x + B_x + C_x + \dots)^2 + (A_y + B_y + C_y + \dots)^2}$$

And $\theta = \tan^{-1} \left(\frac{A_x + B_x + C_x + \dots}{A_y + B_y + C_y + \dots} \right)$

Summary

- Find the x and y components of all given vectors.
- Add x components of all the vectors to find the x-component R_x of the resultant vector.
- Add y components of all the vectors to find the y-component R_y of the resultant vector.
- Find the magnitude of resultant vector \vec{R} by using $R = \sqrt{R_x^2 + R_y^2}$.
- Find the direction of resultant vector \vec{R} by using $\theta = \tan^{-1} \left| \frac{R_y}{R_x} \right|$.

Q 12 How can you determine the angle θ of the vector \vec{R} by its rectangular components.

LINE 9 **Determination of Angle (θ_R) OR Direction of Resultant Vector**

It is the angle which resultant vector makes with positive x-axis in anti-clockwise direction.

- 1) First find ϕ by the following relation

$$\phi = \tan^{-1} \left| \frac{R_y}{R_x} \right|$$

Where ϕ = the angle which \vec{R} makes with positive x-axis and while calculating ϕ ignore the signs of R_x and R_y .

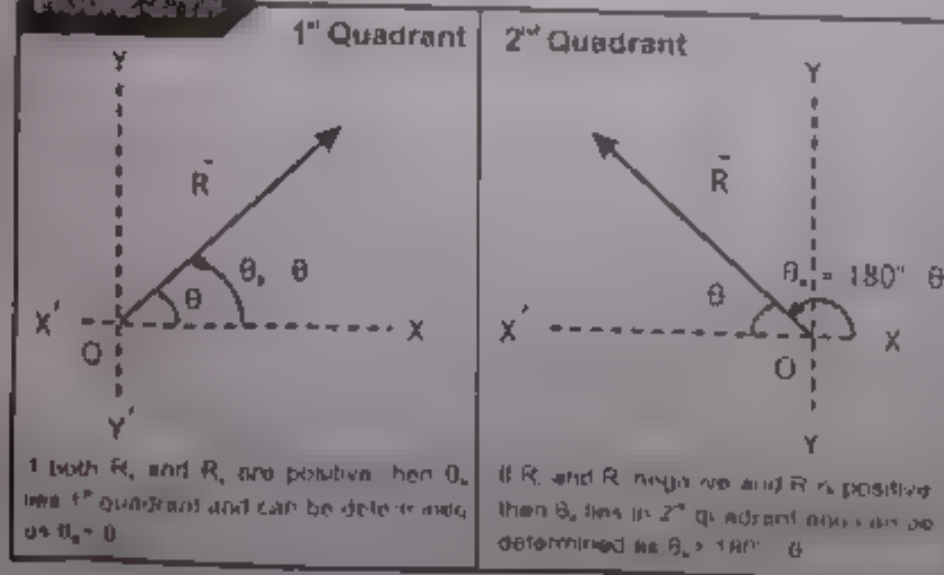
- 2) By the signs of R_x and R_y find the quadrant in which \vec{R} lies as follows

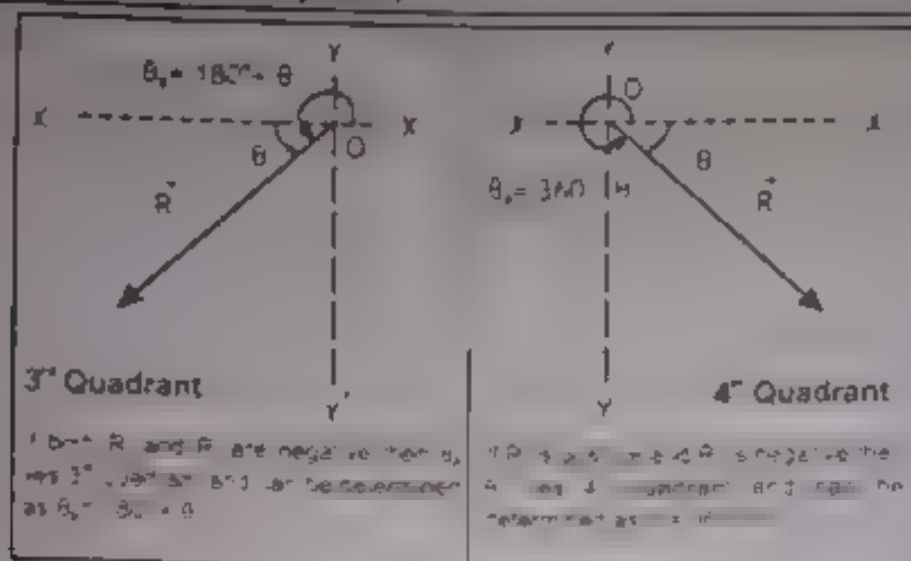
- If both R_x and R_y are positive, the resultant lies in the 1st quadrant and its direction $\theta_R = \phi$.
- If R_x is -ve and R_y is +ve, the resultant lies in 2nd quadrant and its direction is $\theta_R = 180^\circ - \phi$.
- If both R_x and R_y are -ve, the resultant lies in 3rd quadrant and its direction is $\theta_R = 180^\circ + \phi$.
- If R_x is +ve and R_y is -ve, the resultant lies in 4th quadrant and its direction is $\theta_R = 360^\circ - \phi$.

Table 2.1

	II	Y	I	
	R_x		R_x	
	R_y		R_y	
X				X
	R_x		R_x	
	R_y		R_y	
	III	Y'	IV	

FIGURE 2.13



**Assignment 2.2:**

A Force $F_1 = 20\text{ N}$ making an angle $\theta_1 = 30^\circ$ with positive x-axis and Force $F_2 = 30\text{ N}$ making an angle $\theta_2 = 150^\circ$ with positive x-axis acts at a point. Calculate the resultant force

Solution

Given Data:

$F = 20\text{ N}$ at angle $\theta_1 = 30^\circ$ with positive x-axis

$F = 30\text{ N}$ at angle $\theta_2 = 150^\circ$ with positive x-axis

Required

Resultant force = $F = ?$

Calculation

The magnitude of resultant force is given by

$$F = \sqrt{F_1^2 + F_2^2} \quad \dots$$

First we resolve F and F_2 into components as shown. For F and F_2 we have

$$F_x = F \cos \theta_1 = 20 \cos 30^\circ = 17.32\text{ N}$$

$$\Rightarrow F_y = 10\text{ N}$$

$$\text{And } F_2 = F_2 \sin \theta_2 = 30 \sin 30^\circ = 15\text{ N}$$

$$\Rightarrow F_2 = 15\text{ N}$$

For F_2 we have

$$F_x = F_2 \cos \theta_2 = 30 \cos 30^\circ = 25.98\text{ N}$$

$$\Rightarrow F_{2y} = 25.98\text{ N}$$

Now the y-component (vertical component) of the resultant force F is given by

$$F_y = F_{1y} + F_{2y} = 10\text{ N} + 15\text{ N}$$

$$\Rightarrow F_y = 25\text{ N}$$

Putting the values of F_x and F_y in equation (1) we get

$$F = \sqrt{(17.32\text{ N})^2 + (25)^2} = \sqrt{499.6 + 625}\text{ N}$$

$$\Rightarrow F = \left[\sqrt{1124.6} \right] \text{ N}$$

$$\Rightarrow F = 33.54\text{ N or } F = 33.5\text{ N}$$

Direction of \vec{F}

We have $\theta = \tan^{-1} \frac{F_y}{F_x}$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{2.8}{8.66} \right) = \tan^{-1} (0.3232)$$

$$\Rightarrow \theta = 17.8^\circ$$

As x -component of \vec{F} is negative & y -component is positive, so resultant force \vec{F} will be acting in second quadrant by making an angle of 17.8° with negative x -axis as shown in the figure (c). So the actual angle made by \vec{F} with positive x -axis is given by

$$\theta = 180^\circ - 17.8^\circ = 162.2^\circ \approx 162^\circ$$

Assignment 2.3:

Two forces of 20 N and 10 N are making an angle of 120° with each other. Find a single pull that would (a) replace the given forces system (b) balance the given forces system.

Solution.

Let $\vec{F}_1 = 20\text{ N}$ acting along x -axis.

$\vec{F}_2 = 10\text{ N}$ making an angle of 120° with positive x -axis.

(a) (i) Magnitude of resultant of \vec{F}_1 and $\vec{F}_2 = R$

(ii) Direction of $\vec{R} = \theta$

(b) Balance force of $\vec{R} = \vec{F}_3$

(a) First we resolve \vec{F}_1 and \vec{F}_2 into its horizontal and vertical components. As \vec{F}_1 is acting only along x -axis, it has only horizontal component and its vertical component will be equal to zero. So we have,

$$F_{1x} = F_1 \cos 0^\circ = 20 \times \cos 0^\circ$$

$$\Rightarrow F_{1x} = 20 \times 1\text{ N}$$

$$\Rightarrow F_{1x} = 20\text{ N and } F_{1y} = 0\text{ N}$$

Now we resolve \vec{F}_2 into its horizontal and vertical components, as shown in figure (b). So we have

$$F_{2x} = F_2 \cos 60^\circ = 10 \times \cos 60^\circ$$

$$\Rightarrow F_{2x} = 10 \times 0.5 = 5\text{ N}$$

$$\Rightarrow F_{2x} = 5\text{ N}$$

While

$$F_{2y} = F_2 \sin 60^\circ = 10 \times \cos 60^\circ$$

$$\Rightarrow F_{2y} = 10 \times 0.866\text{ N}$$

$$\Rightarrow F_{2y} = 8.66\text{ N}$$

(F_{2y} is in second quadrant & F_{2x} is taken as negative)

Now the x -component of the resultant is given by

$$R_x = F_{1x} + F_{2x}$$

$$\Rightarrow R_x = 20 + (-5) = 20 - 5 = 15$$

$$\Rightarrow R_x = 15\text{ N}$$

Now y -component of \vec{R} (resultant) is given by,

$$R_y = F_{1y} + F_{2y}$$

$$\Rightarrow R_y = 0\text{ N} + 8.66\text{ N}$$

$$\Rightarrow R_y = 8.66\text{ N}$$

(i) Now the magnitude of the resultant is given by

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\Rightarrow R = \sqrt{(15)^2 + (8.66)^2} = \sqrt{225 + 74.9956}$$

Q 13 What are different methods to multiply vectors with vectors?

Ans When two vectors are multiplied then we get either a scalar quantity or vector quantity

- If two vectors are multiplied and we get a scalar quantity then it is called scalar product
- If two vectors are multiplied and we get a vector quantity then it is called vector product

Q 14 Define scalar product of two vectors. Give examples.

Ans **Scalar Product (Dot Product)**

If the product of two vectors be a scalar quantity then the product is called scalar product

$$\text{Vector} \times \text{Vector} = \text{Scalar}$$

Scalar product of two vectors \vec{A} and \vec{B} is defined as

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Where A and B are the magnitudes of vectors \vec{A} and \vec{B} and θ is the angle between them

- Because dot operation (.) is used to represent the two vectors therefore it is also called dot product.

Physical meaning

Dot product of two vectors is equal to product of magnitude of one vector and the component of the second vector in the direction of first vector.

From figure

$$\vec{A} \cdot \vec{B} = A (\text{magnitude of component of } \vec{B} \text{ along } \vec{A})$$

$$\text{OR } \vec{A} \cdot \vec{B} = A (\text{projection of } \vec{B} \text{ on } \vec{A})$$

$$\vec{A} \cdot \vec{B} = A (B \cos \theta)$$

Examples

- 1) Work is scalar product of force and displacement

$$[W = \vec{F} \cdot \vec{d}]$$

- 2) Power is scalar product of force and velocity

$$[P = \vec{F} \cdot \vec{v}]$$

- 3) Electric flux is scalar product of electric intensity and vector area

$$[\phi_e = \vec{E} \cdot \vec{A}]$$

- 4) Magnetic flux is scalar product of magnetic field strength and vector area

$$[\phi_b = \vec{B} \cdot \vec{A}]$$

Q 15 Write down the characteristics of scalar product of two vectors

Ans **Characteristics of Scalar Product**

(i) **Commutative property**

Scalar product of two vectors is commutative

If \vec{A} and \vec{B} be two vectors and θ is the angle between them then

From figure

$$\vec{A} \cdot \vec{B} = A (\text{magnitude of component of } \vec{B} \text{ along } \vec{A})$$

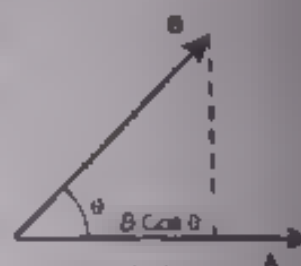
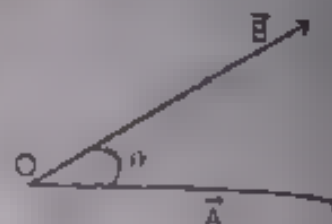


Fig. 2.10 (a)

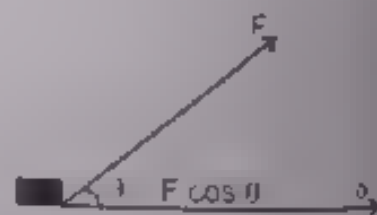


Fig. 2.10 (a)

OR $\vec{A} \cdot \vec{B} = A$ (projection of \vec{B} on \vec{A})

$$\vec{A} \cdot \vec{B} = A (B \cos \theta) = AB \cos \theta \quad (1)$$

Similarly

$\vec{B} \cdot \vec{A} = B$ (Projection of \vec{A} on \vec{B})

$\vec{B} \cdot \vec{A} = B$ (magnitude of component of \vec{A} along \vec{B})

$$\vec{B} \cdot \vec{A} = B A \cos \theta = AB \cos \theta \quad (2)$$

From equations (1) and (2)

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

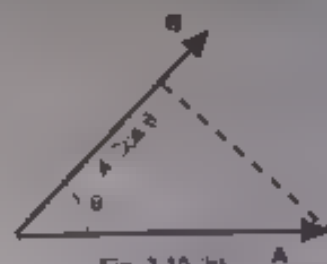
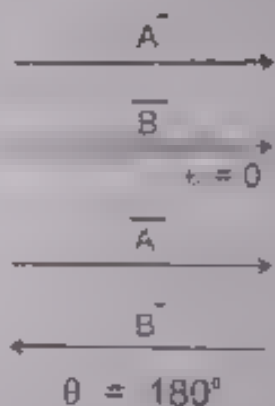
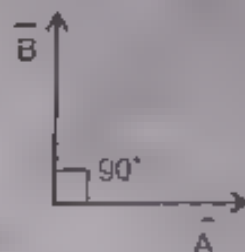


Fig. 2.10 (b)



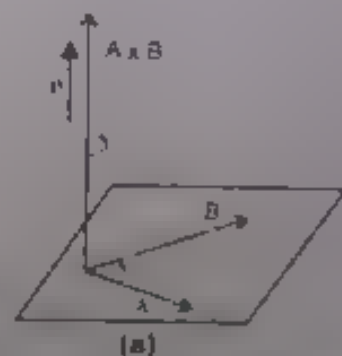
Do You Know?

Magnitude of a vector \vec{A} is $\sqrt{\vec{A} \cdot \vec{A}}$

For Your Information

Why $\vec{A} \cdot \vec{E}$ is scalar quantity even though vector \vec{E} is a vector.

$$A \cdot E = \frac{1}{2} mv^2 = \frac{1}{2} m (\vec{v} \cdot \vec{v})$$



(a)

(iii) Perpendicular vectors

If two vectors are mutually perpendicular ($\theta = 90^\circ$) to each other then their scalar product is zero i.e.

$$\vec{A} \cdot \vec{B} = AB \cos 90^\circ$$

$$\vec{A} \cdot \vec{B} = AB (0)$$

$$\vec{A} \cdot \vec{B} = 0$$

(iii) Parallel Vectors

If two vectors are parallel ($\theta = 0^\circ$) to each other then their scalar product is equal to the product of the magnitudes i.e.

$$\vec{A} \cdot \vec{B} = AB \cos 0^\circ = AB (1) = AB$$

This is the positive maximum value of scalar product

(iv) Anti-Parallel Vectors

If two vectors are anti-parallel ($\theta = 180^\circ$) then their scalar product is negative

$$\vec{A} \cdot \vec{B} = AB \cos 180^\circ = AB (-1) = -AB$$

This is the negative maximum value of scalar product

(iv) Self Scalar product

The self-product of a vector is equal to square of its magnitude i.e.

$$\vec{A} \cdot \vec{A} = AA \cos 0^\circ = A^2 (1) = A^2$$

Q.16 Define and explain vector product of two vectors? Give its physical significance. Give examples

Ans. Vector Product (Cross Product)

If the product of two vectors be a vector quantity then the product of vectors is called vector product.

$$\text{Vector} \times \text{Vector} = \text{Vector}$$

The vector product of two vectors \vec{A} and \vec{B} is defined as

$$\vec{A} \times \vec{B} = AB \sin \theta \vec{n}$$

Where angle between \vec{A} and \vec{B} is θ and \vec{n} is the unit vector perpendicular

the plane containing \vec{A} or \vec{B} (Where θ is smaller angle between the two vectors)

- Vector cross operation (or scalar triple product) will give the direction of the vector $\vec{A} \times \vec{B}$ (cross product)

Direction of vector product

The direction of vector $\vec{A} \times \vec{B}$ can be found by right hand rule

Right hand Rule

1. Curl the fingers of the right hand from the vector \vec{A} into \vec{B} through the smaller angle θ between them. The thumb then points in the direction of the vector $\vec{A} \times \vec{B}$ (out of the page).

2. If the angle between \vec{A} and \vec{B} is θ , then the direction of $\vec{A} \times \vec{B}$ is perpendicular to the plane containing \vec{A} and \vec{B} .

Physical Significance

Consider $\vec{A} \times \vec{B}$. As \vec{A} and \vec{B} are perpendicular, $\vec{A} \times \vec{B}$ is maximum. If \vec{A} and \vec{B} are parallel, $\vec{A} \times \vec{B}$ is zero.

Examples

Example 1: Find the direction of $\vec{A} \times \vec{B}$ if \vec{A} is along the x-axis and \vec{B} is along the y-axis.

Solution: \vec{A} is along the x-axis and \vec{B} is along the y-axis. The direction of $\vec{A} \times \vec{B}$ is along the z-axis.

Q 17 Write down the characteristics of vector product of two vectors.

4.3 Characteristics of Vector Product

- (i) Violation of Commutative law

Consider two vectors \vec{A} and \vec{B} in the xy-plane. The direction of $\vec{A} \times \vec{B}$ is along the z-axis.

But the direction of $\vec{B} \times \vec{A}$ is along the negative z-axis.

Thus, $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

Therefore, the vector product is not commutative.

(ii) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Consider two vectors \vec{A} and \vec{B} in the xy-plane. The direction of $\vec{A} \times \vec{B}$ is along the z-axis.

But the direction of $\vec{B} \times \vec{A}$ is along the negative z-axis.

Thus, $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

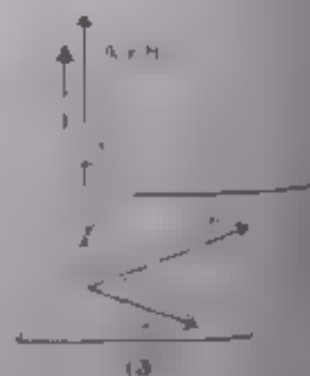
- (ii) Perpendicular Vectors

Consider two perpendicular vectors \vec{A} and \vec{B} . The direction of $\vec{A} \times \vec{B}$ is along the z-axis.

maximum value

For Your Information

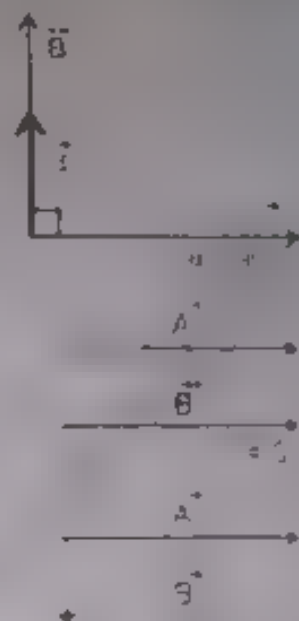
$\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ can be equal in magnitude only when \vec{A} and \vec{B} are perpendicular or parallel.



$$\vec{A} \times \vec{B} = AB \sin 90^\circ \hat{n}$$

$$\vec{A} \times \vec{B} = AB (1) \hat{n}$$

$$\vec{A} \times \vec{B} = AB \hat{n}$$

(iii) **Parallel and Anti-parallel Vectors**

The cross product of two parallel ($\theta = 0^\circ$) or two anti-parallel ($\theta = 180^\circ$) vectors is a **null vector**, i.e.

In case of parallel vectors

$$\vec{A} \times \vec{B} = AB \sin 0^\circ \hat{n} = AB (0) \hat{n} = 0$$

In case of anti-parallel vectors

$$\vec{A} \times \vec{B} = AB \sin 180^\circ \hat{n} = AB (0) \hat{n} = 0$$

(iv) **Self Vector product**

The self product of a vector \vec{A} is **null vector**.

$$\vec{A} \times \vec{A} = AA \sin 0^\circ \hat{n} = AA (0) \hat{n} = 0$$

In case of anti-vectors

(v) **Area of Parallelogram**

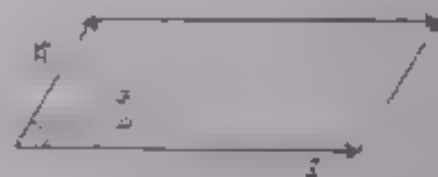
The magnitude of cross product of two vectors represent **an area of parallelogram** formed with these vectors taken as its two adjacent sides.

Area of parallelogram = (length) \times (height)

$$= (A) \times (B \sin \theta)$$

$$= AB \sin \theta$$

$$\text{magnitude of } (\vec{A} \times \vec{B})$$



Area of parallelogram

$$|\vec{A} \times \vec{B}|$$

Assignment 2.4:

Show that $\vec{A} \times \vec{B} = \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Solution:

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

$$|\vec{A} \times \vec{B}| = AB \cos \theta$$

$$|\vec{A} \times \vec{B}| = AB \cos \theta$$

Adding eq (1) and (2)

$$|\vec{A} \times \vec{B}| + |\vec{A} \times \vec{B}| = AB \sin \theta + AB \cos \theta$$

$$= AB (\sin \theta + \cos \theta)$$

$$|\vec{A} \times \vec{B}| + |\vec{A} \times \vec{B}| = AB (1)$$

$$|\vec{A} \times \vec{B}| + |\vec{A} \times \vec{B}| = AB$$

MCQ's

- The dot product of vector A with itself is equal to
(A) Zero (B) A (C) $2A$ (D) A^2
- If the angle between the two vectors with magnitude 12 and 4 is 60° then their scalar product is
(A) 6 (B) 12 (C) 24 (D) None of these
- The scalar product of two vectors is maximum when they are
(A) Parallel (B) Perpendicular (C) Null (D) Anti-parallel
- Projection of B on A is
(A) $A \cos \theta$ (B) $B \sin \theta$ (C) $A \sin \theta$ (D) $B \cos \theta$
- Projection of B along A is given as
(A) $\vec{A} \cdot \vec{B}$ (B) $\vec{B} \cdot \vec{A}$ (C) $\frac{\vec{B} \cdot \vec{A}}{A \cos \theta}$ (D) $\frac{A \cos \theta}{B}$
- If the scalar product of two vectors is $2\sqrt{3}$ and the magnitude of their vector product is 2. The angle between them is
(A) 20° (B) 30° (C) 60° (D) 80°
- The vector product $(\vec{A} \times \vec{A})$ is
(A) 0 (B) 1 (C) A (D) B
- The magnitude of dot and cross product of two vectors are equal when angle between them is
(A) $2e^\circ$ (B) 45° (C) 90° (D) 270°
- An area of parallelogram formed by vectors A and B as its two adjacent sides is given as
(A) $AB \sin \theta$ (B) $AB \cos \theta$ (C) $AB \tan \theta$ (D) $\vec{A} \cdot \vec{B}$
- If $\vec{A} \cdot \vec{B} = 0$, then angle between the vectors is
(A) 30° (B) 45° (C) 0° (D) None of these
- Both the dot product and cross product of two vectors \vec{A} and \vec{B} is zero when
(A) \vec{A} and \vec{B} are parallel to each other (B) \vec{A} and \vec{B} are anti-parallel
(C) \vec{A} and \vec{B} are perpendicular to each other (D) Either the vector is zero

Answers Key

1. D	2. C	3. A	4. D	5. D	6. B	7. A	8. B	9. A	10. C	11. D
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Q 18 Define and explain the terms concurrent forces and equilibrant force

Ans: Concurrent Forces

When two or more forces are acting upon a body and the lines of action of these forces pass through a common point, the forces are said to be concurrent forces.

Example:

As three dogs are pulling a piece of meat with forces F_1 , F_2 and F_3 as shown in the figure the forces are concurrent as their line of action pass through a common point.

FIGURE 2.30

**Equilibrant Force**

Two or more concurrent forces can be balanced by a single force called equilibrant force. OR

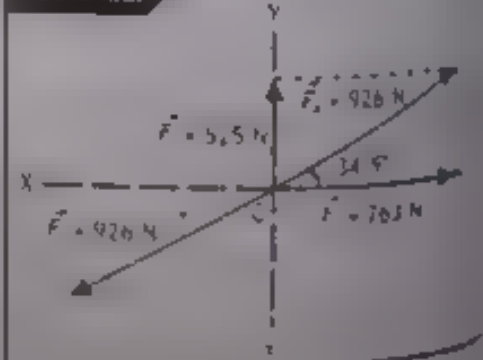
"That single force which is equal in magnitude but opposite to the resultant of different concurrent forces."

Example

Consider two concurrent forces $F_1 = 763 \text{ N}$ and $F_2 = 575 \text{ N}$ which are at right angle to each other with their resultant $F_R = 926 \text{ N}$ at 34° with X-axis.

The equilibrant force is equal in magnitude to that of the resultant force but it acts in the opposite direction as shown in Figure.

FIGURE 2.37



Following diagram may help to understand concurrent forces.

Two or more concurrent forces can be balanced by a single force as shown.

Q 19 Define and explain the term torque or moment of force

Ans Torque or Moment of Force

The turning effect produced in a body about a fixed point due to an applied force is called torque or moment of force.

(OR)

The product of magnitude of force and the perpendicular distance from the axis of rotation to the line of action of force is called moment arm.

$$\tau = F \perp$$

Where

F = perpendicular distance between the line of action of force and axis of rotation.

F = Magnitude of applied force.

(OR)

Torque is the vector product of position vector \vec{r} and force \vec{F} .

OR $\tau = \vec{r} \times \vec{F}$

Unit

The SI unit of torque is newton-metre (Nm).

Its dimension is $[ML^2T^{-2}]$.

Torque is a vector quantity.

Examples of torque

- Torque is the cause of rotation.
- A certain torque is required to start a rotating motion.

Direction

Its direction is determined by Right Hand Rule. Curl the fingers of your right hand in the direction of rotation then your thumb's point out in the direction of torque.

Force and Torque Analogy

1 Just as force determines the linear acceleration produced in a body, the torque acting on a body determines its angular acceleration.

2 Torque is the rotational analogue of force.

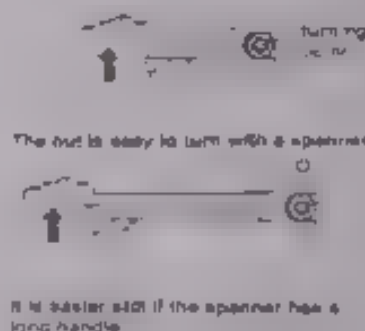
As $F = ma$ similarly $\tau = I\alpha$

where I = Moment of inertia and α is angular rotational acceleration.

3 If the body is at rest or rotating with uniform angular velocity, the angular acceleration will be zero. In this case the torque acting on the body will be zero.

Note

The torque which is perpendicular to the position arm apply torque on the body which is the line which passes through axis of rotation called as moment arm.



Greater the moment arm, easier to rotate the body and vice versa.

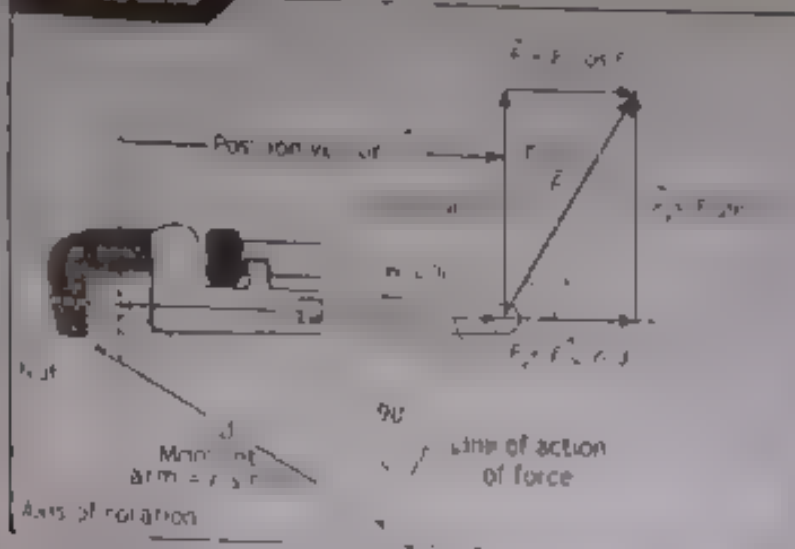
Q 20 Show that torque is the vector product of force and position vector

Ans Torque is an example of vector product of position vector \vec{r} and \vec{F} .

Proof Let us consider a spanner in which we apply a force \vec{F} on its end to loose the nut (where is pivot point) and axis of rotation passes as shown and \vec{r} is the position vector from pivot point to point of action of force.

- Let us resolve the force \vec{F} into its rectangular components F_x and F_y as shown.
- We see that the fraction of $F_x = F \cos \theta$ passes through the pivot point i.e. it is parallel to the position vector \vec{r} . So, it does not apply torque.
- While the fraction of force $F_y = F \sin \theta$ is perpendicular to moment arm or position vector \vec{r} . So, the effective component of force which produces rotation is F_y .
- Since magnitude of torque is given by

FIGURE 2.28



Torque = moment arm (magnitude of perpendicular distance of force)

$$\tau = (r) (F \sin \theta)$$

In vector form,

$$\vec{\tau} = (r) (F \sin \theta) \hat{n}$$

OR

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Another Proof

Instead of resolving force we can project position vector \vec{r} as shown in the figure

We can write the torque as

Torque = (moment arm) (perpendicular distance from axis of rotation to line of action of force) (magnitude of force)

$$\tau = (r \sin \theta) (F)$$

OR

$$\tau = (r F \sin \theta)$$

In vector form

$$\vec{\tau} = (r F \sin \theta) \hat{n}$$

OR

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Dependence of torque

Torque depends on the following factors

- 1) Magnitude of force
- 2) Magnitude of position vector
- 3) Angle between force vector \vec{F} and position vector \vec{r}

Maximum Torque

Magnitude of torque applied on a body is maximum if \vec{F} and \vec{r} are perpendicular ($\theta = 90^\circ$)

$$\tau_{\max} = r F \sin(90^\circ)$$

$$\Rightarrow \tau_{\max} = r F (1) = r F$$

Minimum Torque

Magnitude of torque applied on the body is minimum if \vec{F} and \vec{r} are parallel ($\theta = 0^\circ$) or antiparallel ($\theta = 180^\circ$)

$$\text{If } \theta = 0^\circ \text{ or } 180^\circ$$

$$\tau_{\min} = r F \sin(0^\circ)$$

$$\Rightarrow \tau_{\min} = r F (0) = 0$$

FIGURE 2.29(a)

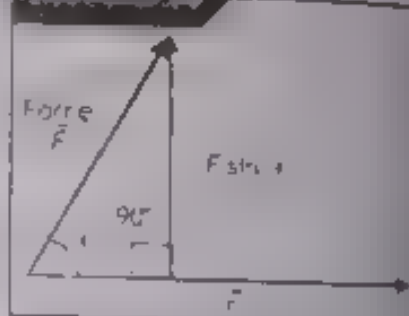
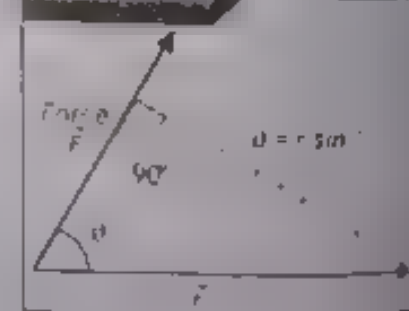


FIGURE 2.29(b)



And if $\theta = 180^\circ$

$$\tau_{\min} = r F \sin(180^\circ)$$

$$\Rightarrow \tau_{\min} = r F \sin \theta = 0$$

DO YOU KNOW?

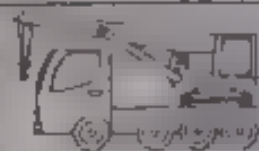
Mobile Crane Counterweights – an Important Safety Factor

Mobile cranes and vehicle-mounted cranes lift, lower and transport heavy loads. A counterweight reduces the heavy loads which operate on the principle of levers. The shorter end of the beam is applied as a fulcrum and the longer end (crane boom) is rotated and move the load either forward or backward to position the object at the desired location.

Since boom is heavy and weight over it is concentrated at its end will produce a great deal of weight with a danger for the crane to topple over.

To overcome this situation cranes have a counterweight at the other side of the boom in a horizontal direction from that which is lifted. The counterweight exerts a torque on the crane in equal and opposite direction to the torque from the load. Mathematically

$$\tau_{\text{boom}} = \tau_{\text{counterweight}}$$



Q 21 What is value of torque if the body is at rest or rotating with uniform angular velocity?

Ans: Torque acting on the body will be zero.

Reason

In this case angular acceleration is zero so torque will be

$$\tau = I\alpha \quad [2^{\text{nd}} \text{ law for rotational motion}]$$

$$\therefore \tau(0) = 0$$

Q 22 What is couple? Derive its formula

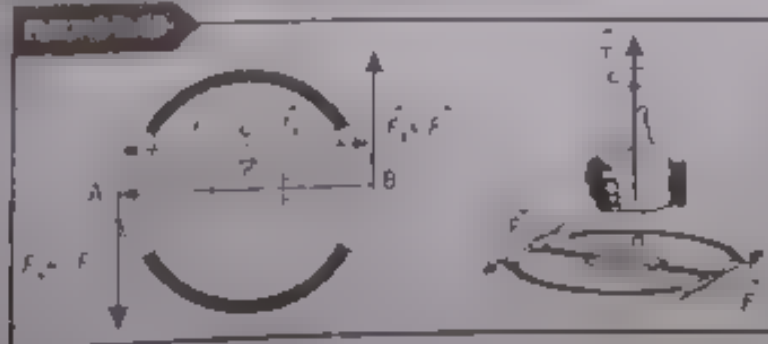
Ans: Moment of A Couple

Two parallel forces F_A and F_B have the same magnitude but opposite in direction and are separated by a perpendicular distance from the couple.

The rotational effect produced by couple is called moment of couple

OR

"The moment produced by a couple is called a couple moment."



Example

For example, imagine that you are driving a car with both hands on the steering wheel and you are making a turn. One hand will push up on the wheel while the other hand pulls down, which causes the steering wheel to rotate as shown in Figure.

Derivation

Consider the steering wheel whose pivot point is O. Let two equal and opposite forces F_A and F_B are acting at points A and B whose position vectors are r_A and r_B respectively from the pivot point O as shown.

A vector whose point of application is not fixed but its magnitude and direction are fixed, is called free vector.

OR

A vector whose point of action is not fixed and can be moved parallel to itself known as free vector.

- Torque applied due to force \vec{F}_A is

$$\tau_A = \vec{r}_A \times \vec{F}_A$$

- Torque applied due to force \vec{F}_B is

$$\tau_B = \vec{r}_B \times \vec{F}_B$$

- Moment of couple i.e. total torque is equal to vector sum of torques due to these forces is

$$\vec{\tau} = \vec{\tau}_A + \vec{\tau}_B$$

$$\Rightarrow \vec{\tau} = (\vec{r}_A \times \vec{F}_A) + (\vec{r}_B \times \vec{F}_B) \quad (1)$$

Since both forces are equal in magnitude but in opposite direction

$$\vec{F}_A = -\vec{F} \quad \text{at } \vec{r}_A = \vec{r}_B = \vec{r}$$

So equation (1) becomes

$$\vec{\tau} = (\vec{r}_A \times (-\vec{F})) + (\vec{r}_B \times \vec{F})$$

$$\Rightarrow \vec{\tau} = (-\vec{r}_A + \vec{r}_B) \times \vec{F}$$

$$\Rightarrow \vec{\tau} = (\vec{r}_B - \vec{r}_A) \times \vec{F}$$

Since $\vec{r}_B - \vec{r}_A = \vec{r} = \text{position vector from point B to A}$

$$\text{So } \vec{\tau} = \vec{r} \times \vec{F}$$

Direction of couple is measured by Right Hand Rule

MCQs

- The direction of torque can be found by
 (A) Head to tail rule (B) Right hand rule (C) Left hand rule (D) Fleming's rule
- Torque of force is given by $\vec{\tau} = \vec{r} \times \vec{F}$. It has maximum value when \vec{r} and \vec{F} are at an angle of
 (A) 90° (B) 0° (C) 30° (D) 180°
- If the position vector \vec{r} and \vec{F} are in same direction, the torque will be
 (A) Maximum (B) Minimum (C) Negative (D) Positive
- If the body is at rest or rotating with uniform angular velocity, then torque will be
 (A) Maximum (B) Negative (C) Zero (D) Positive
- The turning effect of force in a body is called
 (A) Work (B) Moment (C) Acceleration (D) Torque
- The direction of torque is
 (A) Along the position vector \vec{r} (B) Parallel to the plane containing \vec{r} and \vec{F}
 (C) Along the force \vec{F} (D) Perpendicular to the plane \vec{r} and \vec{F}
- The counterpart of force for rotational motion is called
 (A) linear momentum (B) Angular momentum (C) angular acceleration (D) Torque

Answers Key

1 B	2 A	3 C	4 C	5 D	6 D	7 D	
-----	-----	-----	-----	-----	-----	-----	--

Q 23 What is equilibrium? Give its types. Also discuss types of dynamic equilibrium.

Ans

The state of a body under the action of several forces and torques acting together and there is no change in the translational motion as well as rotational motion is called equilibrium.

OR

A body is said to be in equilibrium if it is at rest or moving with uniform velocity under the action of a number of forces.

OR

If a body has no acceleration then it is said to be in equilibrium.



Fig. 2.10

Types of Equilibrium**i. Static Equilibrium**

If a body is at rest then it is in state of static equilibrium.

OR When a body is at rest under the action of forces acting together the body is said to be in static equilibrium.

For example, book placed on the table.

ii. Dynamic Equilibrium

If a body is moving with a constant velocity it is in dynamic equilibrium.

OR When a body is moving with a constant velocity it is in dynamic equilibrium. It is said to be in dynamic equilibrium.

- A paratrooper falling at a constant velocity is in dynamic equilibrium.
- A car moving with a constant velocity is in dynamic equilibrium.
- A wheel at rest is in static equilibrium.

Types of Dynamic Equilibrium**i. Dynamic Translational Equilibrium**

A body is in dynamic translational equilibrium if it is moving with a constant velocity.

For example, a paratrooper falling at a constant velocity is in dynamic translational equilibrium.

ii. Dynamic Rotational Equilibrium

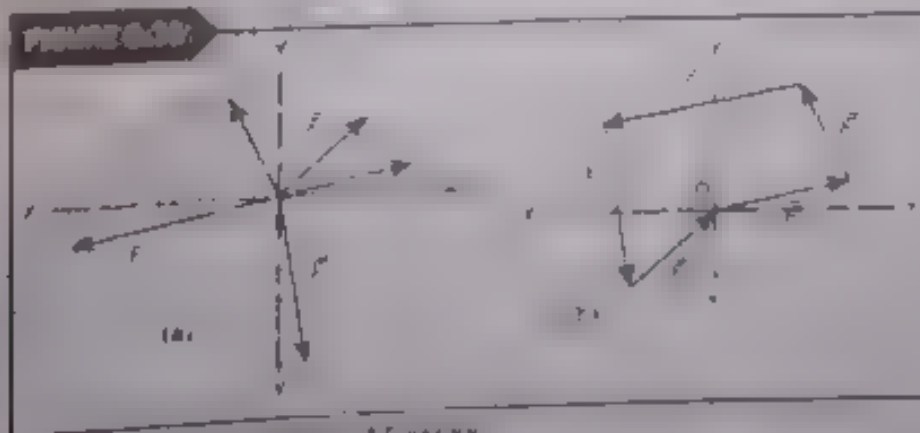
A body is in dynamic rotational equilibrium if it is rotating with a constant angular velocity.

For example, a wheel at rest is in dynamic rotational equilibrium.



Q 24 State the two conditions of equilibrium. What type of equilibrium is guaranteed by each condition of equilibrium?

Ans: For a body to be in static equilibrium, the following conditions must be satisfied:



First condition of equilibrium (Equilibrium of Forces)

The vector sum of all the forces acting on a body must be null vector.

i.e.
$$\sum \vec{F} = \vec{0}$$

In case of coplanar forces, let a point on the x-axis express as:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

And

Where $\sum \vec{F}_x =$ Vector sum of x-directed forces

$\sum \vec{F}_y =$ Vector sum of y-directed forces

Note

- If the rightward forces are taken as positive then leftward forces are taken as negative
- If upward forces are taken as positive, then downward forces are taken as negative
- Forces which lie in a common plane are said to be coplanar

Second Condition of equilibrium (Equilibrium of Torques)

The vector sum of all the torques acting on the body about an axis must be null vector.

i.e. $\sum \vec{\tau} = \vec{0}$

OR $\sum \vec{\tau}_{clockwise} = \sum \vec{\tau}_{anticlockwise}$

There are certain situations in which sum of forces is zero but body is still not in equilibrium e.g. motion of steering wheel. Then we required 2nd condition of equilibrium i.e. sum of torques acting on the must be zero.

Condition for Translational Equilibrium

When sum of all the forces acting the body is zero i.e. $\sum \vec{F} = \vec{0}$ and $\vec{a} = \vec{0}$ then the body is said to be in translational equilibrium.

Condition for Rotational Equilibrium

When sum of torques acting on the body is zero i.e. $\sum \vec{\tau} = \vec{0}$ and $\vec{\alpha} = \vec{0}$ then the body is said to be in rotational equilibrium.

Q 25 What is condition for complete equilibrium?

Ans: For a body to be in complete equilibrium, both conditions must be satisfied i.e. both linear acceleration and angular acceleration must be zero.

OR

For a body to be in complete equilibrium, sum of all forces acting on the body is zero and sum of all torques acting on the body is zero.

Complete Equilibrium:

- When the first condition is satisfied this means that there is no net force acting on the body so it will represent translational equilibrium only.

$$F_{net} = 0 \quad \text{and} \quad a_{net} = 0$$

- When the second condition is satisfied this means that there is no net torque acting on the body so it will represent rotational equilibrium only.

$$\tau_{net} = 0 \quad \text{and} \quad \alpha_{net} = 0 \quad (\alpha_{net} = \text{net angular acceleration})$$

- For complete equilibrium, both the first and second conditions of equilibrium must be satisfied.
- There are certain situations in which one of the two conditions are satisfied, so body will not be in complete equilibrium.

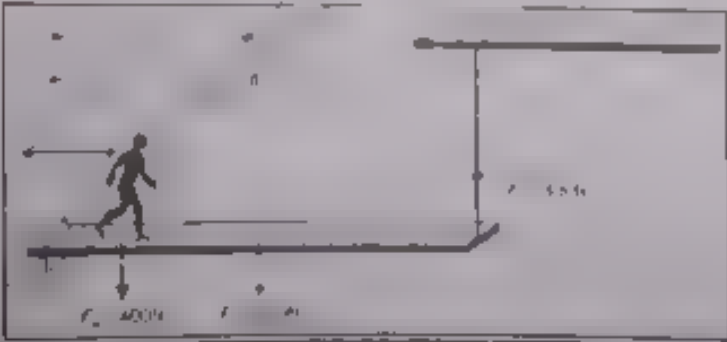
For example in case of couple we see that the first condition is satisfied but still the object can rotate with angular acceleration, therefore the object is not in equilibrium with respect to rotation hence we cannot say that object is in complete equilibrium.

Q 26 Describe different situations which could be helpful in understanding the translational, rotational and complete equilibrium

Ans:

In these diagrams the axis of rotation is at the center of the body	In these diagrams the axis of rotation is at the edge of the body	In these diagrams the axis of rotation is at the end of the body
(a) This body is in equilibrium	(b) This body is in equilibrium	(c) This body is in equilibrium
First Condition Satisfied Net force = 0 so body has no tendency to start moving linearly Second Condition Satisfied Net torque about the axis = 0 so body is at rest and not rotating To be in static equilibrium a body must satisfy both conditions	First Condition Satisfied Net force = 0 so body has no tendency to start moving linearly Second Condition Satisfied Net torque about the axis = 0 so body is at rest and not rotating To be in static equilibrium a body must satisfy both conditions	First Condition NOT Satisfied Net force ≠ 0 so body has a tendency to start moving linearly Second Condition Satisfied Net torque about the axis = 0 so body is at rest and not rotating To be in static equilibrium a body must satisfy both conditions

Assignment 2.5:
 A uniform plank of weight 200 N and length 6 m is supported by a rope as shown in the figure. If the breaking tension in the rope is 400 N. How far can a boy of weight 400 N walk towards the support?



Solution

Apply $\Sigma \tau = 0$ for equilibrium

$\Sigma \tau = 0$

5 Clockwise torque = Anticlockwise torque

$\tau_1 + \tau_2 = \tau_3$

Where τ_1 = torque due to weight of the plank $\tau_2 = 400 \times d$ (Nm)

τ_3 = torque due to weight of the man $\tau_3 = 400 \times 6$ (Nm)

τ_1 = torque due to tension of the rope $\tau_1 = 400 \times d$ (Nm)

Put values in equation (1)

$200 \times 3 + 400 \times d = 400 \times 6$

$600 + 400d = 2400$

$400d = 2400 - 600$

$400d = 1800$

$d = 1800/400$

$d = 4.5 \text{ m}$

MCQ's

- 1 The first condition of equilibrium implies that
 (A) $\Sigma F = 0$ (B) $\Sigma F = 0$ only (C) $F_x = 0$ only (D) $F_x = F_y$
- 2 A body is in static equilibrium only when it is
 (A) At rest (B) Moving with variable velocity
 (C) Moving with uniform acceleration (D) Moving with uniform velocity
- 3 A body will be in translational equilibrium if
 (A) $\Sigma F = 0$ (B) $\Sigma F \neq 0$ (C) $\Sigma p = 0$ (D) $\Sigma p \neq 0$

Answers Key

1 A

2 A

3 B

FORMULAE

1	Commutative law for vector addition	$\vec{A} + \vec{B} = \vec{B} + \vec{A}$
2	Subtraction of vectors	$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$
3	Vector representation	$\vec{A} = A\hat{i}$
4	Unit vector	$\hat{i} = \frac{\vec{A}}{A}$
5	Null vector	$\vec{A} + (-\vec{A}) = \vec{0}$
6	Vector in terms of rectangular components	$\vec{A} = A_x\hat{i} + A_y\hat{j}$
7	x-component of a vector \vec{A}	$A_x = A \cos \theta$
8	y-component of a vector \vec{A}	$A_y = A \sin \theta$
9	Magnitude of vector \vec{A}	$A^2 = A_x^2 + A_y^2$ $A = \sqrt{A_x^2 + A_y^2}$
10	Direction of vector \vec{A}	$\theta = \tan^{-1} \frac{A_y}{A_x}$
11	Position vector of a point P(a, b) in plane	$\vec{r} = a\hat{i} + b\hat{j}$
12	Position vector of a point P(a, b, c) in space	$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$
13	Magnitude of resultant (\vec{R}) of vectors \vec{A} and \vec{B}	$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$
14	Direction of resultant (\vec{R}) of vectors \vec{A} and \vec{B}	$\theta = \tan^{-1} \frac{A_y + B_y}{A_x + B_x}$
15	Scalar product of two vectors	$\vec{A} \cdot \vec{B} = AB \cos \theta$
16	Scalar product of two perpendicular vectors	$\vec{A} \cdot \vec{B} = 0$

17	Scalar product of unit vectors \hat{i} and \hat{k}	$\hat{i} \cdot \hat{i} = 1$	$\hat{j} \cdot \hat{k} = 0$	$\hat{k} \cdot \hat{k} = 1$
18	Scalar product of two parallel vectors	$\vec{A} \cdot \vec{B} = AB$		
19	Scalar product of two anti-parallel vectors	$\vec{A} \cdot \vec{B} = -AB$		
20	Self dot product of vector \vec{A}	$\vec{A} \cdot \vec{A} = A^2$	$A = \sqrt{\vec{A} \cdot \vec{A}}$	
21	Self scalar product of unit vectors \hat{i} and \hat{k}	$\hat{i} \cdot \hat{i} = 1$		$\hat{k} \cdot \hat{k} = 1$
22	Scalar product in terms of rectangular components	$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$		
23	Angle between two vectors \vec{A} and \vec{B}	$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}$	$\theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{AB} \right)$	
24	Projection of vector \vec{A} on \vec{B}	$A \cos \theta$	$\frac{\vec{A} \cdot \vec{B}}{B}$	$A \cos \theta = \frac{\vec{A} \cdot \vec{B}}{B}$
25	Projection of vector \vec{B} on \vec{A}	$B \cos \phi$	$\frac{\vec{A} \cdot \vec{B}}{A}$	$B \cos \phi = \frac{\vec{A} \cdot \vec{B}}{A}$
26	Vector product of two vectors	$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$		
27	Vector product of two perpendicular vectors	$\vec{A} \times \vec{B} = AB \hat{n}$		
28	Vector product of unit vectors \hat{i} and \hat{k}	$\hat{i} \times \hat{k} = -\hat{j}$	$\hat{k} \times \hat{i} = \hat{j}$	$\hat{j} \times \hat{k} = \hat{i}$
29	Vector product of two parallel or anti-parallel vectors	$\vec{A} \times \vec{A} = \vec{0}$		
30	Self cross product of vector \vec{A}	$\vec{A} \times \vec{A} = \vec{0}$		
31	Self Vector product of unit vectors $\hat{i}, \hat{j}, \hat{k}$	$\hat{i} \times \hat{i} = \vec{0}$	$\hat{j} \times \hat{j} = \vec{0}$	$\hat{k} \times \hat{k} = \vec{0}$
32	Vector product in terms of rectangular components	$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$		
33	Angle between two vectors \vec{A} and \vec{B}	$\sin \theta = \frac{ \vec{A} \times \vec{B} }{AB}$		
34	Area of a parallelogram and vector product	Area of a parallelogram = $ \vec{A} \times \vec{B} = AB \sin \theta$		
35	Torque	$\vec{\tau} = \vec{r} \times \vec{F}$	$\tau = r F \sin \theta$	$\vec{\tau} = r F \sin \theta \hat{n}$

36	1 st condition of equilibrium	$\sum \vec{F}$	$\sum \vec{F} = 0, \sum \vec{\tau} = 0$
37	2 nd condition of equilibrium	$\sum \vec{\tau}$	

Key Points

- Cartesian coordinates system provides a framework to sketch vector quantities
- Head to tail rule is a geometrical approach to add vectors
- Addition of vectors by rectangular components method is somehow mathematical method
- Scalar product and vector product helps to understand the physical difference between physical quantities
- e.g. work = Force \times Displacement = $\vec{F} \cdot \vec{s} = F s \cos \theta$
- and torque = Moment arm \times Force = $\vec{r} \times \vec{F}$
- The state in which an object has zero acceleration is called static equilibrium
- If the sum of all the forces acting on an object is equal to zero then the object is said to be in equilibrium
- i.e. $\sum \vec{F} = 0$
- If the sum of all the torques acting on an object is equal to zero then the body is said to be in equilibrium
- i.e. $\sum \vec{\tau} = 0$
- For perfect equilibrium a body must satisfy both the above conditions
- i.e. $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$

Solved Examples

Example 2.1

A ship leaves port and travels 200 km at 30° north of east. Then it changes its direction and travels 350 km in a direction 140° north of east to reach destination. Calculate straight line distance covered by ship?

Given:

Procedure

1. Choosing scale

100 km = 1 cm

200 km = 2 cm \Rightarrow with the north of east

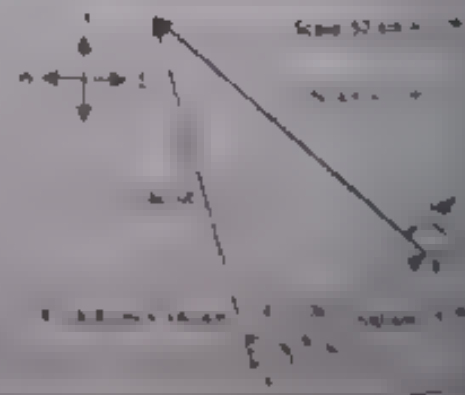
350 km = 3.5 cm \Rightarrow 140° with the north of east

To Find:

Resultant $\vec{R} = ?$

Solution

2. Finding the resultant Using head to tail rule to get the resultant \vec{R} . We measure the length of vector \vec{R} with scales which was about 6.8 cm = 6.8 \times 100 km = 680 km. As I was using protractor we also calculate the value of angle which is 106° with east



Answer:

34 km, 106° with north of east

36	1 st condition of equilibrium	$\sum \vec{F} = 0$	$\sum \vec{F}_x = 0 \quad \sum \vec{F}_y = 0$
37	2 nd condition of equilibrium	$\sum \vec{r} \times \vec{F} = 0$	



Key Points

- ❖ Cartesian coordinates system provides a framework to sketch vector quantities
- ❖ Head to tail rule is a geometrical approach to add vectors
- ❖ Addition of vectors by rectangular components method is somehow mathematical method
- ❖ Scalar product and vector product helps to understand the physical difference between physical quantities i.e.
 - e.g. work = Force \cdot Displacement = $\vec{F} \cdot \vec{s} = F s \cos \theta$
 - and torque = Moment arm \times Force = $\vec{r} \times \vec{F}$
- ❖ The state in which an object has zero acceleration is said to be in equilibrium
- ❖ If the sum of all the forces acting on an object is equal to zero then the object is said to be in equilibrium i.e. $\sum \vec{F} = 0$
- ❖ If the sum of all the torques acting on a body is equal to zero then the body is said to be in equilibrium i.e. $\sum \vec{r} \times \vec{F} = 0$
- ❖ For perfect equilibrium a body must satisfy both the following conditions i.e. $\sum \vec{F} = 0$ and $\sum \vec{r} \times \vec{F} = 0$



Solved Examples

Example 2.1:

A ship leaves port and travels 200 km at 30° north of east. Then it changes its direction and travels 350 km in a direction 140° north of east to reach destination. Calculate straight line distance covered by ship?

Given:

Procedure

1. Selecting scale:



Let 50 km = 1 cm

200 km = 4 cm, $\theta = 30^\circ$ with the north of east

350 km = 7 cm, $\theta = 140^\circ$ with the north of east

To Find:

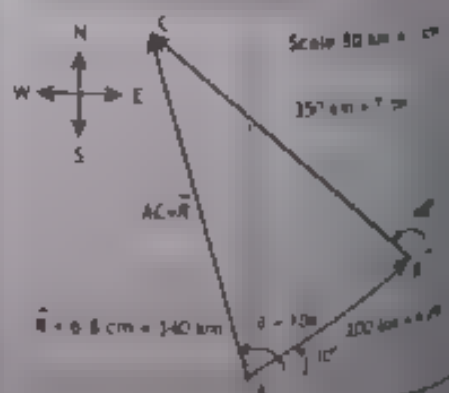
Resultant $\vec{R} = ?$

Solution:

2. Finding the resultant Using head to tail rule to get the resultant \vec{R} . We measure the length of vector \vec{R} (with scale) which was about 6.8 cm ($6.8 \times 50 = 340$ km), and with the protractor we also calculate the value of angle, which is $\theta = 106^\circ$ with east.

Answer:

34 km, 106° with north of east



Example 2.2

Two forces $F_1 = 18 \text{ N}$ making an angle $\theta_1 = 70^\circ$ with positive x axis and Force $F_2 = 25 \text{ N}$ making an angle $\theta_2 = 220^\circ$ with positive x axis. At a point calculate the resultant force.

Given

$$F_1 = 18 \text{ N}$$

$$F_2 = 25 \text{ N}$$

To find

$$R = ?$$

Solution

$$F_1 = 18 \text{ N}$$

$$F_2 = 25 \text{ N}$$

$$R = ?$$

$$\theta_1 = 70^\circ$$

$$\theta_2 = 220^\circ$$

$$R = ?$$

$$\theta_R = ?$$

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**Answer**

$$R = 37 \text{ N}$$

$$\theta_R = 30^\circ$$

$$R = 37 \text{ N}$$

$$\theta_R = 30^\circ$$

$$R = 37 \text{ N}$$

$$\theta_R = 30^\circ$$

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$$R = 37 \text{ N}$$

$$\theta_R = 30^\circ$$

$$R = 37 \text{ N}$$

Example 2.3

Find the angle between two forces of equal magnitude such that the magnitude of their resultant is also equal to either of them.

Solution

Let θ be the required angle between F_1 and F_2 such that $R = F_1 = F_2$.

To find their resultant, first we will find the angle θ between F_1 and F_2 .

$$|F_{1x}| = F \cos 0^\circ = F$$

and $|F_{1y}| = F \sin 0^\circ = 0 \text{ N}$

Similarly $|F_{2x}| = F \cos \theta$

And $|F_{2y}| = F \sin \theta$

It is $K_1 = F + F \cos \theta = F(1 + \cos \theta)$

And $K_2 = F_y + F \sin \theta = 0 + F \sin \theta = F \sin \theta$

Hence the magnitude of the resultant is

$$R = \sqrt{K_1^2 + K_2^2} = \sqrt{(F + F \cos \theta)^2 + (F \sin \theta)^2}$$

$$R = \sqrt{F^2 + F^2 \cos^2 \theta + 2F^2 \cos \theta + F^2 \sin^2 \theta}$$

$$\therefore R = F \sqrt{2 + 2 \cos \theta}$$

As stated in the question, $R = F_1 + F_2 = F + F$

Therefore Eq. (1) becomes as

$$F = \sqrt{F^2 + F^2 + 2F^2 \cos \theta} \quad \sqrt{2} F = F + F \cos \theta$$

$$\therefore 1 = 1 + \cos \theta$$

$$\cos \theta = 0 \quad \theta = 90^\circ$$

$$1 = 1 + \cos \theta$$

$$\cos \theta = 0 \quad \theta = 90^\circ$$

$$\therefore \theta = 90^\circ \quad \therefore \text{The angle between the forces must be } 90^\circ$$

Answer:

$$\theta = 90^\circ$$

Thus the angle between the forces must be 90°

Example 2.4

Vector A having magnitude 3.2 makes 50° with x axis and vector B with magnitude 5.2 makes 110° with x axis. What is the magnitude of the dot and cross products?

Given Vector $A = 3.2$ angle $= 50^\circ$ with x axis

Vector $B = 5.2$ angle $= 110^\circ$ with x axis

Solution

The angle θ is the smaller of the angle between two vectors as shown in Figure below

$$\theta = 110^\circ - 50^\circ = 60^\circ$$

The magnitude of dot product is

$$A \cdot B = AB \cos \theta$$

Putting values

$$A \cdot B = 3.2 \times 5.2 \cos 60^\circ$$

Hence $A \cdot B = 8.2$ **Answer**

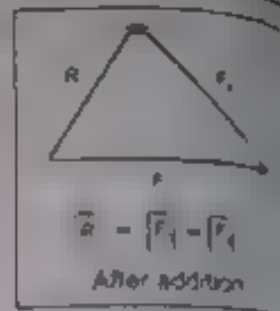
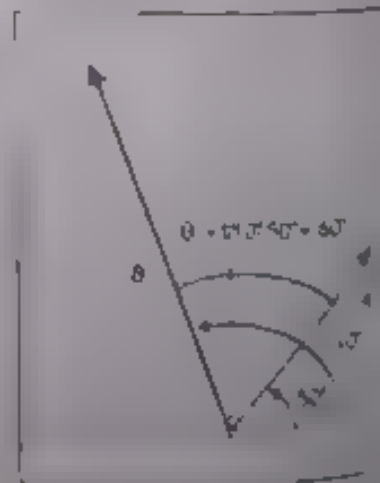


Fig. (a)



Fig. (b)



The magnitude of cross product is

$$|A \times B| = AB \sin \theta$$

Putting values $|A \times B| = (3.2)(5.18 \sin 50^\circ)$

Therefore $|A \times B| = 14$ **Answer:**

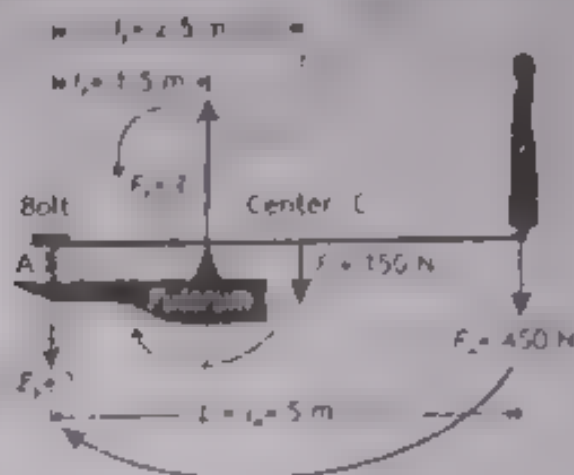
Example 2.6

Hina weighing 450 N is standing at the edge of the uniform diving board 5 m in length. Weight of the board is 150 N, and is bolted down at the left end, while being supported 1.50 m away by a fulcrum as in Figure. Find the forces that the bolt and the fulcrum exert on the board.

Given: Weight of Hina $F_2 = 450 \text{ N}$
 Weight of Board $F_1 = 150 \text{ N}$
 Center of gravity of board $L_1 = 2.5 \text{ m}$
 Distance of fulcrum $L_2 = 1.5 \text{ m}$
 Length of board $L = L_1 = 5 \text{ m}$

Find: (a) Resultant $R = ?$ F_1
 (b) $F_B = ?$

Solution: For the axis of rotation at point A, let the torque produced by support function is τ_1 , the torque produced by weight of board is τ_2 , and the torque produced by weight of Hina is τ_3 .



By second condition of equilibrium $\sum \tau = 0$

Therefore $\tau = \tau_1 = \tau_2 = \tau_3 = 0$

The sign convention is adopted.

Hence $(F_1)(L_1) = (F_2)(L_2) + (F_3)(L_3)$

or $(F_1)(L_1) = (F_2)(L_2) + (F_3)(L_3)$

or $F_1 = \frac{(F_2)(L_2) + (F_3)(L_3)}{L_1}$

Putting values $F_1 = \frac{(150)(2.5) + (450)(5)}{5}$

or $F_1 = \frac{1375 + 2250}{5}$

or $F_1 = \frac{3625}{5}$

Hence $F_1 = 725 \text{ N}$ **Answer:**

Now the force due to bolt F_B can be easily found out by solving 1st condition of equilibrium, that is $\sum F_y = 0$

that $\sum F_y = 0$ or $F_B + F_1 = F_2 + F_3$

Or $F_B = F_2 + F_3 - F_1$

Putting values $F_B = 150 \text{ N} + 450 \text{ N} - 725 \text{ N}$

Hence $F_B = 1150 \text{ N}$ **Answer:**

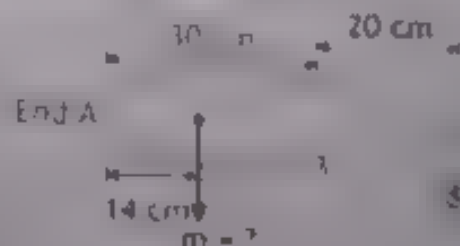


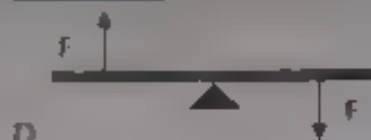
Text Book Exercises

Q 1 Select the correct answer of the following questions

Choose the best possible answer

- Two vectors lie with their tails at the same point. When the angle between them is increased by 20° the scalar product has the same magnitude but changes from positive to negative. The original angle between them was
A 10° B 60° C 70° D 80°
- The minimum number of vectors of unequal magnitude required to produce a zero resultant is
A 1 B 3 C 4 D 5
- If the resultant of two vectors, each of magnitude A is also a magnitude of A the angle between the two vectors will be
A 10° B 45° C 60° D 90°
- The magnitude of vector $\vec{A} = 2\vec{i} + \vec{j} + 2\vec{k}$ is
A 9 B 5 C 3 D 1
- When $P_1 = 3\text{ N}$ and $P = 5\text{ N}$ then $F_1 =$
A 6 N B 4 N C 2 N D 0 N
- A meter stick is supported by a knife edge at the 50-cm mark. And hangs masses of 0.40 kg and 0.60 kg from the 20-cm and 40-cm marks, respectively. Where should And hang a third mass of 0.30 kg to keep the stick balanced?
A 20 cm B 70 cm C 30 cm D 25 cm
- If $\vec{A} = 1.5\text{ cm}$, $\vec{A} \cdot \vec{B} = 1.0\text{ cm}$ into which quadrant does the vector \vec{B} point?
A I B II C III D IV
- $\vec{A} \cdot \vec{B} = ?$
A 0 B 1 C AB D A^2B
- Two forces of magnitude 26 N and 50 N act simultaneously on a body. Which one of the following forces cannot be a resultant of the two forces?
A 20 N B 30 N C 40 N D 70 N
- If the dot product of two nonzero vectors \vec{A} and \vec{B} is zero then the magnitude of their cross product is
A 0 B 1 C AB D $-AB$
- The sum of magnitudes of two forces is 16 N. If the resultant force is 8 N and its direction is perpendicular to the minimum force then the forces are
A 4 N and 12 N B 8 N and 8 N C 4 N and 12 N D 2 N and 14 N
- Find the mass of the uneven rod shown in the figure. If its center of gravity is 14 cm from end A and
A 10 g B 50 g C 10 g D 50 g
- The following diagrams show a uniform rod with its midpoint on the pivot. Two equal forces F are applied on the rod as shown in the figure. Which diagram shows the rod in equilibrium?





14. For which angle the equation $|\vec{A} \cdot \vec{B}| = |\vec{A} \times \vec{B}|$ is correct.

A. 0°

B. 45°

C. 60°

D. 90°

A

B

C

D

15. What is the net torque on wheel radius 2 m as shown?

A. 10 N anti-clockwise

B. 10 Nm anti-clockwise

C. 0 Nm clockwise

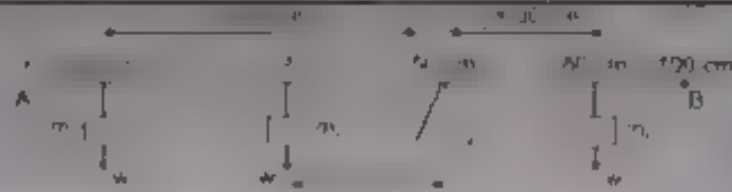
D. 5 Nm clockwise

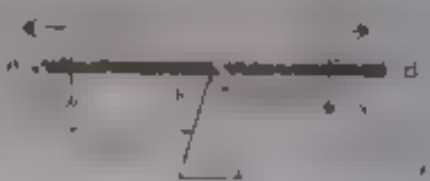


No. Option Answer

EXPLANATION

1	D	80	$ \vec{A} \cdot \vec{B} = \vec{A} \vec{B} \cos \theta$ $ \vec{A} \times \vec{B} = \vec{A} \vec{B} \sin \theta$ $ \vec{A} \vec{B} \cos \theta = \vec{A} \vec{B} \sin \theta$ $\cos \theta = \sin \theta$ $\theta = 90^\circ$
2	B	1	If they form a closed triangle
3	D	20	$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$ $\text{Put } R = B = A$ $A = \sqrt{A^2 + A^2 + 2AA \cos \theta}$ $A^2 = 2A^2 + 2A^2 \cos \theta$ $A^2 = 2A^2(1 + \cos \theta)$ $1 = 2(1 + \cos \theta)$ $1 = 2 + 2 \cos \theta$ $-1 = 2 \cos \theta$ $\cos \theta = -\frac{1}{2}$ $\theta = 120^\circ$
4	C	3	$A = 2 + 4 + 8$ $A = 14$ $A = \sqrt{2^2 + 4^2 + 8^2} = \sqrt{4 + 16 + 64} = \sqrt{84} = 9.16$
5	B	4N	$F_1 = 5 \text{ N and } F_2 = 5 \text{ N}$ $\text{from Pythagoras Theorem}$ $F^2 = F_1^2 + F_2^2$ $F^2 = 5^2 + 5^2 = 25 + 25 = 50$ $F = \sqrt{50} = 7.07 \text{ N}$

6.	c	Diagram	 <p> $\sum \tau = 0$ $\sum \tau_{\text{clockwise}} = \sum \tau_{\text{anticlockwise}}$ $(m_1 g \times r_1) + (m_2 g \times r_2) = (m_2 g \times r_2)$ $(m_1 \times 1.20) + (20 \times 0.40) = (20 \times 0.40)$ $m_1 \times 1.20 = 20 \times 0.40$ $m_1 = \frac{20 \times 0.40}{1.20} = 6.67 \text{ kg}$ </p>
7.	a	a	When x component of a vector lies in 4 th Quadrant,
8.	a		$A \times B = C$ for A and B perpendicular to each other. $A \times B = C$ for A and B perpendicular to each other. $A \times (B \times C) = A \cdot C$
9.	a	a	$A \times B = C$ for A and B perpendicular to each other. $A \times B = C$ for A and B perpendicular to each other.
10.	a	a	$A \times B = C$ for A and B perpendicular to each other. $A \times B = C$ for A and B perpendicular to each other.
11.	a	a	$A \times B = C$ for A and B perpendicular to each other. $A \times B = C$ for A and B perpendicular to each other.

12	A	B	$\Delta x \cdot W_T = \Delta y \cdot W_T$ $p \cos W_T = (Q \sin W_T)$ $(60 \times 10^3) \cos W_T = (Q \sin W_T)$ $100 \times 10^3 \cos W_T = 100 \times 10^3 \sin W_T$ $\tan W_T = 1$ $W_T = 45^\circ$ $m = \frac{23 \times 10^3}{0} = 1000$	
13			$\Delta x \cdot W_T = \Delta y \cdot W_T$	
14	3	4	$\vec{A} \cdot \vec{B} = AB \cos \theta$ $AB \cos \theta = AB \sin \theta$ $\frac{\sin \theta}{\cos \theta} = 1$ $\tan \theta = 1$ $\theta = 45^\circ$	
15		10Nm clockwise	$\text{c.w. torque} = -r \cdot F = -2 \times 10 = -20 \text{ Nm}$ $\text{a.w. torque} = r \cdot F = 2 \times 10 = 20 \text{ Nm}$ $\Sigma \tau = -20 + 20 = 0$ $\text{Net torque} = 0$	



Short Answer Questions

Q 2 Write short answers of the following questions

1. Is it possible to add three vectors of equal magnitude but different directions to get a vector? Illustrate with a diagram.

Ans: Yes, it is possible to add three vectors of equal magnitude but different directions to get a vector. Illustrate with a diagram.

Explanation:

Consider three vectors \vec{A} , \vec{B} and \vec{C} as shown in figure. It is clear that the sum of the vectors is zero because tail of the first vector coincides with the head of the last vector. There is no space to draw the resultant vector, so resultant will be a null vector.



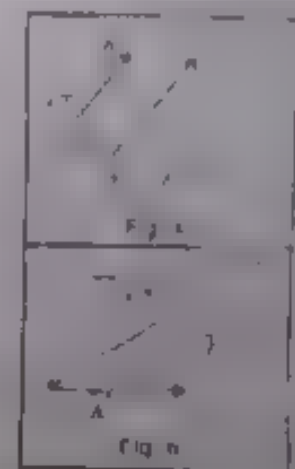
2. The magnitudes of three vectors are 2 m, 3 m and 6 m respectively. The directions are at your disposal. Can these three vectors be added to yield zero? Illustrate with a diagram.

Ans: Yes, it is possible.

Explanation:

If the three vectors are represented by the sides of triangle in order by tail-to-head, then they form a closed triangle. Their vector sum will be zero vector.

Consider three vectors \vec{A} , \vec{B} and \vec{C} as shown in figure. It is clear that sum of the vectors is zero because tail of the first vector coincides with the head of the last vector. There is no space to draw the resultant vector, so resultant will be a null vector.



3. What units are associated with the unit vectors \hat{i} , \hat{j} , and \hat{k} ?

Ans: Unit vectors are unitless and dimensionless vectors. So, no units are associated with unit vectors \hat{i} , \hat{j} , and \hat{k} , they are used only to give direction.

Explanation

Unit vector is given by formula

$$\hat{A} = \frac{\vec{A}}{A}$$

We see that unit vector is obtained by dividing a vector with its magnitude. As both vector and its magnitude have same units, the ratio gives no unit of unit vector.

That is why \hat{i} , \hat{j} and \hat{k} have no units. They give direction of components of vector along axis.

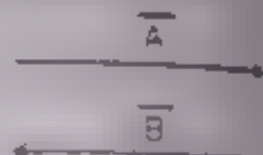
4. Can a scalar product of two vectors be negative? Provide a proof and give an example.

Ans: Yes, scalar product of two vectors can be negative.

Proof:

If two vectors are antiparallel ($\theta = 180^\circ$), then the scalar product is negative.

$$\begin{aligned}\vec{A} \cdot \vec{B} &= AB \cos 180^\circ \\ &= AB(-1) = -AB\end{aligned}$$



Example

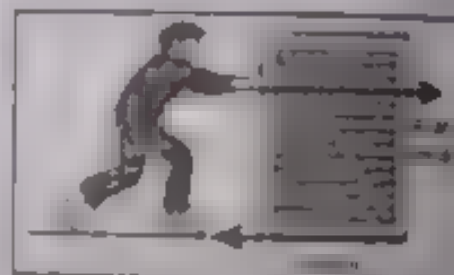
If a body is pushed against the ground, then force \vec{F} acts opposite to the displacement \vec{d} covered by the body. Then work done by the friction is

$$\begin{aligned}W &= \vec{F} \cdot \vec{d} \\ &= Fd \cos 180^\circ\end{aligned}$$

Since friction and displacement oppose each other, $\theta = 180^\circ$.

$$\therefore W = Fd \cos 180^\circ = Fd(-1)$$

$$\Rightarrow W = -Fd$$



5. A and B are two non zero vectors. How can their scalar product be zero? And how can the vector product be zero?

Ans: 1a) Scalar Product Case

Yes, the scalar product of two non zero vectors can be zero if both vectors are perpendicular to each other.

Perpendicular vectors

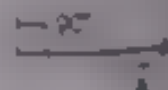
If two vectors are mutually perpendicular ($\theta = 90^\circ$) to each other then their scalar product is zero.

$$\vec{A} \cdot \vec{B} = AB \cos 90^\circ$$

$$\vec{A} \cdot \vec{B} = AB(0)$$

$$\vec{A} \cdot \vec{B} = 0$$

$\vec{B} \uparrow$



1b) Vector Product Case

Yes, if product of two vectors can not be zero because zero is a number, not a vector and vector product always gives vector. It can give null vector.

But magnitude of vector product can be zero if both vectors are parallel or anti parallel.

Parallel and Anti-parallel Vectors

The cross product of two parallel ($\theta = 0^\circ$) or two anti parallel ($\theta = 180^\circ$) vectors is a null vector, i.e.

• In case of parallel vectors

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n} = AB(0) \hat{n} = 0 \hat{n} = \vec{0}$$

So, in this case, magnitude of $\vec{A} \times \vec{B}$ is equal to zero.

- In case of anti-parallel vectors

$$\vec{A} \cdot \vec{B} = AB \sin 180^\circ \hat{n} = AB(0) \hat{n} = 0 \hat{n} = \vec{0}$$

So, in this case, magnitude of $\vec{A} \times \vec{B}$ is equal to zero.

6. Suppose you are given a known non zero vector \vec{A} . The scalar product of \vec{A} with an unknown vector \vec{B} is zero. Likewise, the vector product of \vec{A} with \vec{B} is zero. What can you conclude about \vec{B} ?

Ans: Vector \vec{B} is a null vector.

Reason

Case 1

- If \vec{B} is non-zero vector and \vec{A} is perpendicular to \vec{B} then their scalar product is zero. If two vectors are mutually perpendicular then their dot product is zero.

$$\vec{A} \cdot \vec{B} = AB \cos 90^\circ$$

$$\vec{A} \cdot \vec{B} = AB(0)$$

$$\vec{A} \cdot \vec{B} = 0$$



Case 2

- If \vec{B} is non-zero vector and if \vec{A} is parallel to \vec{B} then the magnitude of their vector product will be zero. In case of parallel vectors

$$\vec{A} \cdot \vec{B} = AB \sin 0^\circ \hat{n} = AB(0) \hat{n} = 0 \hat{n} = \vec{0}$$

So, in this case, magnitude of $\vec{A} \times \vec{B}$ is equal to zero.

$$\vec{A} \times \vec{B} = AB \sin 180^\circ \hat{n} = AB(0) \hat{n} = 0 \hat{n} = \vec{0}$$

So, in this case, magnitude of $\vec{A} \times \vec{B}$ is equal to zero.

- From above studies we conclude that these vector can never be perpendicular or anti-parallel simultaneously. Therefore, vector \vec{B} is a null vector. In vector magnitude, scalar product and vector product with non-zero vector \vec{A} is the zero.

7. Why a particle experiencing only one force cannot be in equilibrium?

Ans: Yes, A particle experiencing only one force cannot be in equilibrium.

Explanation

A body will be in equilibrium if net force is zero, as according to first condition of equilibrium

$$\sum \vec{F} = 0$$

But under action of single force, net force cannot be zero.

As a body will have acceleration. Therefore, body can't be in equilibrium.

8. To open a door that has the handle on the right and the hinges on the left a torque must be applied. Is the torque clockwise or counterclockwise when viewed from above? Does your answer depend on whether the door opens toward or away from you?

Ans: Consider the door that has the handle on the right and the hinges on the left as shown. When we pull the door, it rotates due to applied torque.

When Door is Pulled Towards Us.

When we pull the handle of the door towards us to close it, the door will rotate clockwise (Moment of force will be inward by Right Hand Rule).

When Door is Pulled Towards Us.

When we push the door away from us to open it, the door will rotate anticlockwise (Moment of force will be outward by Right Hand Rule).

Does your answer depend on whether the door opens toward or away from you?

Yes, the direction of torque will be reversed if door opens or closes opposite to the above-mentioned situation.

8 Explain the warning: 'Never use a large wrench to tighten a small bolt.'

Ans: We should not use large wrench to tighten a small bolt because it may damage the nut due to application of large torque.

Explanation:

We know that torque acting on the body is

$$\tau = r \times F$$

$$\Rightarrow \text{Torque} = (\text{Moment arm}) (\text{magnitude of } F)$$

It is obvious from above equation that large wrench will provide greater moment arm. Hence, a pl. greater torque on nut & will damage it. This greater torque acting on the nut is unwanted.

10. A central force is one that is always directed toward the same point. Can a central force give rise to a torque about that point?

Ans: No, central force is a vector point force can not apply torque on the body.

Example: Gravitational force is example of central force. Since Gravitational force between earth and satellite acts in centre-point. Moment arm (displacement) between earth and satellite is perpendicular to the line that makes angle of 180° with the gravitational force.

Formula for torque:

$$\tau = r \times F \sin \theta$$

Since $\theta = 180^\circ$

So, $\tau = r \times F \sin 180^\circ$

$$\tau = 0$$

$$\tau = 0$$

\therefore we conclude that central force can not apply torque on any body.



Comprehensive Questions

Q3. Give a short response to the following questions.

1 Define what is meant by vectors. Give five example of each. Also with the help of an example explain the graphical representation of a vector.

Ans: See Q. # 1, 2

2 Explain in detail whether for two vectors of equal magnitude is it possible to give a resultant of magnitude equal to their individual magnitude. Justify your answer mathematically.

Ans: Yes, it is possible if angle between these vectors of equal magnitudes is 120° .

Reason: The magnitude of the resultant vector is given by:

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

If $F_1 = F_2 = F$, then

$$R = \sqrt{F^2 + F^2 + 2F^2 \cos \theta}$$

$$R = \sqrt{F^2 + 2F^2 \cos \theta}$$

$$R = \sqrt{F^2 (1 + 2 \cos \theta)}$$

$$R = F \sqrt{1 + 2 \cos \theta}$$

$$R = 2F \cos 60^\circ$$

$$\frac{3}{2} = \frac{C}{\sin 60^\circ}$$

$$\frac{3}{2} = \frac{C}{\sin 60^\circ}$$

$$\frac{3}{2} = \frac{C}{\sin 60^\circ}$$

$$C = 1.5 \text{ m}$$

$$B = 20$$

4. Two vectors of equal magnitude will give resultant of the magnitude equal to the magnitude of the vector.

3. What does rectangular components of a vector mean? Explain addition of vectors by rectangular components.

Ans: See Q. 2. (i)

4. Explain scalar product of two vectors.

Ans: See Q. 2. (ii)

5. Explain vector product of two vectors.

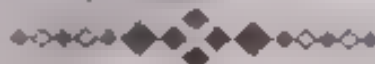
Ans: See Q. 2. (iii)

6. Define torque. Explain why it is equal to the vector product of force and moment arm.

Ans: See Q. 2. (iv)

7. Write the solution steps of a numerical problem of equilibrium.

Ans: (i) First draw vector diagram for all forces involved in problem.
 (ii) Draw their rectangular components.
 (iii) Apply 1st condition of equilibrium i.e. $\Sigma F = 0$
 (iv) Take $\Sigma F_x = 0$ and $\Sigma F_y = 0$ (if required)
 (v) then solve the obtained equation to get the answer.



NUMERICAL QUESTIONS

1. A person throws a ball straight up with a speed of 12 m/s. If the bus is moving at 25 m/s, what is the velocity of the ball to an observer on a ground?

Solution:

Given Data:

$$\text{Velocity of ball} = V_{\text{ball}} = 12 \text{ m/sec}$$

$$\text{Velocity of bus} = V_{\text{bus}} = 25 \text{ m/sec}$$

To Find

$$\text{Velocity of ball to an observer on ground} = V_{\text{observer}} = ?$$

Calculation

Let the ball is thrown straight North and the bus is moving East. Here we add the velocities of ball and bus by head to tail rule as shown in figure.

Now for finding the resultant velocity we combine the tail of first vector OA with the head of last vector AB. Thus OAB gives us the resultant velocity or relative velocity of the ball to an observer on ground. Now

$$(AB) = (OA) + (AB)$$

$$\Rightarrow V_{\text{rel}} = V_{\text{ball}} + V_{\text{bus}} = 25 \text{ m/sec} + (12 \text{ m/sec})$$

$$\Rightarrow V_{\text{rel}} = 37 \text{ m/sec} \quad \text{or} \quad 37 \text{ m/sec}$$



$$V_x = 76 \text{ m/sec}$$

$$V_y = 6.1 \text{ m/sec}$$

$$V = 76.2 \text{ m/sec}$$

$$\theta = 4.6^\circ$$

$$V_x = 76 \text{ m/sec}$$

$$V_y = 6.1 \text{ m/sec}$$

$$\theta = 4.6^\circ$$

$$\theta = 25.1^\circ \text{ or } 26^\circ$$

the vector V is at an angle of 26° with the positive x -axis.

2. A football leaves the foot of a punter at an angle of 54° (positive x -direction) at a speed of 21 m/s . Determine the horizontal and vertical components of the velocity.

Solution

Given Data

$$\text{Velocity of football} = V = 21 \text{ m/sec}$$

$$\text{Angle of } V \text{ with } x\text{-axis} = \theta = 54^\circ$$

To Find

$$\text{Horizontal component of velocity } V_x$$

$$\text{Vertical component of velocity } V_y$$

Calculation

$$\cos V = \frac{V_x}{V} \quad \text{or} \quad V_x = V \cos \theta$$

$$\text{Using the value of } \theta \text{ and } V \text{ we get}$$

$$V_x = (21) \cos 54^\circ = 12.3 \text{ m/sec}$$

$$V_y = V \sin \theta \quad \text{or} \quad V_y = V \sin 54^\circ$$

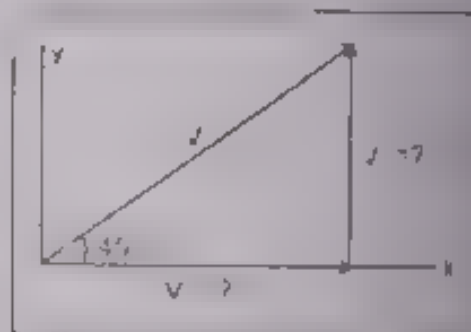
$$\text{We also know that}$$

$$V = V_x \sin \theta \quad \text{or} \quad V_y = 17.7 \text{ m/sec}$$

$$\text{Using the value of } \theta \text{ and } V \text{ we get}$$

$$V_y = (21) \sin 54^\circ = 17.7 \text{ m/sec}$$

$$V_x = 12.3 \text{ m/sec} \text{ and } V_y = 17.7 \text{ m/sec}$$



3. A 1.84-kg school bag hangs in the middle of a clothesline, causing it to sag by an angle $\theta = 35.0^\circ$. Find the tension T in the clothesline. (148 N)

Solution

Given Data

$$\text{Mass of bag } m = 1.84 \text{ kg}$$

$$\text{Weight of bag } w = mg = 1.84 \times 9.8 \text{ N} = 18 \text{ N}$$

$$\text{Angle } \theta = 35^\circ$$

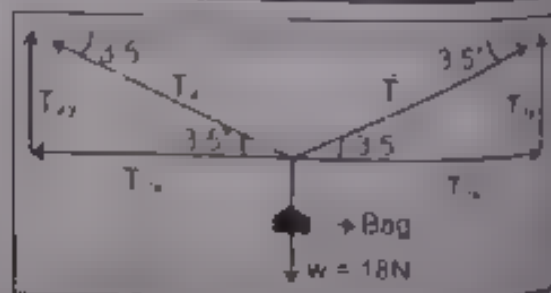
To Find

$$\text{Tension in the clothesline}$$

Calculation

Let $T_1 = T_2 = T$ be the tension in each part of the cloth line. Here we resolve the tension in each part into their horizontal and vertical components, as shown in the figure. As the system is in equilibrium state, we apply the first condition of equilibrium, i.e.

$$\sum F_x = 0 \Rightarrow T_1 \cos \theta + T_2 \cos \theta - 0 \Rightarrow T_1 = T_2 \quad (1)$$



$$\sum F_x = 0 \Rightarrow \sum T = 0 \Rightarrow T_1 + T_2 + (-w) = 0$$

$$\Rightarrow T_1 + T_2 - w = 0 \Rightarrow T \sin \theta + T \sin \theta_2 = w$$

$$\Rightarrow T \sin \theta + T \sin \theta = w \Rightarrow 2T \sin \theta = w$$

$$\theta = \theta = \theta$$

$$T = T = T_1$$

$$\Rightarrow T = \frac{w}{2 \sin \theta} = \frac{8}{2 \sin 35^\circ} \text{ N}$$

$$\Rightarrow T = \frac{8}{2 \times 0.57} \text{ N} = \frac{18}{0.57} \text{ N} = 147 \text{ N}$$

$$[T = 147 \text{ N}] \text{ or } [T = 148 \text{ N}]$$

4. Find the magnitude and direction of vector represented by the following pair of components

(a) $A_x = -2.3 \text{ cm}$, $A_y = +4.1 \text{ cm}$

(b) $A_x = +3.9 \text{ m}$, $A_y = -1.8 \text{ m}$

(a) $A = 4.7$ and $\theta_A = 119.3^\circ$

(b) $A = 4.3$ and $\theta_A = 335.2^\circ$

Solution.

Given Data

(a) $A_x = -2.3 \text{ cm}$, $A_y = +4.1 \text{ cm}$

(b) $A_x = +3.9 \text{ m}$, $A_y = -1.8 \text{ m}$

To Find:

Magnitude of vector in each case $A = ?$

Direction of A in each case $\theta = ?$

Calculation:

(a) We know that the magnitude of a vector is given by

$$A = \sqrt{A_x^2 + A_y^2} \quad (1)$$

Putting $A_x = -2.3 \text{ cm}$ and $A_y = +4.1 \text{ cm}$ in equation (1), we get

$$A = \sqrt{(-2.3 \text{ cm})^2 + (4.1 \text{ cm})^2} = \sqrt{5.29 \text{ cm}^2 + 16.81 \text{ cm}^2}$$

$$\Rightarrow A = \sqrt{5.29 + 16.81 \text{ cm}^2} = \sqrt{22.1 \text{ cm}^2} = 4.7 \text{ cm}$$

$$\Rightarrow [A = 4.7 \text{ cm}]$$

We also know that

$$\theta = \tan^{-1} \frac{A_y}{A_x} = \tan^{-1} \frac{4.1}{-2.3} = \tan^{-1} (-1.78) \Rightarrow \theta = 60.7^\circ$$

As the x-component of A is negative and y-component is positive, so vector A lies in second quadrant. Thus the actual angle made by the vector A with positive x-axis is given by

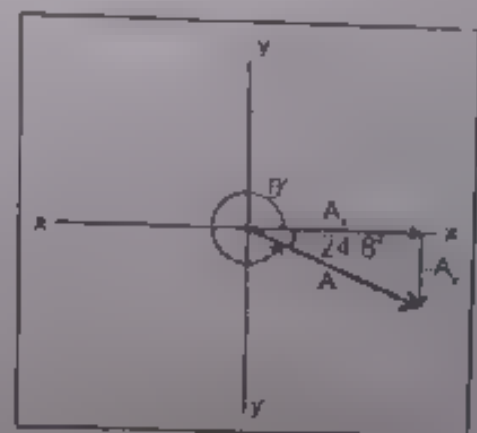
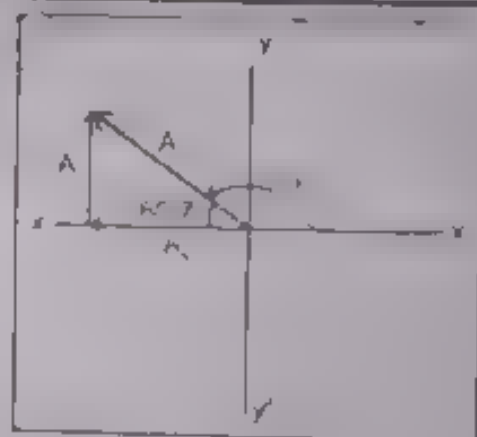
$$\theta' = 180^\circ - \theta \Rightarrow \theta = 180^\circ - 60.7^\circ \Rightarrow \theta' = 119.3^\circ$$

(b) Now putting $A_x = +3.9 \text{ m}$ and $A_y = -1.8 \text{ m}$ in equation (1), we get,

$$A = \sqrt{(3.9 \text{ m})^2 + (-1.8 \text{ m})^2} = \sqrt{15.21 \text{ m}^2 + 3.24 \text{ m}^2}$$

$$\Rightarrow A = \sqrt{18.45 \text{ m}^2}$$

$$\Rightarrow A = 4.3 \text{ m}$$



$$\therefore 1 = -\sin \theta = 24.8^\circ$$

As the x component of A is positive and y component is negative so the vector A lies in fourth quadrant. Therefore, the actual angle made by the vector A with positive x axis is given by,

$$\theta = 360^\circ - 24.8^\circ$$

$$\theta = 335.2^\circ$$

$$\theta = 335^\circ$$

5. Vector F having magnitude 5.5 N makes 10° with x-axis and vector r with magnitude 4.3 N makes 80° with x-axis. What is the magnitude of their dot and cross products?

Solution

Given Data

Magnitude of vector F = $F = 5.5 \text{ N}$, $\theta_F = 10^\circ$

Magnitude of vector r = $r = 4.3 \text{ N}$, $\theta_r = 80^\circ$

To Find

(a) $F \cdot r = ?$

(b) $F \times r = ?$

Calculation

The angle between F and r is given by

$$= \theta_r - \theta_F$$

$$= 80^\circ - 10^\circ$$

(a) The magnitude of dot product of F and r is given by,

$$F \cdot r = F r \cos \theta$$

Putting the values in eqn (1) we get

$$\begin{aligned} F \cdot r &= 5.5 \times 4.3 \times \cos 70^\circ \\ &= 8.09 \text{ Nm} \end{aligned}$$

$$F \cdot r = 8.09 \text{ Nm} \quad \text{or} \quad F \cdot r = 8.1 \text{ Nm}$$

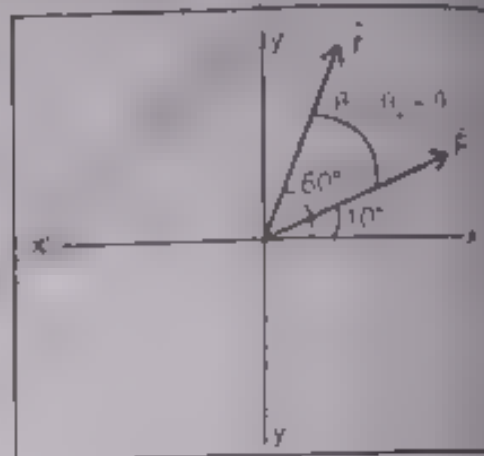
(b) The magnitude of cross product of F and r is given by

$$F \times r = F r \sin \theta$$

Putting the values in equation (2) we get

$$F \times r = 5.5 \times 4.3 \times \sin 70^\circ = 22.2 \text{ Nm}$$

$$F \times r = 22.2 \text{ Nm}$$



6. The magnitude of dot and cross product of two vectors A, B and C respectively. Find the angle between the vectors.

Given data Let A and B be two given vectors.

$$\text{Magnitude of dot product of two vectors} = |\vec{A} \cdot \vec{B}| = 6\sqrt{3}$$

$$\text{Magnitude of cross product of two vectors} = |\vec{A} \times \vec{B}| = 6$$

To Find The angle between two vectors = $\theta = ?$

Calculation

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta = 6\sqrt{3} \quad (1)$$

$$10 \times 1 = 10 \text{ N}$$

$$15 \times 1 = 15 \text{ N}$$

$$10 \times 1 = 10 \text{ N}$$

$$15 \times 1 = 15 \text{ N}$$

$$10 \times 1 = 10 \text{ N}$$

$$15 \times 1 = 15 \text{ N}$$

$$10 \times 1 = 10 \text{ N}$$

$$15 \times 1 = 15 \text{ N}$$

$$10 \times 1 = 10 \text{ N}$$

$$15 \times 1 = 15 \text{ N}$$

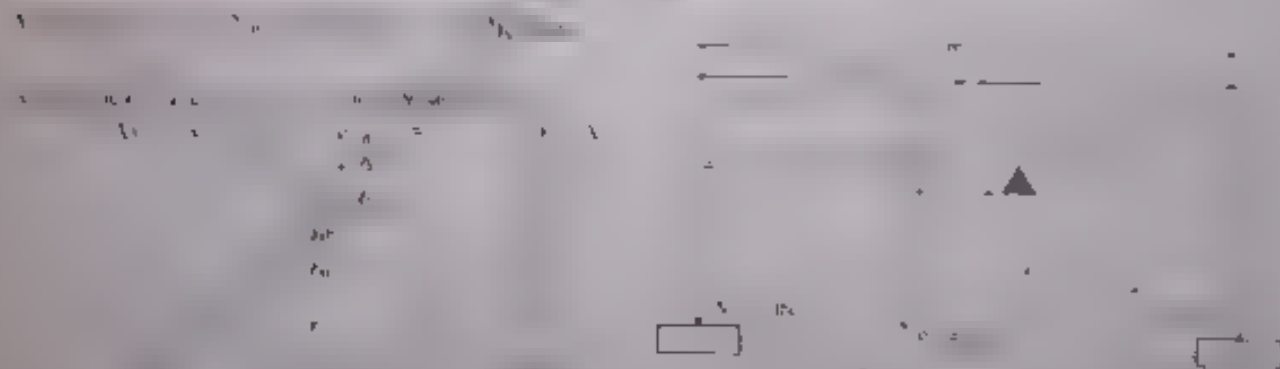
7. A uniform rod 1 m long with weight 6 N can be supported in a horizontal position on a sharp edge with weights of 10 N and 15 N suspended from its ends. What is the position of point of balance?

Solution

Given Data



To Find



8. A 4.0 m long uniform ladder with weight of 120 N leans against a wall making 70° above a cement floor as shown in Figure. Assuming the wall is frictionless but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.

Solution

Given Data

$$\text{Length of ladder} = 4.0 \text{ m}$$

$$\text{Weight of ladder} = 120 \text{ N}$$

$$\text{Angle with floor} = 70^\circ$$

Required

Force exerted by the wall on the ladder

Force exerted by the floor on the ladder

Calculation

$$\text{Force exerted by the wall} = F_w$$

$$\text{Force exerted by the floor} = F_f$$

$$\text{Weight of ladder} = 120 \text{ N}$$

$$\text{Length of ladder} = 4.0 \text{ m}$$

$$\text{Angle with floor} = 70^\circ$$

$$\Rightarrow F_x - F_{\text{wall}} = 0 \quad \Rightarrow F_x = F_{\text{wall}} \quad (1)$$

$$\text{And } \sum F_y = 0 \Rightarrow F_y - w = 0 \Rightarrow F_y = w = 0$$

$$\Rightarrow F_y = 120 \text{ N} = 0 \quad \Rightarrow F_y = 120 \text{ N} \quad (2)$$

Now applying second condition of equilibrium we get Sum of anticlockwise moment = Sum of clockwise moment

$$\Rightarrow AE \cdot F_{\text{wall}} = AD \cdot w$$

$$\Rightarrow F_{\text{wall}} = \frac{AD \cdot w}{AE} \quad (3)$$

Now from the figure we have $AD = AC \cos \theta$

$$\Rightarrow AD = \frac{1}{2} \cos 70^\circ = \frac{1}{2} \cos 70^\circ = 2 \cos 70^\circ = 2 \cdot 0.342 = 0.684$$

$$\Rightarrow AD = 0.684 \text{ m}$$

$$\text{And } AE = CB = AB \sin \theta = 1.5 \sin 70^\circ = 1.423 \text{ m} \Rightarrow AE = 1.423 \text{ m}$$

Putting the values of AD , AE and w in equation (3) we get

$$F_{\text{wall}} = \frac{0.684 \cdot 120}{1.423} = 58.2 \text{ N}$$

Putting the value of F_{wall} in equation (1) we get

$$F_x = 58.2 \text{ N}$$

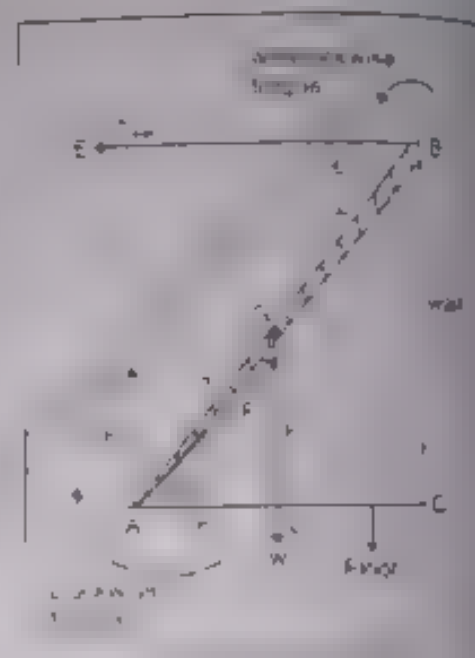
$$(b) \text{ Now } F = \sqrt{F_x^2 + F_y^2}$$

Putting equation (2) and equation (4) in equation (5), we get,

$$F = \sqrt{(21.8)^2 + (120)^2} \text{ N}$$

$$\Rightarrow F = \sqrt{475.24 + 14400} \text{ N} = \sqrt{14875.24} \text{ N}$$

$$\Rightarrow F = 121.96 \text{ N} \approx F = 122 \text{ N}$$



9. The 450-kg uniform I-beam supports the load of 220 kg as shown. Determine the reactions at the supports (2850 N)

Solution:

Given Data

$$\text{Mass of beam} = M = 450 \text{ kg}$$

$$\text{Weight of beam } w = mg = 450 \cdot 9.8 \text{ N}$$

$$\Rightarrow w = 4410 \text{ N}$$

$$\text{Mass of load } m = 220 \text{ kg}$$

$$\text{Weight of load } w_1 = mg = 220 \cdot 9.8 \text{ N}$$

$$\Rightarrow w_1 = 2156 \text{ N}$$

$$\text{Moment arm of } w = r = 4 \text{ m}$$

$$\text{Moment arm of } w_1 = r_1 = 6 \text{ m}$$

$$\text{Moment arm of } R_B = r_2 = 8 \text{ m}$$

Required:

$$(a) \text{ Reaction at 'B'} = R_B = ?$$

$$(b) \text{ Reaction of 'A'} = R_A = ?$$

Calculation:

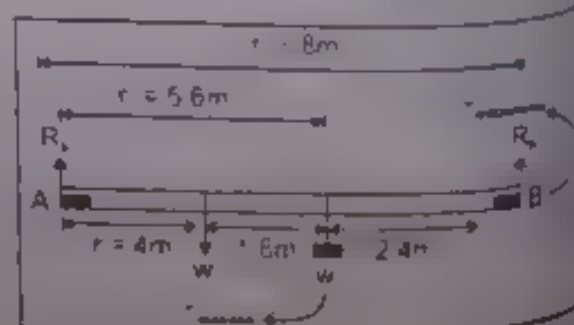
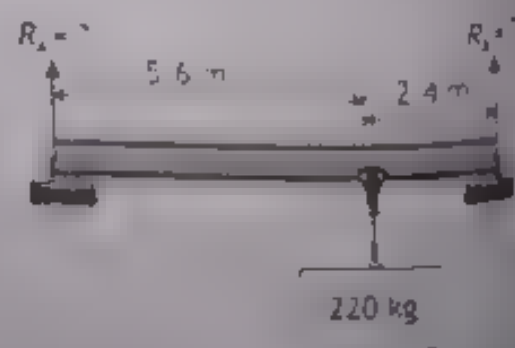
(a) Applying second condition of equilibrium we get

$$\tau_{\text{anticlockwise}} = \tau_{\text{clockwise}}$$

$$\Rightarrow R_B \cdot r_2 = (w \cdot r_1 + w_1 \cdot r_1)$$

$$\Rightarrow R_B = \frac{(w \cdot r_1 + w_1 \cdot r_1)}{r_2} \quad (1)$$

Putting the values in equation (1), we get,



$$\Rightarrow R_A = \frac{29713.6}{8} \text{ N} \Rightarrow R_A = 3714.2 \text{ N} \text{ or } R_B = 3714 \text{ N}$$

Now applying first condition of equilibrium, we get,

$$\sum F_x = 0 \Rightarrow R_A + R_B + (-W) + (-W_1) = 0$$

$$\Rightarrow R_A + R_B - W - W_1 = 0 \Rightarrow R_A = W + W_1 - R_B$$

$$\Rightarrow R_A = (4410 + 2156 - 3714) \text{ N} = 6566 - 3714 \text{ N}$$

$$\Rightarrow R_A = 2852 \text{ N}$$



Additional Conceptual Short Questions With Answers

1. A plane is moving at 100 km hr^{-1} at an angle of 60° with the ground. Then what is speed of its projection (shadow) on the ground if the sun is just above the plane at noon?

Ans: It means that projection of plane is moving on ground along x-axis. So, we are to find x-component of the velocity of the plane i.e., V_x is the velocity of the projection.

Velocity of plane = $V = 100 \text{ km h}^{-1}$

So, velocity of projection is

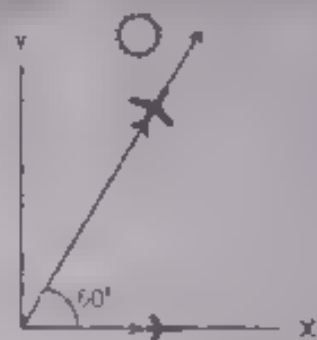
$$V_x = V \cos \theta$$

$$\Rightarrow V_x = 100 \text{ h}^{-1} (\cos 60^\circ)$$

$$= (100) \times \frac{1}{2}$$

$$V_x = 50 \text{ km h}^{-1}$$

So, velocity of projection on the ground is 50 km h^{-1}



2. Show that $A \times B + A \cdot B = A \cdot B$

Ans: $\left| \vec{A} \times \vec{B} \right| = AB \sin \theta$

$$\vec{A} \times \vec{B} = AB \sin \theta \quad (1)$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\vec{A} \cdot \vec{B} = AB \cos^2 \theta \quad (2)$$

Adding eq (1) and (2)

$$\vec{A} \times \vec{B} + \vec{A} \cdot \vec{B} = AB \sin^2 \theta + AB \cos^2 \theta$$

$$= AB (\sin^2 \theta + \cos^2 \theta)$$

$$\vec{A} \times \vec{B} + \vec{A} \cdot \vec{B} = AB (1)$$

$$\vec{A} \times \vec{B} + \vec{A} \cdot \vec{B} = AB$$

3. Under what circumstances would a vector has components that are equal in magnitude?

Ans: It is possible only when the vector makes an angle of 45° with x-axis

Proof

Let A_x and A_y be the magnitude of rectangular components of vector \vec{A}

If $A_x = A_y$

OR $A \sin \theta = A \cos \theta$

OR $\sin \theta = \cos \theta$

OR $\sin \theta = \cos \theta$

OR $\tan \theta = 1$

$\theta = 45^\circ$



OR $\tan \theta = 1$

OR $\theta = \tan^{-1}(1)$

$\Rightarrow \theta = 45^\circ$

4. Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length?

Ans Consider two vectors \vec{A} and \vec{B} as shown in figure.
By using head-to-tail rule

$$\vec{R} = \vec{A} + \vec{B} \quad \text{and} \quad \vec{R}' = \vec{A} - \vec{B}$$

Given Data, $A = B$

Angle between two vectors \vec{A} and \vec{B} is 90°

Proof

Magnitude of \vec{R}

$$R = \sqrt{A^2 + B^2} = \sqrt{A^2 + A^2} \quad \dots$$

Magnitude of \vec{R}'

$$R' = \sqrt{A^2 + B^2} = \sqrt{A^2 + A^2} \quad (2)$$

From equations (1) & (2) it is clear

$$R = R' \quad (1)$$

Since $A = B$

$$\angle M = \angle N = 45^\circ$$

Therefore

the angle between \vec{R} and \vec{R}' is $\angle N = 90^\circ$

$$\therefore \angle M + \angle N = 45^\circ + 45^\circ$$

$$\therefore \angle N = 90^\circ$$

$\therefore \vec{R}$ and \vec{R}' are perpendicular to each other

Hence proved.



5. Name the three different conditions that could make $\vec{A} \times \vec{A} = \vec{0}$?

Ans If \vec{A} and \vec{A} are two vectors then

$$\vec{A} \times \vec{A} = \vec{0} \text{ or } 0$$

Conditions

$\vec{A}_1 \times \vec{A}_2$ is zero if

1) \vec{A}_1 or \vec{A}_2 is a null vector

2) \vec{A}_1 and \vec{A}_2 are parallel ($\theta = 0^\circ$ or 180°)

3) \vec{A}_1 and \vec{A}_2 are anti-parallel ($\theta = 0^\circ$ or 180°)

4) \vec{A}_1 and \vec{A}_2 are equal vectors

$$\vec{A} \times \vec{A} = A A \sin 0^\circ$$

$$= A A \sin 0^\circ = 0$$

$$\vec{A}_1 \times \vec{A}_2 = A_1 A_2 \sin 180^\circ$$

$$= A_1 A_2 \sin 0^\circ = 0$$

6. Can a body rotate about its center of gravity under the action of its weight?

Ans No, it is not possible

Reason

In this case, the line of action of force (weight) passes through pivot point (center of gravity), so the moment becomes zero.

$$\text{As } \tau = rF$$

$$\text{So } \tau = (0)(1) \quad (\text{as } r = 0)$$

$$\text{OR } \tau = 0$$

Hence the torque will also be zero.

7. If $\vec{A} + \vec{B} = \vec{A} - \vec{B}$ then what is the angle between \vec{A} and \vec{B} ?

Ans As $\vec{A} + \vec{B} = \vec{A} - \vec{B}$
 $(\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})$
 $A^2 + B^2 + 2\vec{A} \cdot \vec{B} = A^2 + B^2 - 2\vec{A} \cdot \vec{B}$
 $4\vec{A} \cdot \vec{B} = 0$
 $\vec{A} \cdot \vec{B} = 0$

Since the scalar product of \vec{A} and \vec{B} is equal to zero, \vec{A} and \vec{B} are at right angles to each other.

8. If $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ are perpendicular to each other, show that \vec{A} and \vec{B} are of same magnitude?

Ans As $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ are mutually perpendicular to each other then

$$(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$$

$$\vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - \vec{B} \cdot \vec{B} = 0$$

Or $\vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - \vec{B} \cdot \vec{B} = 0$
 or $A^2 - B^2 = 0$
 or $A^2 = B^2$
 or $A = B$

So \vec{A} and \vec{B} are of equal magnitude.

9. Explain dot product of unit vectors.

Ans $\hat{i} \cdot \hat{j} = (\hat{i} \cdot \hat{j}) \cos 90^\circ$
 $\hat{i} \cdot \hat{j} = (1)(1) \cos 90^\circ = (1)(1)(0) = 0$
 Similarly $\hat{j} \cdot \hat{k} = 0$ and $\hat{k} \cdot \hat{i} = 0$
 Thus $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} + \hat{k} \cdot \hat{i} = 0$
 $\hat{j} \cdot \hat{j} = \hat{j} \cdot \hat{j} \cos 0^\circ$
 $\hat{j} \cdot \hat{j} = (1)(1) \cos 0^\circ = (1)(1)(1) = 1$

Similarly

$$\hat{j} \cdot \hat{j} = 1 \text{ and } \hat{k} \cdot \hat{k} = 1$$

Thus

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

10. Explain cross product of unit vectors.

Ans $\hat{i} \times \hat{j} = (\hat{i} \times \hat{j}) \sin 90^\circ$
 $\hat{i} \times \hat{j} = (1)(1) \sin 90^\circ \hat{k} = (1)(1)(1) \hat{k} = \hat{k}$

Similarly,

$$\hat{j} \times \hat{k} = \hat{i} \text{ and } \hat{k} \times \hat{i} = \hat{j}$$

Hence

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

$$\hat{i} \times \hat{j} = \hat{i} \sin 0^\circ$$

$$\hat{i} \times \hat{i} = (1)(1) \sin 0^\circ \hat{n} = (1)(1)(0) \hat{n} = \vec{0}$$

Similarly

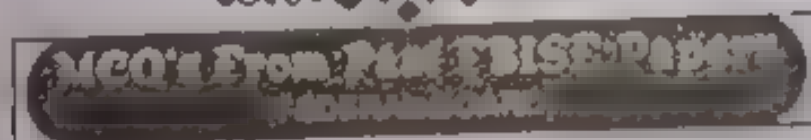
$$\hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$$

So,

$$\hat{i} \times \hat{i} = \vec{0}$$

$$\hat{j} \times \hat{j} = \vec{0}$$

$$\hat{k} \times \hat{k} = \vec{0}$$



1. Minimum number of forces of different magnitudes which can give zero resultant are _____
A 2 B 3 C 4 D 5
2. If $\vec{V} = 4\hat{i} - 4\hat{j}$ then the angle of vector with positive x - axis is _____
A 315° B 180° C 90° D 45°
3. Angle at which magnitude of dot product and cross product, are equal is _____
A 60° B 60° C 45° D 19°
4. A force of 5N acts parallel to moment arm of 5m, then torque is _____
A 25 Nm B 10 Nm C 5 Nm D zero
5. $\hat{i} \cdot (\hat{k} \times \hat{j}) + \hat{j} \cdot (\hat{k} \times \hat{i}) = ?$
A -1 B 2 C 2 D 0
6. If $\vec{A} = 5\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{B} = 2\hat{i} + 2\hat{j} - a\hat{k}$ are perpendicular vectors then which of the following is value of 'a' ?
A -2 B -7 C 8 D 8
7. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ then angle between A and B is _____
A 30 B 60 C 90 D 180
8. Moment of force is called _____
A Torque B Impulse C Force D Angular Momentum
9. x-component of a force of 10N acting along horizontal direction is _____
A 5N B 10N C 15N D Zero
10. If $A_x = -1$, $A_y = -1$ then resultant vectors make angle _____
A 45° B 180° C 90° D 225°

11. At which angle two vectors of equal magnitude are oriented such that, their resultant is also equal to magnitude of either of the vector
A 60° B 120° C 150° D 30° (ANNUAL 2017)
12. Two vectors will have maximum resultant when angle between them is
A 0° B 90° C 45° D 30° (ANNUAL 2017)
13. When angle between vectors is 45° . The magnitude between of their resultant is
A $\sqrt{A^2 + B^2}$ B $\sqrt{A^2 + B^2 + \frac{1}{\sqrt{2}}AB}$ C $\sqrt{A^2 + B^2 + \sqrt{2}AB}$ D Zero (ANNUAL 2017)
14. Two forces are acting together on an object. The magnitude of their resultant is minimum, when the angle between the forces is
A 120° B 180° C 45° D 60° (ANNUAL 2017)
15. If the scalar product of two vectors is $2\sqrt{3}$ and the magnitude of their vector product is 2. The angle between them is
A 30° B 60° C 90° D 120° (ANNUAL 2017)
16. When two reference lines are drawn at right angles to each other, their point of intersection is called.
A Coordinate system B Cartesian C Coordinate Axis D Rectangular components (ANNUAL 2017)
17. Maximum number of components of a vector may be
A Infinite B One C Two D Three (ANNUAL 2017)
18. Counter clockwise Torque is
A Zero B Infinite C Negative D Positive (ANNUAL 2018)
19. If $A = a\hat{i}$ and $B = b\hat{j}$ then
A $A \cdot B = a$ B $A \cdot B = 0$ C $A \cdot B = b$ D $A \cdot B = a$ (ANNUAL 2018)
20. SI unit of Torque is
A N B J C Nm D Nm (ANNUAL 2018)
21. In case of unit vectors \hat{i} , \hat{j} and \hat{k} Which of the following is valid?
A $\hat{i} \times \hat{j} = 0$ B $\hat{j} \times \hat{k} = \hat{i}$ C $\hat{k} \times \hat{i} = \hat{k}$ D $\hat{i} \times \hat{k} = \hat{j}$ (ANNUAL 2018)
22. A person walks first 10 km north and 20 km east. The magnitude of the resultant vector is
A 22.36 km B 22.46 km C 25.23 km D 20.16 km (ANNUAL 2018)
23. For which angle the equation $A \sin \theta = B$ is correct
A 45° B 60° C 30° D 90° (ANNUAL 2018)
24. If the scalar product $A \cdot B \neq 0$ then which of the following is NOT correct?
A $|A| \neq 0$ B $|B| \neq 0$ C $\theta \neq 0^\circ$ D $\cos \theta \neq 0$ (ANNUAL 2018)

Answers Key

B

D

C

B

C

A

C

A

A

A

C

A

C

D

A

D

B

B

B

C

B

D

A

2

SELF-ASSESSMENT PAPER

Total Mark 40

Question No 1 Choose the correct answer from the given options

(1 x 6 = 6)

SECTION - A

1. If a vector of magnitude 10 N is along y-axis, then which of the following is its component along x-axis?
 (A) 0 N (B) 5 N (C) 8.66 N (D) 10 N
2. The magnitude of resultant of two vectors of 6 N and 8 N at right angles is
 (A) 8 N (B) 10 N (C) 14 N (D) 100 N
3. If the magnitude of $\vec{A} + \vec{B} = \frac{1}{2} AB$ then an angle between \vec{A} and \vec{B} is
 (A) 30° (B) 45° (C) 60° (D) 45°
4. If the body is at rest or rotating with uniform angular velocity, then torque will be
 (A) Maximum (B) Negative (C) Zero (D) Positive
5. A meter stick is supported by a knife-edge at the 50 cm mark. Arif hangs masses of 0.40 kg and 0.60 kg from the 20 cm and 80 cm marks respectively. Where should Arif hang a third mass of 0.30 kg to keep the stick balanced?
 (A) 20 cm (B) 70 cm (C) 30 cm (D) 25 cm
6. For which angle the equation $A \cdot B = A \times B$ is correct?
 (A) 30° (B) 45° (C) 60° (D) 90°

Question No 2 Give short answers of following questions (3 x 7 = 21)

- (i) Is it possible to add three vectors of equal magnitude but different directions to get a vector? Illustrate with a diagram.
- (ii) A and B are two nonzero vectors. How can their scalar product be zero? And, how can their vector product be zero?
- (iii) A central force is one that is always directed toward the same point. Can a central force give rise to a torque about that point?
- (iv) A uniform rod 1 m long with weight 8 N can be supported in a horizontal position on a sharp edge with weights of 10 N and 15 N suspended from its ends. What is the position of point of balance?
- (v) Show that $\vec{A} \times \vec{B} + \vec{A} \times \vec{B} = \vec{A} \times \vec{B}$.
- (vi) State the two conditions of equilibrium. Name the type of equilibrium that is guaranteed by each condition of equilibrium?
- (vii) Write down steps to add vectors by rectangular components vectors.

Question No 3 Extensive Questions

SECTION - C

- (a) Define Scalar product. Give two examples. Write down its four properties.
- (b) Define moment of force. On what factors it depends? Show that moment of force acting on the rigid body is $\vec{r} \times \vec{F} = \vec{F} \times \vec{r}$.

~~~~~The End~~~~~

## CHAPTER

## 3

## MOTION &amp; FORCE

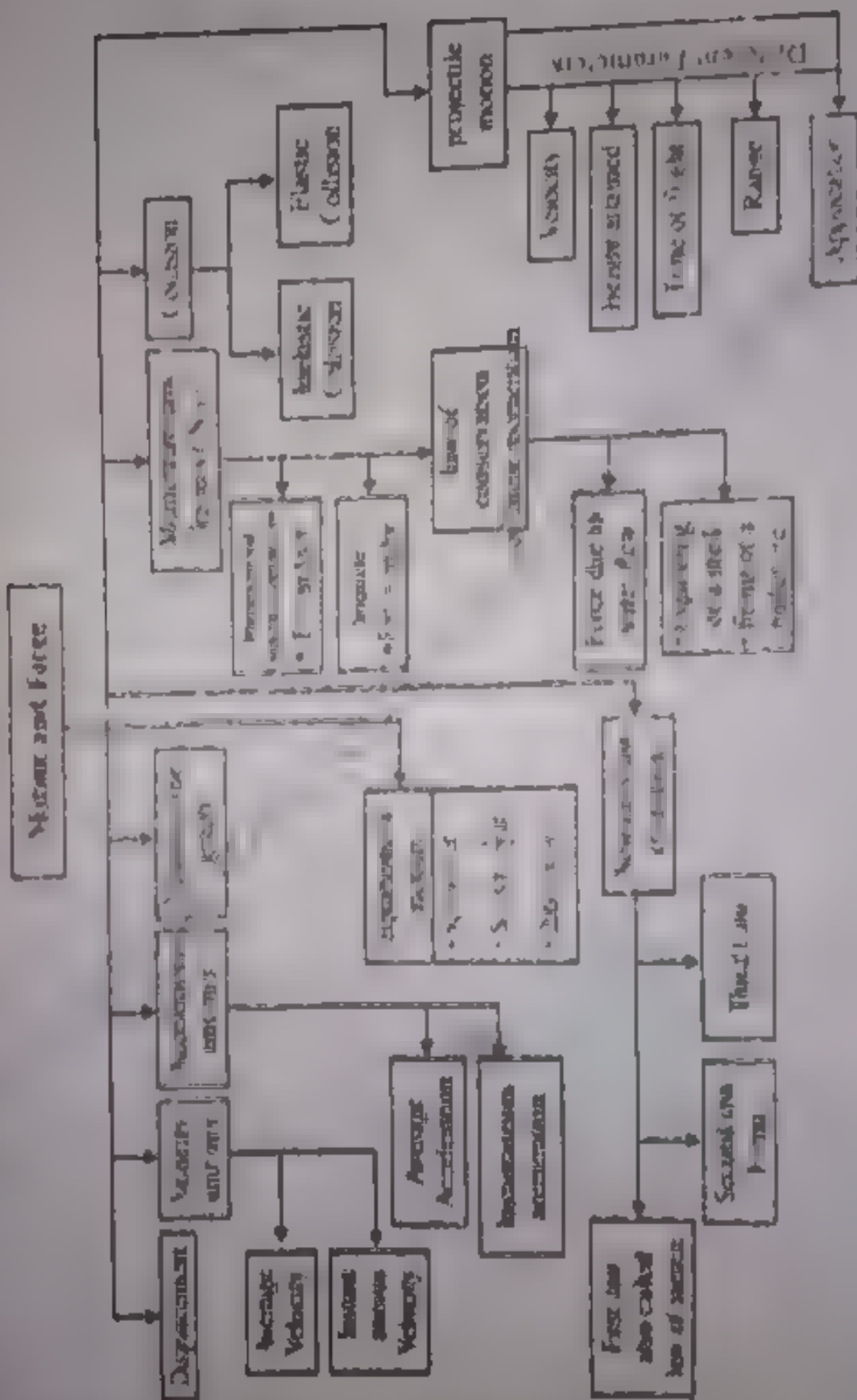
Learning Objectives

- ◆ Describe the nature of displacement
- ◆ Describe average and instantaneous velocities of objects
- ◆ Calculate average and instantaneous speeds with average and instantaneous velocities
- ◆ Plot displacement-time and velocity-time graphs of objects moving along the same straight line
- ◆ Calculate the average velocity of an object moving along the same straight line by measuring its displacement-time graph
- ◆ Calculate the instantaneous rate of change of velocity, average and instantaneous acceleration as the slope of the velocity-time graph or when the velocity-time graph is zero
- ◆ Describe the positive and negative acceleration, uniform and variable acceleration
- ◆ Calculate the acceleration of an object measuring the slope of velocity-time graph
- ◆ Manipulate equations of uniformly accelerated motion to solve problems
- ◆ Explain that projectile motion is two dimensional motion in a vertical plane
- ◆ Calculate the time of a projectile in the absence of air resistance that
  - (i) Horizontal component of velocity is constant
  - (ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object
  - (iii) The horizontal motion and vertical motion are independent of each other
- ◆ Evaluate using equations of uniformly accelerated motion that for a given initial velocity of projectile
  - (i) How higher does it go?
  - (ii) How far would it go along the level land?
  - (iii) What is it after a given time?
  - (iv) How long will it remain in air?
- ◆ Determine for a projectile launched from ground height
  - (i) Launch angle that results in the maximum range
  - (ii) Relation between the launch angles that result in the same range

- ◆ Describe how air resistance affects both the horizontal component and vertical component of velocity and hence the range of the projectile
- ◆ Apply Newton's Laws to explain the motion of objects in a variety of context
- ◆ Define mass (as the property of a body which resists change in motion)
- ◆ Describe the Newton's second Law of motion as rate of change of momentum
- ◆ Co-relate Newton's third Law of motion and conservation of momentum
- ◆ Show awareness that Newton's Laws are not exact but provide a good approximation unless an object is moving close to the speed of light or is small enough that quantum effects become significant
- ◆ Define impulse (as a product of impulsive force and time)
- ◆ Describe the effect of an impulsive force on the momentum of an object and the effect of lengthening the time stopping or rebounding from the collision
- ◆ Describe that while momentum of a system is always conserved in interaction between bodies some change in K.E. usually takes place
- ◆ Solve different problems of elastic and inelastic collisions between two masses in one dimension by using law of conservation of momentum
- ◆ Describe that momentum is conserved in all situations
- ◆ Identify that for a perfectly elastic collision the relative speed of approach is equal to the relative speed of separation
- ◆ Differentiate between explosion and collision

# Chapter No. 3

## CONCEPT MAP



Q 1 What is mechanics? What are its branches?

Ans **Mechanics**

It is the branch of physics in which we study about motion of bodies and laws and effects of motion.

**Types of Mechanics**

- i. **Kinematics** It deals with the motion of bodies without reference to the cause of motion.
- ii. **Dynamics** It deals with the motion of bodies with reference to the cause of motion.

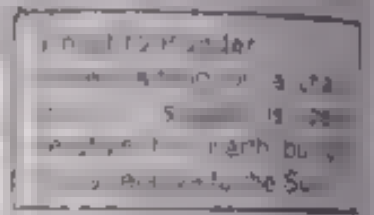
Q 2 Define Rest and Motion. With the help of an example explain that motion and rest are relative?

Ans **Rest** A body is said to be at rest if its position does not change with respect to an observer.

**Motion** A body is said to be in motion if its position changes with respect to an observer.

**Rest and motion are relative.** For example, a person sitting in a train moving with a constant velocity can describe the state of motion or rest.

For example, a person sitting in a train moving with a constant velocity relative to the ground is at rest with respect to the train but in motion with respect to the ground. This means that the state of motion or rest is relative to the observer.



Q 3 What is displacement? Describe its vector nature?

Ans **Displacement**

The change in position of a body is called displacement.

**Unit** The SI unit of displacement is metre (m). It is a vector quantity as it has both magnitude and direction. A body has displaced.

**Dimension**

**Magnitude**

Displacement is a vector quantity. Its magnitude is the length of the straight line joining the initial and final positions.

**Vector Nature of Displacement**

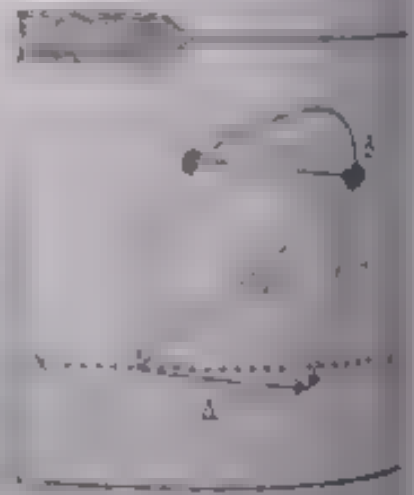
Displacement is a vector quantity. It is represented by a straight line joining the initial and final positions.

- Displacement is represented by the vector  $\vec{d}$  and its magnitude is denoted by  $d$ .
- Displacement is a vector quantity. It is represented by the vector arrow.

$$d = |\vec{d}| = r_f - r_i$$

**Note**

- Displacement is the shortest distance covered by the body. It is a vector quantity.
- Displacement is represented by a straight line in a specific direction from the initial position to the final position.
- Displacement is a vector quantity. It is represented by the vector arrow.



Q 4 What is velocity? Define average and instantaneous velocity.

Ans **Velocity**

The time rate of change of displacement of a body is called velocity.

It is a vector quantity. It is represented by the vector arrow.

$$v = \frac{\Delta x}{\Delta t}$$

### Unit and Direction

- SI unit of velocity is m/s.
- It is a vector quantity.

### Average Velocity

The ratio of the total displacement covered by body to the total time taken to cover this displacement is called its average velocity.

$$v_{avg} = \frac{\text{Total displacement}}{\text{Total time taken}} = \frac{\Delta x}{\Delta t}$$

### Instantaneous Velocity

The velocity of a body at a particular instant of time is called its instantaneous velocity.

- It is a vector quantity.
- Its SI unit is m/s.
- It is denoted by  $v$ .

### Mathematically

$$v = \frac{dx}{dt}$$

#### Remember

1. Speed is always positive.
2. Average velocity can be positive or negative for average displacement.
3. Instantaneous velocity can be positive or negative.

### Q 5 Define uniform and variable velocity.

#### Ans Uniform velocity

A body is said to be moving with uniform velocity if it covers equal displacement in equal intervals of time.

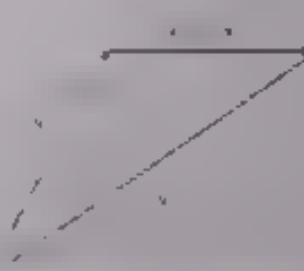
#### Non uniform velocity (Variable Velocity)

A body is said to be moving with non uniform velocity if it covers unequal displacement in equal intervals of time.

#### Condition for uniform velocity

If the magnitude and direction of velocity of a body are equal, the body is moving with uniform velocity.

If the direction of velocity does not change, the body is moving with uniform velocity.



### Q 6 Differentiate between speed and velocity

| Speed                       | Velocity                    |
|-----------------------------|-----------------------------|
| 1) It is a scalar quantity. | 1) It is a vector quantity. |
| 2) It is denoted by $s$ .   | 2) It is denoted by $v$ .   |
| 3) Its symbol is $s$ .      | 3) Its symbol is $v$ .      |
| 4) $s = \frac{d}{dt}$       | 4) $v = \frac{dx}{dt}$      |

- (4) Its SI unit is metre per second ( $\text{ms}^{-1}$ )
- (5) Its SI unit is metre per second ( $\text{ms}^{-1}$ )
- (6) It is scalar
- (6) It is vector
- (7) Average Speed is always positive
- (7) Average Velocity can be positive or negative

7. What is acceleration? Explain its different terms

## Acceleration

The time rate of change in velocity of a body is called acceleration.

Unit

SI unit of acceleration is  $\text{ms}^{-2}$ . The dimensions of acceleration are  $[\text{L T}^{-2}]$

Direction

Acceleration is also a vector quantity and the direction of acceleration is along the direction of change in velocity.

Acceleration is a measure of how rapidly the velocity is changing.

## Average Acceleration

The ratio of the total change in velocity to the total time taken is called average acceleration.

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

## Instantaneous Acceleration

Acceleration of the body at any instant of time is called instantaneous acceleration.

OR

The limiting value of  $\frac{\Delta \vec{v}}{\Delta t}$  as the time interval  $\Delta t$  following the time  $t$ , approaches to zero is called instantaneous acceleration.

Mathematically

$$a_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

## Deceleration or Retardation:

If the velocity of body is decreasing then it has retardation or deceleration. Under this condition velocity and acceleration are in opposite direction i.e. ( $\vec{v}$  anti  $\parallel$   $\vec{a}$ ).

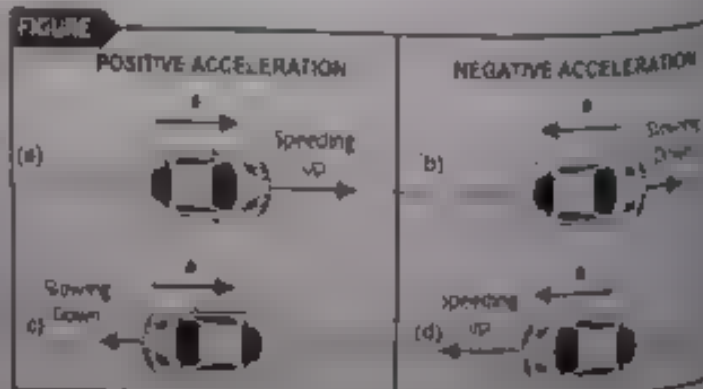
## Positive and Negative Accelerations

Since acceleration is a vector quantity, it has a direction associated with it. The direction of the acceleration vector depends on two things.

- whether the object is speeding up (velocity increasing) or slowing down (velocity decreasing)
- whether the object is moving in the + or - direction as defined for displacement.  
(e.g. towards right or along positive x-axis, velocity is taken positive and towards left or along negative x-axis, velocity is taken negative)

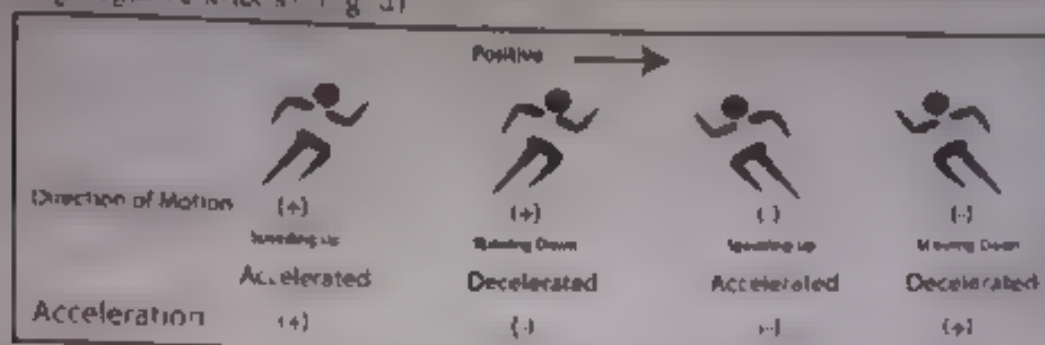
**Case 1.** If the car is moving towards right (along positive x-axis) with increasing velocity, it has accelerated motion, then it has positive acceleration towards right (towards positive direction) (fig. a)

**Case II.** If the car is moving towards right (along positive x-axis) with decreasing velocity (slowing down), it has retarded motion, then it has negative acceleration towards left (along negative x-axis) (fig. b)



**Case III** If the car is moving towards left (along negative  $x$ -axis) with decreasing velocity (slowing down), it has retarded motion, then it has positive acceleration opposite to motion towards right along positive  $x$ -axis, Fig. (c)

**Case IV** If the car is moving towards left (along negative  $x$ -axis) with increasing velocity (speeding up), it has accelerated motion, then it has negative acceleration in the direction of motion towards left along negative  $x$ -axis, Fig. (d)



### Uniform Acceleration

If velocity of the body changes equally in equal intervals of time then it has uniform acceleration.

### Variable Acceleration

If velocity of the body does not change equally in equal intervals of time then it has variable acceleration.

### Condition for uniform acceleration

For a body moving with uniform acceleration its average and instantaneous accelerations are equal.

**Q.8** (a) Discuss the displacement – time graph

(b) Consider a car moving back and forth along the straight line as shown in figure and also consider data of its position of the car every 10 s, as shown in table. Draw its displacement-time. Discuss its motion on different parts of the graph

(c) What should you do to calculate the instantaneous velocity of the car (as mentioned above)

**Ans** **Displacement - Time Graph**

It is the graph plotted between displacement of the body and time taken by it during its journey.

Displacement of the body is taken along  $y$ -axis and time is taken along  $x$ -axis.

### Slope of a graph

Slope at a point is the ratio of change in value on  $y$ -axis to the change in value on  $x$ -axis between two points on the tangent line at that point.

$$\text{Slope} = \frac{\text{change in value on } Y\text{-axis}}{\text{change in value on } X\text{-axis}}$$

### Slope or gradient of Displacement – time graph

$$\text{Slope of displacement – time graph} = \frac{\Delta d}{\Delta t} = \text{Velocity}$$

### Slope or gradient of Displacement – time graph

$$\text{Slope of distance – time graph} = \frac{\Delta s}{\Delta t} = \text{Speed}$$

### Difference between Displacement – Time and Distance – Time Graphs

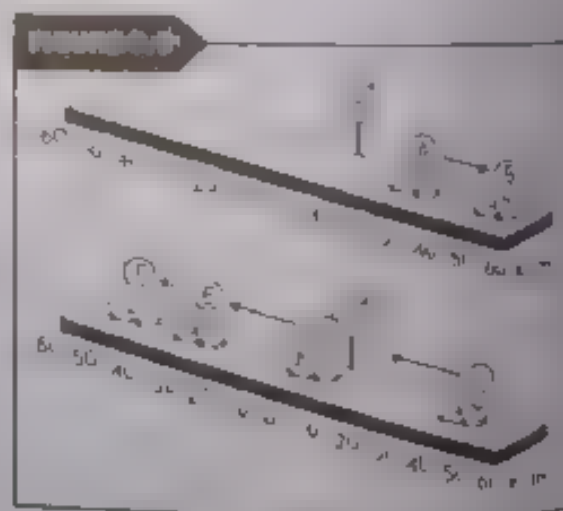
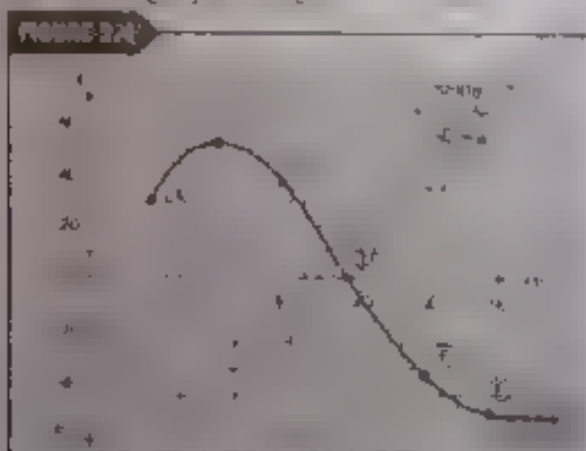
- As displacement can be positive and negative, so, slope of displacement time graph may be negative which indicates reverse journey.
- But distance is always positive so slope of distance - time graph can never be negative, which means distance is always positive and increases during motion, no matter whichever is the direction of motion.

## Graph Between Displacement and Time

|     |  |                                                                                                                                                                                                                                               |
|-----|--|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (1) |  | The graph is a straight line parallel to the time axis. The displacement 'd' does not change with time. Hence the body is at rest. Velocity is zero. As $a = 0$ .                                                                             |
| (2) |  | It shows constant velocity as slope of graph is constant. So acceleration is zero.                                                                                                                                                            |
| (3) |  | It shows velocity is increasing (body is speeding up), as slope is increasing and acceleration is positive.                                                                                                                                   |
| (4) |  | It shows velocity is decreasing (body is slowing down), as slope decreases and acceleration is negative.                                                                                                                                      |
| (5) |  | Displacement is decreasing (body is moving in negative direction) with a constant slope, i.e. uniform velocity. Graph is extending in negative direction, object has not only approached the reference or starting point but also crossed it. |

## (b) Motion of Car Moving Forward and Backward on A Straight Road

Let us draw the graph as follows:



Let us motion of the car on different parts of the graph:

- From A to B, it has positive velocity as displacement is positive. Its average velocity is calculated as
- At point B, car comes to rest for a moment. If we draw tangent at point B, it is parallel to time axis, so its velocity is zero at point B.
- After B it has reversed its direction and is heading back towards the reference (0).
- Between B and C, the average velocity is

$$v = \frac{r_c - r_b}{t_c - t_b} = \frac{0 - 52}{30 - 20} = -\frac{52}{10} = -5.2 \text{ m/s}$$

Negative sign indicates the reverse direction.

**Table 3.2: POSITION OF CAR AT VARIOUS TIMES**

| Position | Time (s) | Velocity (m/s) |
|----------|----------|----------------|
| A        | 0        | 0              |
| B        | 20       | 0              |
| C        | 30       | 0              |
| D        | 40       | 0              |
| E        | 50       | 0              |
| F        | 60       | 0              |

(c) Calculation of instantaneous velocity

Why do we need instantaneous velocity?

Calculating the average velocity of the car over relatively long time intervals will not give us the complete description of motion as shown in Figure 3.5(b). Since the car was not moving all the way through this speed, to describe the motion exactly, we need to know the velocity at every instant of time.

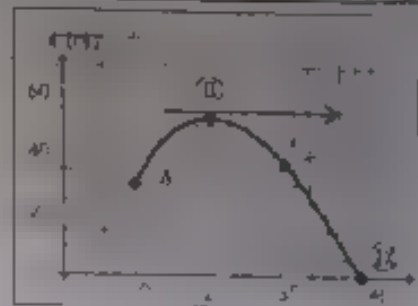
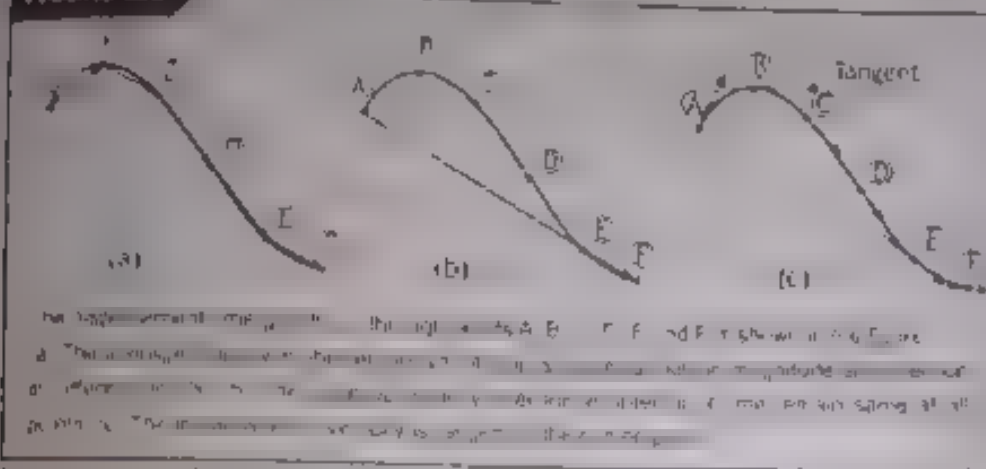


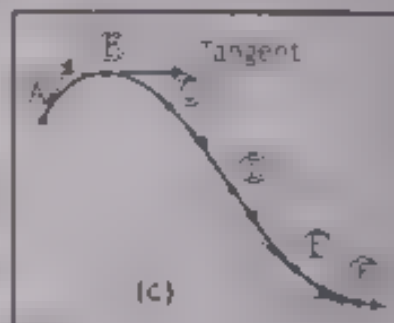
FIGURE 3.5



How to calculate instantaneous velocity?

The instantaneous velocity is obtained by taking the time intervals shorter and shorter. In other words, we say that the limit in which time approach to zero, i.e.,  $\Delta t \rightarrow 0$ , in a time graph.

This gives us a series of smaller straight line segments which have the same direction as the tangent to the curve as shown in Figure 3.5(c).



Q9 Discuss the Velocity – time graph to discuss motion of the body

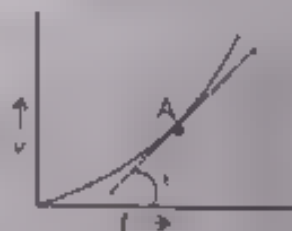
Velocity - Time Graph

"The graph which represents the variation of velocity with time is called velocity-time graph."

Slope or gradient of Velocity – time graph

Slope of velocity-time graph  $\frac{dv}{dt} = \text{Acceleration}$

- ❖ Slope of Velocity – Time graph is equal to acceleration.
- ❖ Instantaneous acceleration can be calculated by finding the slope of a tangent to the v-t graph at that required point on the curve, as shown in Fig.
- ❖ Area under velocity-time graph gives distance covered by the body.



|       |                                                                                                                                                             |  |
|-------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| (i)   | A. Uniform velocity<br>B. acceleration is zero                                                                                                              |  |
| (ii)  | A. Velocity is increasing<br>B. acceleration is positive<br>C. Acceleration is also uniform as slope of velocity-time graph is constant                     |  |
| (iii) | A. Velocity is increasing<br>B. Acceleration is positive<br>C. Also it is variable acceleration<br>D. Its acceleration is increasing as slope is increasing |  |

|        |                                                                                                                                                                             |  |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| (iv)   | <p>A. Velocity is increasing</p> <p>B. Acceleration is positive</p> <p>C. Also it has variable acceleration</p> <p>D. Acceleration is decreasing as slope is decreasing</p> |  |
| (v)    | <p>A. Velocity is decreasing</p> <p>B. Uniform deceleration</p>                                                                                                             |  |
| (vi)   | <p>A. Velocity is decreasing</p> <p>B. Retardation is decreasing as slope is decreasing</p>                                                                                 |  |
| (vii)  | <p>A. Velocity is decreasing</p> <p>B. Retardation is increasing as slope is increasing</p>                                                                                 |  |
| (viii) | Infinite acceleration                                                                                                                                                       |  |

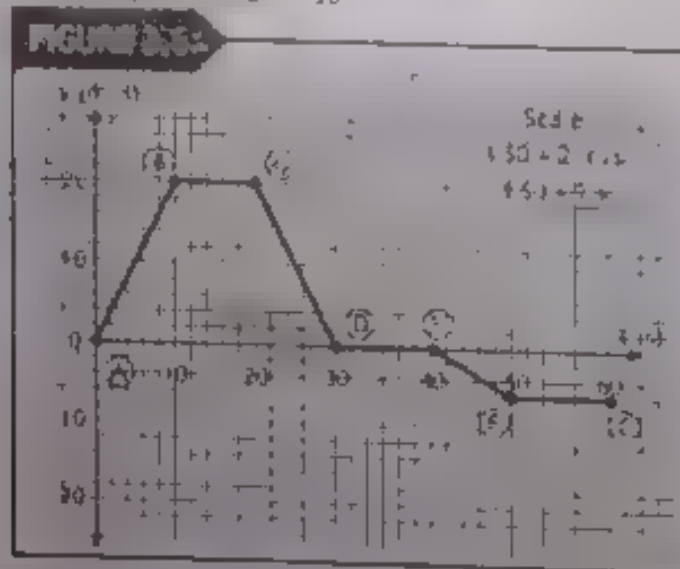
**Q 10** Discuss the motion of a body as described by the velocity time graph

**Ans** Describing the Motion Using Velocity – Time Graph

Consider the velocity time graph of an object as shown.

i From A to B, its velocity is increasing and it has uniform acceleration (as its slope is constant)

$$a = \frac{v_B - v_A}{t_B - t_A} = \frac{20 - 0}{10 - 0} = \frac{20}{10} = 2 \text{ m/s}^2$$



ii From B to C, object has uniform velocity i.e. zero acceleration

iii From C to D, its velocity is decreasing, it has uniform retardation (it has constant negative slope)

$$a = \frac{v_D - v_C}{t_D - t_C} = \frac{0 - 20}{10 - 0} = -\frac{20}{10} = -2 \text{ m/s}^2$$

iv From D to E, object is in rest as velocity of the object is zero here

v From point E to F, the object accelerates in the opposite direction (opposite to initial direction of its motion from A to C), its acceleration in this part is

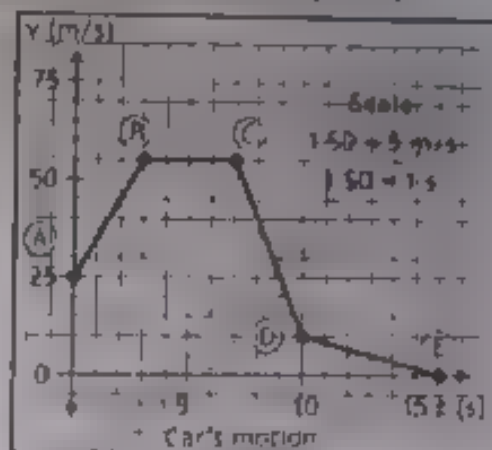
$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{0 - 6}{10 - 0} = -\frac{6}{10} = -0.6 \text{ m/s}^2$$

Object is accelerating still, but its acceleration is negative. This negative sign with acceleration shows that object has reversed its direction of its motion.

From F to G, object has constant speed but in opposite direction (as velocity is negative).

### Assignment 3.1:

The Velocity time graph shows the motion of car in a straight line. By reading the scale carefully, calculate (a) the acceleration of the car between segment A and B, B and C, C and D and D and E, from the slope of the graph. Also (b) Calculate the car's average acceleration for the complete journey.



**Solution.**

i. Acceleration of car from A to B

$$a = \frac{v_B - v_A}{t_B - t_A}$$

$$a = \text{slope of line AB} = \frac{55 - 25}{4 - 0} = \frac{30}{4} = 7.5 \text{ m/s}^2$$

ii. Acceleration of car from B to C is zero, because it has uniform velocity of 55 m/s.

$$a = \text{slope of line BC} = \frac{55 - 55}{8 - 4} = 0$$

iii. Acceleration of car from C to D

$$a = \text{slope of line CD} = \frac{10 - 55}{10 - 8} = -22.5 \text{ m/s}^2$$

$$a = \frac{v_D - v_C}{t_D - t_C} = -22.5 \text{ m/s}^2$$

Car has retardation in this part as it has negative value of acceleration, it is slowing down.

iv. Acceleration from D to E

$$a = \frac{v_E - v_D}{t_E - t_D}$$

$$a = \frac{0 - 10}{15 - 10} = -2 \text{ m/s}^2$$

v. Average acceleration of whole motion is equal to slope of line from A to E

$$a = \frac{v_E - v_A}{t_E - t_A} = \frac{0 - 25}{15 - 0} = -\frac{5}{3} = -1.67 \text{ m/s}^2$$

### MCQs

- The slope of velocity time graph shows the
  - Total distance covered by the body
  - Average acceleration of body
  - Average force by acting on body
  - Total work done on the body
- When a body moves with constant acceleration, the velocity time graph is
  - Parabola
  - Hyperbola
  - inclined straight line
  - Curve
- The area under the velocity time graph is
  - Force
  - Acceleration
  - Distance
  - Torque
- The average acceleration of the car during the interval of time is given by the slope of its
  - Displacement time graph
  - Force time graph
  - Velocity time graph
  - Acceleration time graph
- The slope of velocity time graph at any instant represents
  - instantaneous velocity
  - instantaneous acceleration
  - Power
  - Force
- An ant walk with speed
  - 0.1 ms<sup>-1</sup>
  - 1.0 ms<sup>-1</sup>
  - 0.01 ms<sup>-1</sup>
  - 100 ms<sup>-1</sup>
- When a body is moving with uniform velocity its?
  - Speed changes
  - Direction of motion change
  - Displacement from origin changes
  - Acceleration changes
- If the slope of the Velocity Time Graph remains constant then body is moving with
  - Uniform velocity
  - Negative variable Acceleration
  - Variable Acceleration
  - Uniform Acceleration

- 9 When average velocity becomes equal to instantaneous velocity then the body is said moving with  
 A. instantaneous acceleration B. Constant acceleration C. Instantaneous velocity D. None

### Answers Key

|     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 B | 2 C | 3 C | 4 C | 5 B | 6 C | 7 C | 8 D | 9 C |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|

**Q 11** Write down the equations of motion for uniformly accelerated bodies?

### Ans: Equation of Motion for Uniformly Accelerated Bodies

Consider a body is moving with uniform acceleration  $a$  along a straight line. Its initial velocity is  $v_i$  and after time interval  $t$  its final velocity becomes  $v_f$  then

$$\begin{aligned} \text{(i)} \quad v_f &= v_i + at & \text{(ii)} \quad S &= v_i t + \frac{1}{2} at^2 \\ \text{(iii)} \quad 2aS &= v_f^2 - v_i^2 & \text{(iv)} \quad S &= \frac{(v_i + v_f)}{2} \cdot t \end{aligned}$$

### How to apply these equations in problems

- These equations are useful only for **linear motion** i.e. **uniform acceleration**.
- When the object moves along a **straight line** the **direction of motion** does not change. In such cases all vectors can be treated as scalars.
- In problems where vectors can be treated as scalars, the **direction of motion** velocity is taken **positive**.
- A **negative** sign is applied to a vector quantity if its direction is opposite to that of initial velocity.
- The above equations can also be applied to **free fall motion** by replacing ' $a$ ' by ' $g$ '.

### Acceleration due to Gravity ( $g$ )

In the absence of air all bodies fall freely near the surface of earth under the action of gravity with uniform acceleration called as **acceleration due to gravity**. It is denoted by  $g$ .

- Its average value near the surface of earth is taken as  $9.8 \text{ m/s}^2$  or  $32 \text{ ft/s}^2$ .

### MCQ's

- A bullet shot straight up, returns to its starting point in 10 sec. Find its initial speed?  
 (A) 9.8 m/s (B) 24.5 m/s (C) 40 m/s (D) 98 m/s
- A cricket ball is hit so that it travels straight up in air and it requires 3 seconds to reach the maximum height. Its initial velocity is  
 (A) 10 m/s (B) 15 m/s (C) 29.4 m/s (D) 1.2 m/s
- The equation of motion are not useful for objects moving with:  
 (A) uniform velocity (B) uniform acceleration (C) Variable acceleration (D) Variable velocity
- A ball is allowed to fall freely from certain height. How much distance will it cover in first second?  
 (A) 2g (B) g (C)  $g/2$  (D) None
- The distance covered by free falling body in two seconds is  
 (A) 9.8 m (B) 19.6 m (C) 44.4 m (D) 49 m
- If the force acting on a body is doubled, then the acceleration becomes  
 (A) double (B) half (C) one fourth (D) constant
- The velocity of a free falling body just before hitting the ground is  $9.8 \text{ m/s}^2$  what will be the height through which it falls?  
 (A) 98 m (B) 19.6 (C) 4.9 (D) 1 m
- A body having uniform acceleration of  $10 \text{ m/s}^2$  has a velocity of  $100 \text{ m/s}$ . In what time its velocity will be doubled?  
 (A) 8 sec (B) 10 sec (C) 12 sec (D) 14 sec

9. What is the value of  $g$  at the centre of the earth?  
 A.  $4g$  B.  $2g$  (C)  $g$  (D) Zero
10. Acceleration of bodies of different masses allowed to fall freely is (air friction is negligible)  
 A. Same in magnitude and direction B. Same in magnitude only  
 C. Different for different bodies
11. A ball is thrown up vertically. It takes 5 sec to reach maximum height. What was its initial velocity?  
 A.  $5 \text{ m/s}$  B.  $12.2 \text{ m/s}$  (C)  $15 \text{ m/s}$  (D)  $49 \text{ m/s}$
12. If a body is moving with constant velocity of  $10 \text{ m/s}^2$  towards west, then what is its acceleration?  
 A.  $10 \text{ m/s}$  B.  $10 \text{ m/s}^2$  (C)  $30 \text{ m/s}^2$  (D) Zero
13. A car starts from rest and covers a distance of  $100 \text{ m}$  in one second with uniform acceleration. Its acceleration is  
 A.  $200 \text{ m/s}^2$  B.  $1 \times 10^3 \text{ m/s}^2$  (C)  $200 \text{ m/s}^2$  (D)  $300 \text{ m/s}^2$
14. When a body is thrown straight up at highest point, its velocity becomes zero and its acceleration will be  
 A.  $9.8 \text{ m/s}^2$  B.  $7.5 \text{ m/s}^2$  (C)  $9.8 \text{ m/s}^2$  (D) Cannot be determined

### Answer Key

|      |      |     |     |     |     |     |     |     |      |      |      |
|------|------|-----|-----|-----|-----|-----|-----|-----|------|------|------|
| 1 C  | 2 C  | 3 C | 4 C | 5 B | 6 A | 7 C | 8 B | 9 D | 10 A | 11 D | 12 D |
| 13 C | 14 B |     |     |     |     |     |     |     |      |      |      |

### Additional Questions

A proton moving with a speed of  $1.0 \times 10^6 \text{ m/s}$  passes through a  $0.020 \text{ cm}$  thick sheet of paper and emerges with a speed of  $2.0 \times 10^5 \text{ m/s}$ . Assuming uniform deceleration, find retardation and time taken to pass through the paper.

### Given Data

Initial speed  $u = 1.0 \times 10^6 \text{ m/s}$   
 Final speed  $v = 2.0 \times 10^5 \text{ m/s}$   
 Thickness of paper  $s = 0.020 \text{ cm}$

### To Find

Retardation  $a$   
 Time taken  $t$

### Calculation

$$v^2 = u^2 + 2as$$

$$(2.0 \times 10^5)^2 = (1.0 \times 10^6)^2 + 2a(0.020)$$

$$4 \times 10^{10} = 1 \times 10^{12} + 0.04a$$

$$4 \times 10^{10} - 1 \times 10^{12} = 0.04a$$

$$-9.6 \times 10^{11} = 0.04a$$

$$a = \frac{-9.6 \times 10^{11}}{0.04}$$

$$a = -2.4 \times 10^{13} \text{ m/s}^2$$

$$v = u + at$$

$$2.0 \times 10^5 = 1.0 \times 10^6 + (-2.4 \times 10^{13})t$$

$$2.0 \times 10^5 - 1.0 \times 10^6 = -2.4 \times 10^{13}t$$

$$-8.0 \times 10^5 = -2.4 \times 10^{13}t$$

$$t = \frac{8.0 \times 10^5}{2.4 \times 10^{13}}$$

$$t = 3.3 \times 10^{-8} \text{ s}$$

Retardation  $a = -2.4 \times 10^{13} \text{ m/s}^2$   
 Time taken  $t = 3.3 \times 10^{-8} \text{ s}$

$$2.4 \times 10^{17} \text{ s} = 8 \times 10^6$$

$$t = \frac{8 \times 10^6}{2.4 \times 10^7} = 3.33 \times 10^{-1}$$

$$t = 3.33 \times 10^{-1} \text{ sec}$$

**Q.12** Discuss Newton's laws of motion briefly.  
(Comprehension Q.3) Explain three Newton's laws of motion. Give example in each case for further elaboration.

**Ans** There are three laws of motion given by Newton  
**Newton's First Law of Motion / Law of Inertia**  
**Statement**

"In the absence of external force, A body at rest will remain at rest and a body moving with uniform velocity will continue to do so forever."

This is also known as law of inertia

Mathematically  $\vec{F}_{\text{ext}} = 0$  then  $\vec{a} = 0$

**Inertia**

The property of a body due to which it tends to maintain its state of rest or uniform motion is called inertia

(OR)

It is the opposition offered by the body to any change in its state of rest or of uniform motion

**Explanation**

It is a natural resistance to acceleration and all objects have greater the object's mass the greater this resistance is.

The mass of the object is a quantitative measure of inertia.

The larger the mass the greater is the inertia. A greater net force is required to change the velocity of objects with large mass.

**Examples**

- When you make a turn while driving a car, you have the tendency to continue in which the car turns the corner.
- Passenger get pressed back in your seat when an airplane takes off. It is due to inertia. Body wants to remain in rest and feels get back pressed with the seat.
- Driver's face is smashed against the windshield if your car suddenly stops against a brick wall. Due to inertia, driver continues its motion in forward direction and hits the windshield.

**Newton's Second Law of Motion**

**Statement**

"When a force is applied on a body, it produces the acceleration in its own direction. The magnitude of acceleration is directly proportional to applied force and inversely proportional to its mass."

Mathematical form  $\vec{a} = \frac{\vec{F}}{m}$

OR

$$\vec{F} = m\vec{a}$$

SI unit of force is newton

$$1 \text{ N} = 1 \text{ kg (1 m/s}^2\text{)}$$

**For Your Information**

First law of Newton gives the definition of force while second law gives the measurement of force and give definition of acceleration.



Sir Isaac Newton (1642-1726) was born in England. He developed a theory of the universe of motion in a book written in Latin called Philosophiæ Naturalis Principia Mathematica.

**For Your Information**

A force is a push or pull that can cause an object to move.

**Newton:** The force which produces acceleration of  $1 \text{ m s}^{-2}$  in a body of mass  $1 \text{ kg}$  is called  $1 \text{ N}$  force

In C.G.S system unit of force is dyne

$1 \text{ dyne} = 1 \text{ g } 1 \text{ cm s}^{-2}$

$$1 \text{ N} = 10^5 \text{ dy}$$

### Newton's Third Law of Motion

#### Statement

*"Action and reaction are equal in magnitude but opposite in direction."*

#### Explanation

- > When two bodies interact with each other action and reaction forces act for the same length of time
- > They never act on the same body but always act on different bodies. That's why car not balance each other

If a body A exerts force  $F_{AB}$  on body B then body B exerts the reaction force  $F_{BA}$  on body A which is equal in magnitude but opposite in direction, so mathematically

$$F_{AB} = -F_{BA}$$

#### FOR YOUR INFORMATION

Newtonian mechanics are limited to situations where speeds are less than about 1% of the speed of light that is less than  $3,000 \text{ km/s}$ . Most things we encounter in daily life move much slower than this speed, therefore we can safely apply Newton's laws. However, they were not applicable to the world of the 20th century when Einstein developed the theories of relativity. His theories showed that Newtonian mechanics to be applied to all objects in the universe, including those at the speed of light.

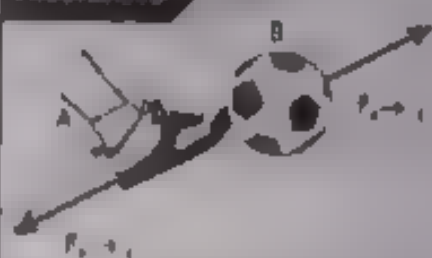
#### Point to Ponder

A car accelerates along a road. Which force actually moves the car?

#### EXPLANATION

Reactional force of the road is actually responsible to move the car. The engine of the car simply rotates the wheels but does not move it.

#### Illustration



#### For your information

Newtonian mechanics are limited to situations where speeds are less than about 1% of the speed of light that is less than  $3,000 \text{ km/s}$ .

Most things we encounter in daily life move much slower than this speed, therefore we can safely apply Newton's laws. However, they were refined further at the beginning of the 20th century when Einstein developed his theories of relativity. His theories actually extended the concept of Newtonian mechanics to be applied to all objects even objects traveling close to the speed of light.

#### Assignment

Suppose that the mass of the spacecraft ' $m_s$ ' is  $11,000 \text{ kg}$  and that the mass of the astronaut ' $m_A$ ' is  $92 \text{ kg}$ . In addition, assume that the astronaut pushes with a force of  $F = +36 \text{ N}$  (along x-axis) on the spacecraft. Find the accelerations of the spacecraft and the astronaut.

#### Solution.

Given Data:  $m_s = 11000 \text{ kg}$

$m_A = 92 \text{ kg}$

$F = 36 \text{ N}$

#### To Find:

Acceleration of spacecraft =  $a_s = ?$

Acceleration of astronaut =  $a_A = ?$

#### Solution.

From Newton's 1<sup>st</sup> law, magnitude of force on both is same but opposite in direction i.e.  $F_s = -F_A$

$$F_s = 36 \text{ N}$$

and

$$F_A = -36 \text{ N}$$

Acceleration of spacecraft is:

$$\text{From Newton's 2<sup>nd</sup> law, } a_s = \frac{F_s}{m_s} = \frac{36}{11000} = 0.0033 \text{ ms}^{-2}$$

Acceleration of astronaut is:

$$\text{From Newton's 2<sup>nd</sup> law, } a_A = \frac{F_A}{m_A} = \frac{-36}{92} = -0.39 \text{ ms}^{-2}$$

Negative sign is due to the forces which are acting in opposite direction.

#### MCQs

1. The mass of an object is the quantitative measure of its \_\_\_\_\_

(A) Mass only

(B) Acceleration

(C) Force

(D) Energy

2. If the force acting on a body is doubled then the acceleration becomes \_\_\_\_\_  
(A) Double (B) Half (C) One fourth (D) Constant
3. When a ball is thrown straight up, what is direction of acceleration at its highest point?  
(A) Upward (B) Downward (C) Zero (D) 2 g (up)
4. The law of inertia was first formulated by \_\_\_\_\_  
(A) Aristotle (B) Galileo (C) Newton (D) Einstein
5. An object of mass 1 kg moving with acceleration of  $1 \text{ ms}^{-2}$  will experience a force of \_\_\_\_\_  
(A)  $1 \text{ ms}^{-2}$  (B)  $10 \text{ N}$  (C)  $1 \text{ N}$  (D)  $1 \text{ gms}$
6. The weight of the body at the centre of the earth is \_\_\_\_\_  
(A) Slightly less (B) Slightly greater (C) zero (D)  $10^3$
7. If the mass of a body is doubled and F constant then acceleration becomes \_\_\_\_\_  
(A) Double (B) Half (C) one fourth (D)  $10^3 \text{ N}$
8. A mass of 1Kg is freely falling. The force of gravity is \_\_\_\_\_  
(A)  $1 \text{ N}$  (B)  $9.8 \text{ N}$  (C)  $98 \text{ N}$  (D)  $9.8 \times 10^3 \text{ N}$
9. Relativistic Mechanics was developed by \_\_\_\_\_  
(A) Newton (B) Faraday (C) Einstein (D) Bohr

**Answers Key**

- |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 C | 2 A | 3 B | 4 C | 5 C | 6 C | 7 B | 8 B | 9 C |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|

**Q 13 Define and explain linear momentum****Ans** Momentum

The quantity of moving body which depends on the mass of the body and its velocity is called linear momentum.

The product of mass and velocity of moving body is called linear momentum.

**Mathematically**

$$\vec{p} = m \vec{v}$$

Where  $\vec{p}$  = Momentum of the body

$m$  = Mass of the body

$\vec{v}$  = Velocity of the body

**Direction**

Since  $\vec{v}$  is a vector quantity, so momentum is also a vector quantity having same direction is same as that of velocity

**Unit**

SI - unit of momentum  $m$  is  $\text{kg ms}^{-1}$  or  $\text{Ns}$ . Its dimension is  $[M][L][T]^{-1}$

**Q 14 Show that N s is equivalent to  $\text{kg ms}^{-1}$** **Ans** Proof

$$\begin{aligned} \text{Ns} &= \text{kg} \frac{\text{m}}{\text{s}^2} \times \text{s} \\ &= \text{kg} \frac{\text{m}}{\text{s}} = \text{kg ms}^{-1} \end{aligned}$$

$$\left[ 1 \text{ N} = \text{kg} \frac{\text{m}}{\text{s}^2} \right]$$

$$\begin{aligned} &\text{kg} \frac{\text{m}}{\text{s}} \\ \text{Multiplying and dividing by N} &= \text{kg} \frac{\text{m}}{\text{s}^2} \times \text{N} \\ &= \text{kg} \frac{\text{m}}{\text{s}^2} \times \text{N} \\ &= \text{Ns} \end{aligned}$$

**Q 15 How force and momentum are related to each other? State Newton's second law in terms of momentum****Ans** Momentum and Newton's Second Law of Motion

Consider a body of mass  $m$  moving with velocity  $\vec{v}$ . A force  $\vec{F}$  is applied on the body for a time  $t$  and its velocity changes to  $\vec{v}'$ .

Acceleration produced by the force is

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t} \quad (1)$$

According to Newton's second law of motion

$$\vec{F} = m\vec{a} \quad (2)$$

Using equation (1) in (2), we have

$$\vec{F} = m \frac{\vec{v}_2 - \vec{v}_1}{t}$$

OR 
$$\vec{F} = \frac{m\vec{v}_2 - m\vec{v}_1}{t} \quad (3)$$

Where

$m\vec{v}_1$  = initial momentum of body =  $\vec{p}_1$

$m\vec{v}_2$  = final momentum of body =  $\vec{p}_2$

$$\Rightarrow \vec{F} = \frac{\vec{p}_2 - \vec{p}_1}{t}$$

$$\Rightarrow \vec{F} = \frac{\Delta \vec{p}}{t}$$

## 2<sup>nd</sup> Law in terms of momentum

*The rate of change of momentum of body is equal to the applied force.*

This is more general form of Newton's second law of motion. Because it can easily be applicable for the cases where mass is changing.

**Q 16** Define Impulse how it is related to momentum?

**Ans** Impulse

*When large force acts on a body for a very short interval of time, then the product of force and time for which the force acts, is called impulse of force.*

Mathematically

$$\boxed{\vec{J} = \vec{F} \times \Delta t} \quad (1)$$

Unit

- > SI unit of impulse is kg-m/s or Ns. It is same as that of linear momentum
- > The dimensions of impulse are  $[M \cdot L \cdot T^{-1}]$
- > It is a vector quantity

## Relation between Impulse and momentum

According to Newton's second law of motion

$$\vec{F} = \frac{m\vec{v}_2 - m\vec{v}_1}{t} \quad (2)$$

Using equation (2) in (1), we have

$$\vec{J} = \frac{m\vec{v}_2 - m\vec{v}_1}{t} \times t$$

OR

$$\boxed{\vec{J} = m\vec{v}_2 - m\vec{v}_1} \quad (3)$$

Thus, Impulse = change in momentum of the body

*Instantaneous change in momentum of body due to impulsive force is called impulse.*

**on Your Information**

$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$  is m's general form of force than  $\vec{F} = ma$

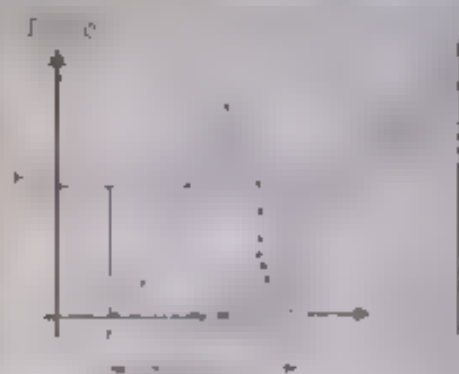
**Concept of impulse**

Some times a applied force is not constant and it acts for some time. The impulse is the time integral of the force over the time interval during which it acts.

$$\vec{J} = \int_{t_1}^{t_2} \vec{F} dt$$

**Impulsive Force**

Force acting on a body for a very short time is called impulsive force.

**WORKED EXAMPLE**

A girl of mass  $48.0 \text{ kg}$  is rescued from a burning fire by falling into a firefighters' net. The window from which she fell was  $12 \text{ m}$  above the net. She lands in the net so that she is brought to a complete stop in  $0.45 \text{ s}$ . During this interval (a) what is her change in momentum? (b) What is the impulse on the net due to the girl? (c) What is the average force on the net due to the girl?

**Given Data**  $m = 48.0 \text{ kg}$   
 $u = 0$   
 $h = 12 \text{ m}$   
 $t = 0.45 \text{ s}$

**To find**

- $\Delta p = ?$
- $J = ?$
- $F = ?$

**Solution**

(a) To find the velocity when the girl just reaches the level of net.

$$v^2 - u^2 = 2gh$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

Thus the velocity of the girl just before she hits the net and final velocity is  $v = \sqrt{2gh}$ .

Therefore the change in momentum when she strikes the net is

$$\Delta p = mv - mu$$

$$= 48.0 \times \sqrt{2 \times 9.8 \times 12}$$

$$= 48.0 \times \sqrt{235.2}$$

$$= 1176.48 \text{ kg m/s}$$

So the girl exerts her force upwards (opposite to motion).

(b) Total impulse applied by the net on the girl is

$$J = \Delta p = mv - mu$$

$$= 1176.48 \text{ kg m/s}$$

$$= (-48 \times 9.8 + 0.45) + (-734.4)$$

$$J_{\text{net}} = -2.17 - 734.4 = -946.1 \text{ Ns}$$

This is the impulse applied by net in upward direction by net

Therefore, impulse applied by girl on net in downward direction is

$$J_{\text{net}} = J_{\text{net}} = (946.1 \text{ Ns} - 946.1 \text{ Ns, Down})$$

$$(c) \quad J = I_{\text{net}} \times t$$

$$\Rightarrow I_{\text{net}} = \frac{J}{t} = \frac{946}{0.5} = 2102.4 \text{ N (Down)}$$

### MCQ's

- The rate of change in momentum of a body is equal to  
(A) Displacement (B) Velocity (C) Acceleration (D) Applied force
- Impulse can be defined as  
(A)  $J = F \times t$  (B)  $J = F \times v$  (C)  $J = F \times v$  (D)  $J = F \times p$
- If a force of 10 N acts on a body of mass 5 kg for one second, what is its rate of change of momentum?  
(A) 10 kg m s<sup>-2</sup> (B) 50 kg m s<sup>-2</sup> (C) 5 kg m s<sup>-2</sup> (D) 4 kg m s<sup>-2</sup>
- Total change in momentum of moving body is equal to its  
(A) KE (B) Impulse (C) Force (D) net force
- An alternate unit to kg m s<sup>-1</sup> is  
(A) J s (B) N s (C) N m (D) N
- SI unit of impulse is  
(A) kg m s (B) N m (C) N s (D) N m<sup>2</sup>
- At what speed the momentum and kinetic energy of a body having the same value?  
(A) 1 ms (B) 2 ms (C) 4 ms (D) 8 ms
- What are the dimensional unit of impulse?  
(A)  $[M L T^{-1}]$  (B)  $[M L T^{-2}]$  (C)  $[M L T^{-1}]$  (D)  $[M L T^{-2}]$

### Answers

1 (C) 2 (B) 3 (A) 4 (B) 5 (B) 6 (C) 7 (B) 8 (D)

Q 17 What is isolated system? Give its examples

**Ans** Isolated system

*It is a system in which no external agency exerts any force.*

**Example**

The molecules of a gas enclosed in a gas vessel at a constant temperature

**Explanation**

An isolated system is a collection of particles that can interact with each other but whose interactions with the environment outside the collection have a negligible effect on their motions. It is such a system in which external forces like friction, air resistance or any other forces don't have any effect on particles of the system.

(Note: If external forces like friction etc are negligibly smaller than the forces between the particles then we can take such a system as isolated system)

Q 18 State Law of conservation of Momentum.

**Ans** Law of conservation of Momentum

**Statement**

*The total linear momentum of an isolated system remains constant*

OR

*It states that if there is no external force applied to a system, the total linear momentum of that system remains constant in time.*


According to Newton's 2<sup>nd</sup> law of motion in term of momentum,

$$\frac{dp}{dt} = F_{\text{ext}}$$

$$F_{\text{ext}} = 0 \text{ ou}$$

$$\text{or } \Delta p = 0$$

$$\mu = \mu = 0$$



Where  $P$  is the probability of a word  $p$  in the local neighborhood

Q 18 Define elastic and plastic collision?

Collection:

An even stronger union barrier can be set up when  
interact by means of peer-to-peer interaction

The forces due to the  $\sigma_{ij}$  are assumed to be much larger than external forces per unit volume.

### Example

- i. ( )
- ii. ( )
- iii. ( )

For collection to be successful, the collector must not encounter a "bushy" type of terrain.

For example, a person who is a member of the  
religion may be a member of the religion, or positively  
charged, they represent a religion. A person has occurred,  
but the religion part was never mentioned.

### Elastic and inelastic Collisions

### Elastic Collision

The collision in which momentum and kinetic energy of the system is conserved, is called elastic collision.

### Example

House of, back of a hand ball from a more a floor to a, an object on a

### Instantia Collection

The collision in which only momentum of system is conserved but the kinetic energy of the system is not conserved, is called inelastic collision.

Example

- Scoops up back of a hard hat from sandy floor
- Combination of two germicidal

### Note

Momentum and total energy are conserved in all types of collisions.

Rejection of the null hypothesis cannot be possible

**Solid reasons for loss of kinetic energy:**

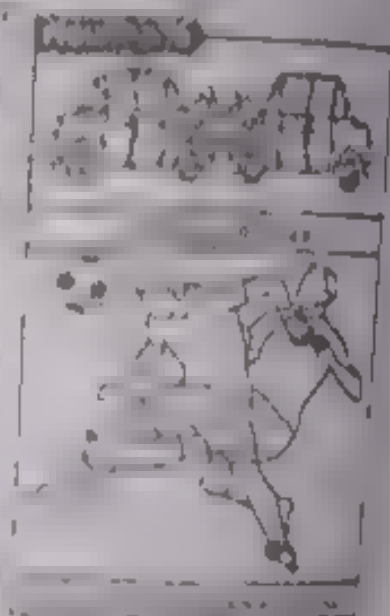
In case of elastic collision, the loss of kinetic energy is due to

- friction of ball with clay
- friction of ball and air
- work done by ball produced during collision

Q 22 Show that relative speed of approach is equal to relative speed of separation for elastic collision in one dimension.

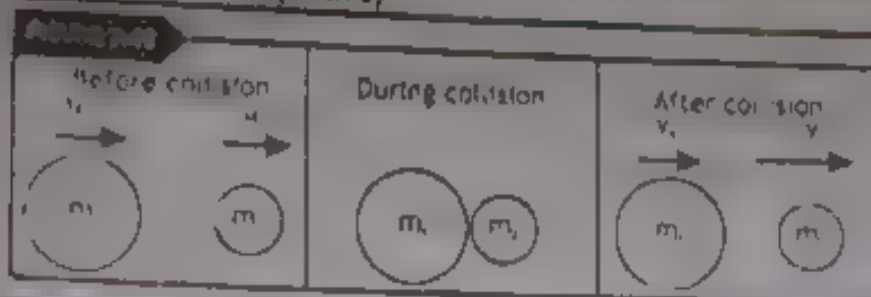
### Ans Elastic Collision in One Dimension

Let us let two smooth, non-rotating hard balls of masses  $m_1$  and  $m_2$  moving in such a way so that their centers lie along the same straight line with initial velocities  $u_1$  and  $u_2$  towards the  $x$ -axis. When they make head on collision with each other their velocities become  $v_1$  and  $v_2$ ,  $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$  as shown in figure.

[illegible]

Stand On Call on

1. የጥንታዊ የግብርና ስርዓት  
 2. የጥንታዊ የግብርና ስርዓት  
 3. የጥንታዊ የግብርና ስርዓት  
 4. የጥንታዊ የግብርና ስርዓት  
 5. የጥንታዊ የግብርና ስርዓት  
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 10. የጥንታዊ የግብርና ስርዓት



By Law of conservation of momentum,  $P_i = P_f$

or  $m_1 u + m_2 u = m_1 v_1 + m_2 v_2$  \_\_\_\_\_ (1)

rearranging  $m_1 u - m_1 v_1 = m_2 v_2 - m_2 u$

$m_1 (u - v_1) = m_2 (v_2 - u)$  \_\_\_\_\_ (2)

Since for elastic collision KE is conserved therefore  $KE_i = KE_f$

$$\frac{1}{2} m_1 u^2 + \frac{1}{2} m_2 u^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

rearranging

$$\frac{1}{2} m_1 u^2 - \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2 - \frac{1}{2} m_2 u^2$$

or  $\frac{1}{2} m_1 (u^2 - v_1^2) = \frac{1}{2} m_2 (v_2^2 - u^2)$

therefore

$$m_1 (u - v_1)(u + v_1) = m_2 (v_2 - u)(v_2 + u) \quad (3)$$

Dividing equation 1 by equation 2 we get

$$\frac{m_1 (u - v_1)}{m_1 (u^2 - v_1^2)} = \frac{m_2 (u - v_2)}{m_2 (v_2^2 - u^2)}$$

$$\frac{1}{u + v_1} = \frac{v_2 - u}{v_2 + u} \quad [\text{As } a^2 - b^2 = (a + b)(a - b)]$$

Therefore

$$\frac{v_2 - u}{(u + v_1)} = \frac{v_2 + u}{v_2 + u}$$

Rearranging

$$v_2 - u = \frac{v_2 + u}{v_2 + u} \quad \text{or} \quad v_2 - u = v_2 + u \quad (4)$$

Therefore

$$\boxed{u_1 - u_2 = -(v_1 - v_2)}$$

$$u_{rel} = -v_{rel}$$

Speed of approach = Speed of separation

Negative sign shows that faster body has become slower and slower has become faster

**Q.21** Find the expressions for the velocities of two bodies  $m_1$  and  $m_2$  after elastic collision in one dimension

**Ans** **Determination of velocities after collision**

We can calculate the velocities of the masses after collision by solving equations (2) and (4)

**Velocity of mass  $m_1$  (i.e.  $v_1$ )**

From equation (4),  $u_1 + v_1 = v_2 + u_2$

$$v_2 = u_1 - u_2 + v_1 \quad (5)$$

Using equation (5) in (1) we have

$$m_1 (u_1 - v_1) = m_2 [(u_1 - u_2 + v_1) - u_2]$$

$$\Rightarrow m_1 u_1 - m_1 v_1 = m_2 u_1 - m_2 u_2 + m_2 v_1 - m_2 u_2$$

$$\Rightarrow m_1 v_1 + m_2 v_1 = m_1 u_1 - m_2 u_1 + m_2 u_2 + m_2 u_2$$

$$\Rightarrow (m_1 + m_2) v_1 = (m_1 - m_2) u_1 + 2m_2 u_2$$

Dividing by  $(m_1 + m_2)$  on both sides

Therefore

$$v = \frac{1}{m_1 + m_2} \left( \frac{m_1}{m_1 + m_2} u_1 + \frac{m_2}{m_1 + m_2} u_2 \right) \quad (6)$$

From equation (4),

$$u = v_1 + v_2 + u_2$$

$$\Rightarrow v = v_1 + u - u_1$$

Put this value in eq (1), given by

$$m_1(u_1 - v) = m_2(v_2 - u_2)$$

$$\Rightarrow m_1 u_1 - (v_1 + u - u_1) = m_2(v_2 - u_2)$$

$$\Rightarrow m_1 u_1 - v_1 - u_2 + u = m_2(v_2 - u_2)$$

$$\Rightarrow m_1 u_1 - v_1 - u_2 + m_2 u_2 = m_2 v_2 - m_2 u_2$$

$$\Rightarrow 2m_1 u_1 - m_1 v_2 - m_1 u_2 = m_2 v_2 - m_2 u_2$$

$$\Rightarrow 2m_1 u_1 + m_2 u_2 - m_1 u_2 = m_1 v_2 + m_2 v_2$$

$$\Rightarrow 2m_1 u_1 + (m_2 - m_1) u_2 = v_2 (m_1 + m_2)$$

$$\Rightarrow \frac{2m_1}{m_1 + m_2} u_1 + \frac{(m_2 - m_1)}{m_1 + m_2} u_2 = v_2$$

$$\Rightarrow v = \frac{m_1}{m_1 + m_2} u_1 + \frac{(m_2 - m_1)}{(m_1 + m_2)} u_2 \quad (7)$$

**Assignment 3.5:**

On a highway a car of mass 1500 kg is stopped at traffic signal. A pickup of mass 2000 kg comes up from behind and hits the stopped car. Assuming the collision is elastic, the pickup stops with collision and push the car ahead onto the highway at 10.0 m/s. How fast was the pickup going just before the collision?

**Solution**Mass of Car =  $m_c = 1500$  kgMass of pickup =  $m_p = 2000$  kgVelocity of car before collision =  $u_c = 0$ Velocity of car after collision =  $v_c = 10$  m/sVelocity of pickup after collision =  $v_p = 0$ Velocity of pickup before collision =  $u_p = ?$ 

From law of conservation of momentum

$$m_c u_c + m_p u_p = m_c v_c + m_p v_p$$

$$1500(0) + 2000(u_p) = 1500(10) + 2000(0)$$

$$0 + 2000(u_p) = 15000 + 0$$

$$2000(u_p) = 15000$$

$$u_p = 15000/2000$$

$$u_p = 7.5 \text{ m/s}$$

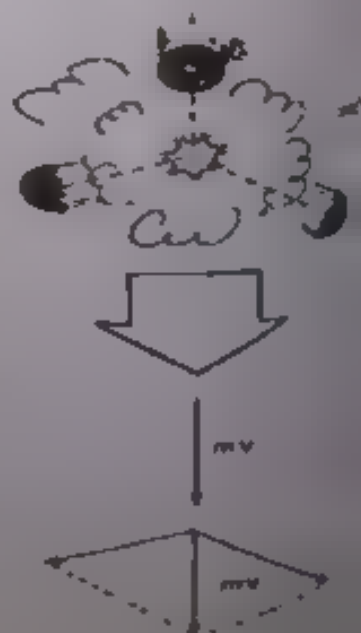
**Q 22** Is momentum conserved, when momentum changes are produced by explosive forces? Explain.

**Ans** Momentum and Explosive Forces

An explosion is a sudden, intense release of energy that often produces a loud noise, high temperature, and flying pieces, and generates a pressure wave. If the system is isolated, its total momentum during the explosion will be conserved.

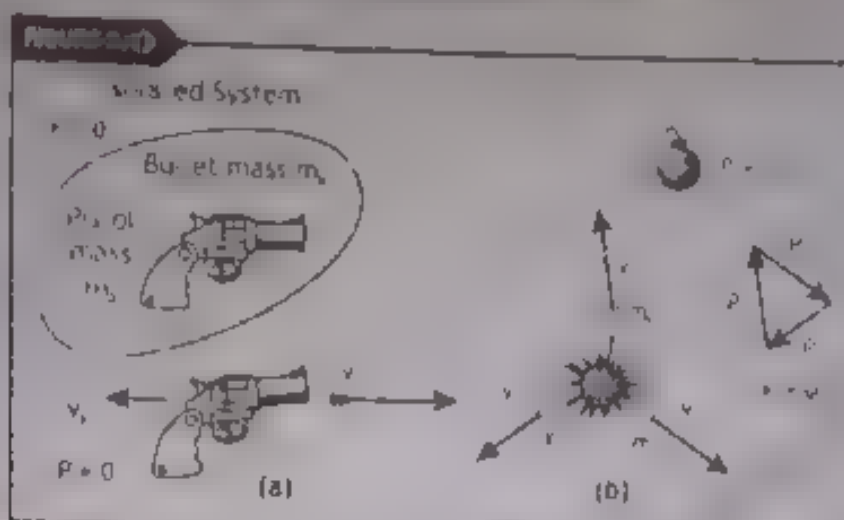
Mathematically  $P_i = P_f$ **Examples****1- Explosion of a shell or bomb**

Suppose a bomb is falling in its way it explodes into two pieces. The momentum of the bomb fragments combined by vector addition is equal to the original momentum of falling bomb, as shown in figure



## 2- Firing of Bullet from Pistol

Consider an isolated system of pistol of mass  $m_p$  and bullet of mass  $m_b$ . Initial momentum is zero as both bullet and pistol are initially at rest  $p = 0$ .



When the bullet is fired from the gun, the gun moves with velocity  $v_p$  and bullet moves with velocity  $v_b$  in opposite direction such that the total momentum is  $p = 0$  again. As  $p$  is constant, momentum of the system is conserved. The velocity with which gun moves in the backward direction is called recoil velocity.

$$P_f = P_i$$

$$m_p v_p + m_b v_b = 0$$

$$m_p v_p = -m_b v_b$$

(Momentum of gun is equal and opposite to that of bullet)

This is to maintain recoil of the pistol after firing.

## MCQ's

- If the body of mass 2 kg moving with 16 ms<sup>-1</sup> collides with stationary body of same mass, then after elastic collision the 2nd body will move with the velocity of \_\_\_\_\_  
 A. 16 ms<sup>-1</sup>      B. 30 ms<sup>-1</sup>      C. 20 ms<sup>-1</sup>      D. None of these
- As the rocket moves upward, firing its jets, its acceleration goes on \_\_\_\_\_  
 A. increasing      B. decreasing      C. Remains same      D. moves with uniform velocity
- A force of 10 N acts on a body of mass 1 kg for 5 sec. to a distance of 10 m. What is the rate of change of momentum of the body?  
 A. 50 N      B. 25 N      C. 20 N      D. 10 N
- In the absence of external force the change in momentum is \_\_\_\_\_  
 A. zero      B. 20 ms<sup>-1</sup>      C. decreasing      D. increasing

## Answers Key

|      |      |      |      |
|------|------|------|------|
| 1. A | 2. A | 3. D | 4. A |
|------|------|------|------|

Q 23 What is projectile motion? Give examples. Find out the expression of instantaneous velocity for a projectile

## Ans: Projectile Motion

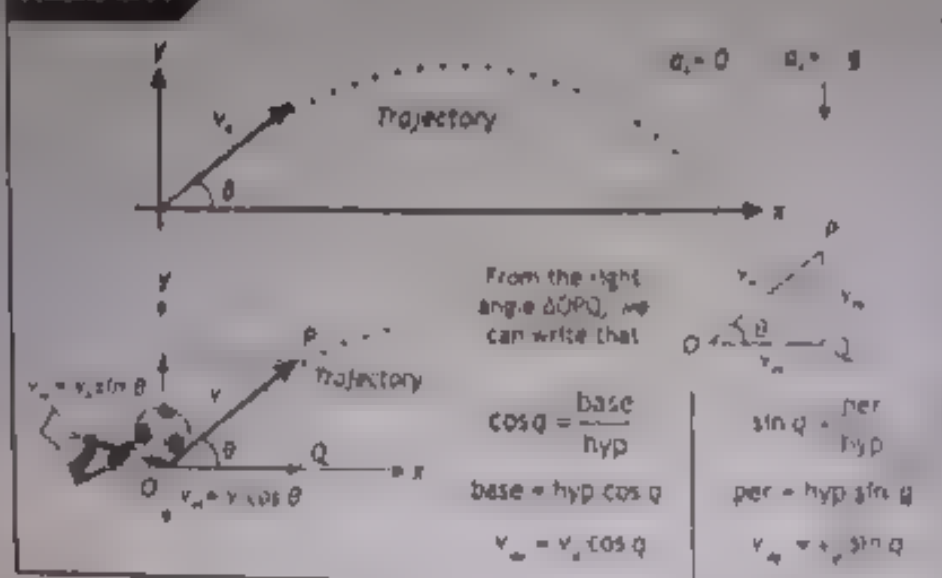
Two dimensional motion of the body under the action of gravity and inertia is called projectile motion.

(OR)

Two dimensional motion under the constant acceleration due to gravity and inertia is called projectile motion.



FIGURE 3-14

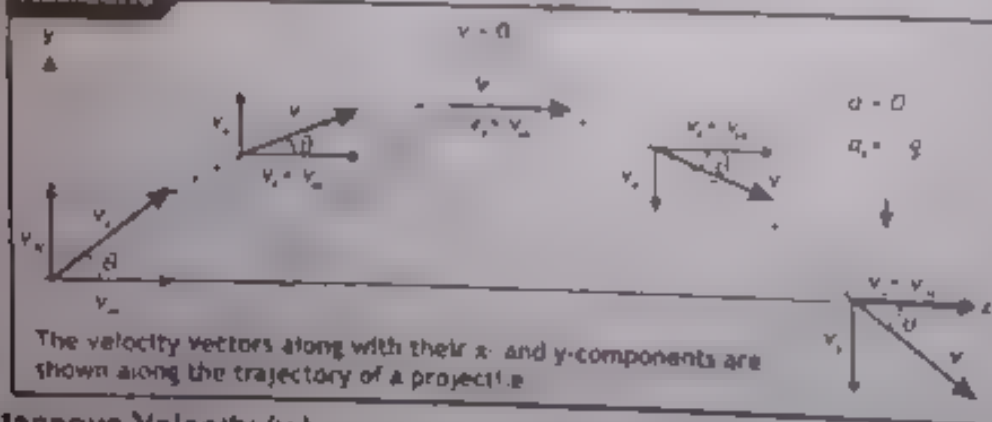
**Example of projectile motion**

- A football kicked off by a player
- Bullet fired from gun
- A missile fired from a launching pad
- A ball thrown by a cricketer
- A ball thrown from some height

**Trajectory**

The path followed by the projectile is called its trajectory. The trajectory of a projectile is usually a *parabola*.

FIGURE 3-15

**Sign Conventions**

In projectile motion a special convention is used, i.e. whichever quantity is directed upward is considered positive and whichever quantity is directed downward is considered negative. Therefore acceleration of projectile motion is equal to  $-g$ , and that is along  $y$  axis.

**Instantaneous Velocity ( $v_t$ )**

Suppose a projectile is fired with initial velocity  $v$  at an angle  $\theta$  with horizontal ( $0 < \theta < 90^\circ$ ).

Let

Horizontal component of initial velocity  $= v_x = v \cos \theta$

Vertical component of initial velocity  $= v_y = v \sin \theta$

**Horizontal component of velocity**

Since there is no horizontal force along horizontal axis so acceleration  $a_x = 0$

$$F_x = 0$$

So,  $a_x = 0$

So, final horizontal component of velocity at any instant  $t$  is

$$v_{tx} = v_{ix} + a_x t$$

$$v_{tx} = v_{ix} + (0)t \quad \text{as} \quad a_x = 0$$

$$v_{tx} = v_x$$

OR

$$v_x = v_{ix} = v \cos \theta \quad (1)$$

So, horizontal component of the  $v$  only remains constant during the whole motion

### Vertical component of velocity

The only force acting on projectile is gravity, so it has gravitational acceleration

$$F_y = F_g$$

So,

$$a_y = -g$$

Final vertical component of velocity at any instant  $t$  is

$$v_y = v_{y0} + a_y t$$

OR

$$v_y = v \sin \theta - gt$$

$$2) \text{ [since } a_y = -g \text{ and } v_{y0} = v \sin \theta]$$

### Magnitude of velocity

Final velocity  $v$  of projectile at any instant can be calculated by formula

$$v = \sqrt{v_x^2 + v_y^2}$$

Putting

$$v_x = v_{x0} = v \cos \theta$$

$$v_y = v \sin \theta - gt$$

$$\Rightarrow v = \sqrt{(v \cos \theta)^2 + (v \sin \theta - gt)^2}$$

$$\Rightarrow v = \sqrt{v^2 \cos^2 \theta + v^2 \sin^2 \theta - 2vgt \sin \theta + g^2 t^2}$$

$$\Rightarrow v = \sqrt{v^2 (\cos^2 \theta + \sin^2 \theta) - 2vgt \sin \theta + g^2 t^2}$$

$$\Rightarrow v = \sqrt{v^2 - 2vgt \sin \theta + g^2 t^2}$$

$$\Rightarrow v = \sqrt{v^2 - 2vgt \sin \theta + g^2 t^2}$$

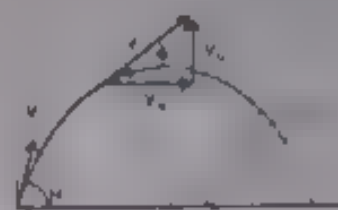
Direction of velocity

velocity of projectile makes an angle  $\phi$  with the horizontal. Then

$$\tan \phi = \frac{v_y}{v_x}$$

$$\phi = \tan^{-1} \frac{v_y}{v_x}$$

$$\Rightarrow \phi = \tan^{-1} \frac{v \sin \theta - gt}{v \cos \theta}$$



**Q.24** A projectile is thrown with initial velocity  $v$  making an angle  $\theta$  with the horizontal. Find its

(a) Maximum height

(b) Time of flight

(c) Range

(d) Maximum range

### Ans. Max-Height of the projectile

The maximum vertical distance covered by the projectile is called the maximum height of the projectile.

For the determination of height we take

Initial vertical velocity of projectile  $= v_y = v \sin \theta$

Vertical acceleration  $= a_y = -g$

Vertical velocity at highest point  $= v_y = 0$

Maximum height  $= v_y^2 - H = ?$

Now according to equation of motion,

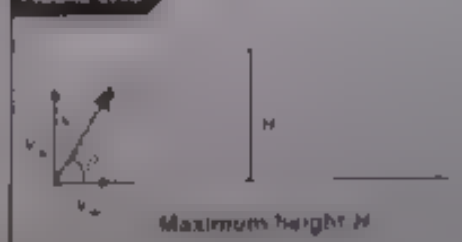
$$2as_y = v_y^2 - v^2$$

$$\text{OR } 2as_y = v_y^2 - v^2$$

$$2(-g)H = 0 - (v \sin \theta)^2$$

$$2gH = v^2 \sin^2 \theta$$

FIGURE 2.16



$$H = \frac{v^2 \sin^2 \theta}{2g} \quad (1)$$

### Time of flight

The time taken by body to cover the distance from the place of projection to the place where it hits the ground at same level is called the time of flight

OR

It is the time during which projectile remains in air

As the body goes up and comes back to same level so it covers no vertical displacement

$$\text{i.e. } S_y = 0$$

Initial vertical velocity of projectile  $= v_{iy} = v \sin \theta$

acceleration due to gravity  $= a_y = -g$

time of flight  $= T$

$$\text{As } S_y = v_{iy} T + \frac{1}{2} a_y T^2$$

$$\text{OR } 0 = v \sin \theta T - \frac{1}{2} g T^2$$

$$\text{OR } T = \frac{2(v \sin \theta)}{g} \quad (2)$$

$$\text{OR } \frac{1}{2} g T^2 = (v \sin \theta) T$$

$$T = \frac{2v \sin \theta}{g}$$

(2)

### Time to reach maximum height (Time of Summit)

It is the time in which projectile reaches maximum height from point of projection

It is equal to half of the time of flight

$$T = \frac{T}{2}$$

$$T = \frac{2v \sin \theta}{2g} = \frac{v \sin \theta}{g}$$

### 2<sup>nd</sup> method

From 1<sup>st</sup> equation of motion

$$v_f = v_i + at$$

$$v_f = v_i + a_y t \quad (3)$$

Put following values in equation (3)

$$v_f = 0 \text{ (at highest point)}$$

$$v_i = v \sin \theta$$

$$a_y = -g$$

$$t = T$$

$$\text{We have, } 0 = v \sin \theta - g T$$

$$g T = v \sin \theta$$

$$T = \frac{v \sin \theta}{g}$$

### For Your Information

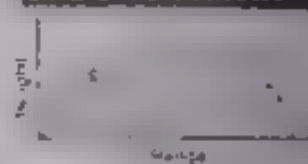
1. The time of flight of a projectile is independent of the angle of projection.

2. The time of flight of a projectile is maximum when the angle of projection is  $45^\circ$ .

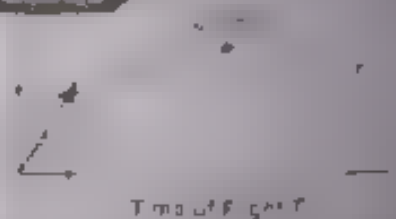
### For Your Information

$$H = R$$

### Do You Know?



### Figure 2.9

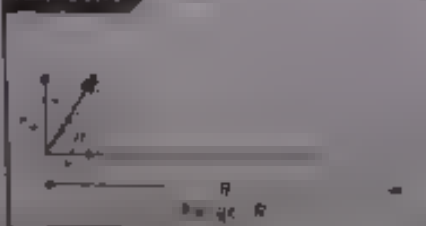


| Time (t)      | Vertical displacement (y) |
|---------------|---------------------------|
| 0             | 0                         |
| $\frac{T}{2}$ | $H$                       |
| $T$           | 0                         |

### EXPLANATION

The time of flight of a projectile is the time taken by the projectile to return to the same level from which it was launched. The time of flight is independent of the angle of projection. The time of flight is maximum when the angle of projection is  $45^\circ$ .

### Figure 2.10



**Range of projectile**

Maximum distance which a projectile covers in the horizontal direction is called the range of projectile.

Then  $x = v_{ix}t + \frac{1}{2}a_x t^2$  becomes

$$R = v \cos\theta \times \frac{2v \sin\theta}{g} + 0 \quad [a_x = 0]$$

$$\left[ t = \frac{2v \sin\theta}{g} \right]$$

OR  $R = \frac{v^2 (2 \sin\theta \cos\theta)}{g}$

$$R = \frac{v^2 \sin 2\theta}{g}$$

— (4) [Since  $2 \sin \theta$

$\cos \theta = \sin 2\theta$ ]

The range of the projectile will be maximum when  $\sin 2\theta$  has maximum value i.e.

$\sin 2\theta = 1$

OR  $2\theta = 90^\circ$  (1)

OR  $2\theta = \pi$

OR  $\theta = 45^\circ$

So, a projectile will have its maximum range when it is launched at an angle of projection of  $45^\circ$ .

**Maximum Range**

So, equation  $R = \frac{v^2 \sin 2\theta}{g}$  becomes

$$R_{\max} = \frac{v^2 \sin 90^\circ}{g}$$

OR  $R_{\max} = \frac{v^2 \sin 90^\circ}{g}$

$$R_{\max} = \frac{v^2}{g}$$

— (5) [  $\sin 90^\circ = 1$  ]

**Note:**

We can express the range of the projectile in terms of maximum range as

$$R = R_{\max} \sin 2\theta$$

**Q.25 For which pair of angles, ranges of projectile will be same?**

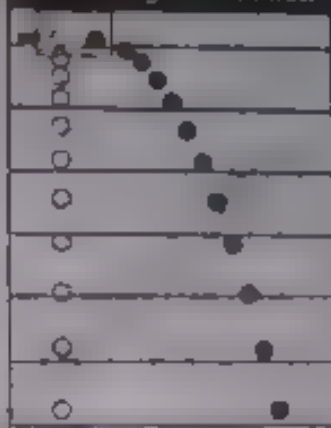
**Ans:** If initial velocity of projectile and value of  $g$  is same then ranges of a Projectile will be same for pair of angles whose sum is  $90^\circ$  i.e. these are Complementary angles of one another e.g.  $(10^\circ, 80^\circ)$ ,  $(15^\circ, 75^\circ)$ ,  $(30^\circ, 60^\circ)$  etc.

For example the range at  $75^\circ$  &  $15^\circ$  is the same

$$R_{75} = \frac{v^2}{g} \sin 2(75^\circ)$$

Or  $R_{15} = \frac{v^2}{g} \sin 150^\circ$

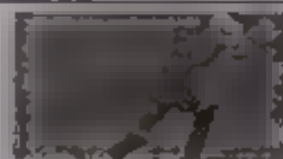
Therefore  $R_{75} = R_{15}$

**Interesting Information**

A photograph of two balls released simultaneously from a mechanical launcher. The balls follow parabolic paths and hit the ground at the same time. The balls are released from the same height and follow the same path.

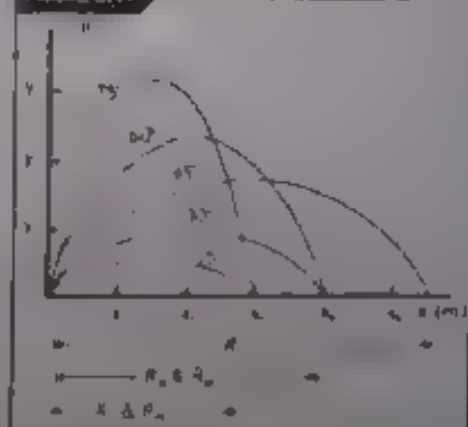
**EXPLANATION**

Since both balls are released from the same height and follow the same path, they will hit the ground at the same time. The balls are released from the same height and follow the same path.

**To R<sub>1</sub>**

The figure shows a person in a dynamic pose, possibly a dancer or athlete, illustrating a concept related to the text.

FIGURE 2.19





As the range is less than the length of boundary (72m), so for  $\theta = 30^\circ$  the ball will not cross the boundary

(b) For  $\theta = 45^\circ$ , equation (1) becomes

$$R = \frac{28^2 \times \sin(2 \times 45^\circ)}{9.8} = \left( \frac{28^2 \times 1}{9.8} \right) \text{m}$$

$R = 80 \text{ m}$

In this case the range is greater than the length of the boundary (72m), so the ball will cross the boundary

(c) Put  $\theta = 70^\circ$  in equation (1),

$$R = \frac{28^2 \times \sin(2 \times 70^\circ)}{9.8} = \left( \frac{28^2 \times 0.941}{9.8} \right) \text{m}$$

$R = 91.4 \text{ m}$

The range is less than the length of boundary so in this case the ball will not cross the boundary

MCQ's

- 1. Motion of projectile is  
(A) 1 dimensional (B) 2 dimensional (C) 3 dimensional (D) 4 dimensional
- 2. The horizontal range is maximum when it is projected at an angle of  
(A)  $0^\circ$  (B)  $30^\circ$  (C)  $45^\circ$  (D)  $60^\circ$
- 3. For which pair of angles the range of projectile is equal?  
(A)  $15^\circ$  &  $45^\circ$  (B)  $30^\circ$  &  $45^\circ$  (C)  $15^\circ$  &  $60^\circ$  (D)  $30^\circ$  &  $60^\circ$
- 4. The horizontal component of velocity of projectile  
(A) increases (B) decreases (C) Remains same (D) Decreases then increases
- 5. The velocity of projectile is maximum  
(A) At the highest point (B) At the end of ascending (C) At half of the height (D) After striking the ground
- 6. The horizontal range of projectile at  $30^\circ$  with horizontal is the same as that at an angle of  
(A)  $45^\circ$  (B)  $60^\circ$  (C)  $90^\circ$  (D)  $120^\circ$
- 7. A projectile is thrown upward with the velocity "v" making an angle  $\theta$  with the horizontal. The maximum horizontal range is  
(A)  $\frac{v^2}{g}$  (B)  $\frac{v^2}{g} \sin 2\theta$  (C)  $\frac{v^2}{g} \sin 2\theta$  (D)  $\frac{v^2}{2g} \sin 2\theta$
- 8. The time of flight of a projectile when it is projected from the ground is  
(A)  $\frac{v}{g} \cos \theta$  (B)  $\frac{v}{g} \sin \theta$  (C)  $\frac{v}{g} \sin 2\theta$  (D)  $\frac{v}{g} \sin \theta$
- 9. In the projectile motion, the vertical component of velocity  
(A) Remains constant (B) Increases to a point (C) Remains zero (D) Increases with time
- 10. The angle of projection for which the maximum height and horizontal range are equal  
(A)  $45^\circ$  (B)  $30^\circ$  (C)  $60^\circ$  (D)  $90^\circ$
- 11. A bomber drops its bomb when it is vertically above the target. It misses the target due to  
(A) Horizontal component of velocity (B) Vertical component of velocity (C)  $\frac{v}{g}$  of the body (D) Acceleration due to gravity
- 12. A ball is thrown up with  $30 \text{ ms}^{-1}$  at an angle of  $60^\circ$  with axis. The velocity of the ball at the top position is  
(A)  $0 \text{ ms}^{-1}$  (B)  $10 \text{ ms}^{-1}$  (C)  $20 \text{ ms}^{-1}$  (D)  $15 \text{ ms}^{-1}$
- 13. The shape of trajectory of short range projectile is  
(A) circular (B) parabolic (C) elliptic (D) hyperbolic
- 14. The path followed by a projectile is known as its  
(A) Range (B) Trajectory (C) Cycle (D) Height
- 15. For which pair of angles the range of projectile are equal?  
(A)  $20^\circ$  &  $30^\circ$  (B)  $70^\circ$  &  $20^\circ$  (C)  $80^\circ$  &  $40^\circ$  (D)  $60^\circ$  &  $40^\circ$
- 16. The velocity of projectile at maximum height is  
(A)  $v \cos \theta$  (B) Zero (C) Maximum (D)  $v \sin \theta$
- 17. Ranges of projectile are equal for pair of angles  
(A)  $45^\circ$  &  $60^\circ$  (B)  $40^\circ$  &  $60^\circ$  (C)  $40^\circ$  &  $60^\circ$  (D)  $40^\circ$  &  $60^\circ$
- 18. Horizontal range is equal for the angles  
(A)  $30^\circ$  and  $45^\circ$  (B)  $10^\circ$  and  $60^\circ$  (C)  $30^\circ$  and  $60^\circ$  (D)  $10^\circ$  and  $60^\circ$

- 18 The acceleration along x-axis directed at an angle of projection is  
 A)  $g \sin \theta$  B)  $g \cos \theta$  C) Maximum D) Constant
- 19 A ball rolls off the edge of a table. The horizontal component of the ball's velocity remains constant during its entire trajectory because  
 (A) the ball is acted upon by a force in the only vertical direction  
 (B) the net force acting on the ball is zero  
 (C) the ball is acted upon by a force in the vertical direction  
 (D) the ball is acted upon by a force in the horizontal direction

|    |    |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |

### FORMULAE

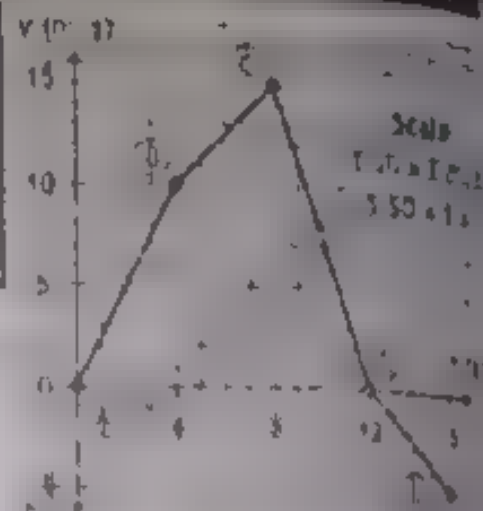
|    |                                                                                    |                                                     |
|----|------------------------------------------------------------------------------------|-----------------------------------------------------|
| 1  | Displacement of a particle                                                         | $s = ut + \frac{1}{2}at^2$                          |
| 2  | Average velocity                                                                   | $v_{avg} = \frac{s}{t}$                             |
| 3  | Instantaneous velocity                                                             | $v = \frac{ds}{dt}$                                 |
| 4  | Average acceleration                                                               | $a_{avg} = \frac{v - u}{t}$                         |
| 5  | Instantaneous acceleration                                                         | $a = \frac{dv}{dt}$                                 |
| 6  | 2 <sup>nd</sup> law of motion                                                      | $F = ma$                                            |
| 7  | Linear momentum                                                                    | $p = mv$                                            |
| 8  | 2 <sup>nd</sup> law of motion in terms of momentum                                 | $F = \frac{dp}{dt} = m \frac{dv}{dt}$               |
| 9  | Impulse                                                                            | $I = \Delta p$<br>$I = F \Delta t = m \Delta v$     |
| 10 | Law of conservation of linear momentum                                             | $m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4$             |
| 11 | Relation between relative velocity of approach and relative velocity of separation | $v_1 - v_2 = -(v_3 - v_4)$                          |
| 12 | Velocity of mass $m_1$ after collision in one dimensional elastic collision        | $v_1 = \frac{(m_1 - m_2)u_1 + 2m_2 u_2}{m_1 + m_2}$ |
| 13 | Velocity of mass $m_2$ after collision in one dimensional elastic collision        | $v_2 = \frac{2m_1 u_1 + (m_2 - m_1)u_2}{m_1 + m_2}$ |
| 14 | Force due to water flow                                                            | $F = \rho A v^2$                                    |

|    |                                                                    |                                  |
|----|--------------------------------------------------------------------|----------------------------------|
| 15 | Reco. velocity of a rifle                                          | $v = \frac{mv}{M}$               |
| 16 | Acceleration of rocket                                             | $a = \frac{mv}{M}$               |
| 17 | Horizontal distance of an object thrown horizontally from height h | $x = v_x t$                      |
| 18 | Vertical distance of an object thrown horizontally from height h   | $y = \frac{1}{2} g t^2$          |
| 19 | x component of instantaneous velocity of a projectile              | $v_x = v_{ix} = v \cos \theta$   |
| 20 | y component of instantaneous velocity of a projectile              | $v_y = v_{iy} + g t$             |
| 21 | Instantaneous velocity of a projectile                             | $v = \sqrt{v_x^2 + v_y^2}$       |
| 22 | Height of projectile                                               | $H = \frac{v_{iy}^2}{2g}$        |
| 23 | Time flight of projectile                                          | $T = \frac{2 v_{iy}}{g}$         |
| 24 | Range of projectile                                                | $R = \frac{v^2 \sin 2\theta}{g}$ |
| 25 | Maximum range of projectile                                        | $R_{max} = \frac{v^2}{g}$        |

- ◆ A body will remain at rest or a body in motion will remain in uniform motion unless an external force is applied on it.
- ◆ A body will accelerate if a body produces an acceleration in the direction of the net force. Acceleration is directly proportional to the force and inversely, proportional to the mass of the body  $F = ma$ .
- ◆ For every action force there is an equal and opposite reaction force. The action and reaction forces are equal and opposite.
- ◆ The linear momentum of a body is defined as the product of the mass  $m$  of that body and its velocity  $v$   $p = mv$ . Newton's second law states that the rate of change of momentum of a body is equal to the force acting on the body.
- ◆ Impulse is a process in which momentum changes but the forces are very short lived forces, large varying over wide limits and instantaneously not measurable. The change in momentum is a measurable. It is therefore convenient to introduce a physical quantity known as impulse, defined as the product of an average force  $\bar{F}$  and the duration of time  $\Delta t$  during which the force  $F$  acts  $J = \bar{F} \Delta t$ .
- ◆ The principle of conservation of linear momentum. This principle states that if there is no external force applied to a system, the linear momentum of that system remains constant in time. It is true in collisions of particles, decay of nuclei, collision of atoms and molecules etc.
- ◆ Displacement: Shortest distance between two points is called displacement.
- ◆ Speed: Rate of change of distance is called speed.
- ◆ Velocity: Rate of change of displacement is called velocity.
- ◆ Acceleration: Rate of change of velocity is called acceleration.

## Solved Examples

The velocity time graph shows the motion of bicyclist in a straight line (a) From the slope of the graph calculate the acceleration of the bicyclist between segment A and B, B and C, C and D and D and E. (b) Calculate the average acceleration of the bicyclist. Also (c) Plot the acceleration time graph for this motion.



**Solution:**

(a) The acceleration from point A to B can be calculated by measuring the slope as

$$a = \frac{v_2 - v_1}{t_2 - t_1} \quad \text{or} \quad a = \frac{10 \text{ m/s} - 0}{4 \text{ s} - 0} \quad \text{or} \quad a = \frac{10}{4} \text{ m/s}^2$$

Therefore  $a = 2.5 \text{ m/s}^2$

The acceleration from point B to C by measuring the slope is

$$a = \frac{v_2 - v_1}{t_2 - t_1} \quad \text{or} \quad a = \frac{15 \text{ m/s} - 10 \text{ m/s}}{8 \text{ s} - 4 \text{ s}} \quad \text{or} \quad a = \frac{5}{4} \text{ m/s}^2$$

Therefore  $a = 1.25 \text{ m/s}^2$

The acceleration from point C to D can be calculated as

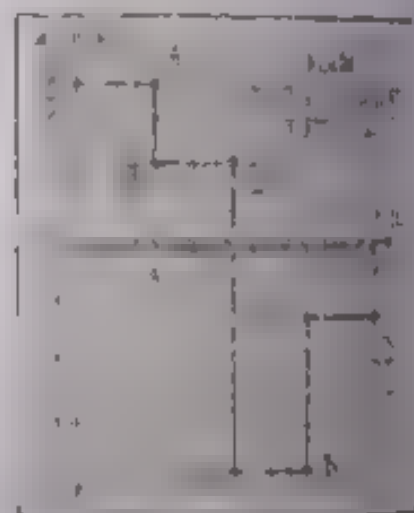
$$a = \frac{v_2 - v_1}{t_2 - t_1} \quad \text{or} \quad a = \frac{0 \text{ m/s} - 15 \text{ m/s}}{12 \text{ s} - 8 \text{ s}} \quad \text{or} \quad a = -\frac{15}{4} \text{ m/s}^2$$

Therefore  $a = -3.75 \text{ m/s}^2$

The acceleration from point D to E can be calculated as

$$a = \frac{v_2 - v_1}{t_2 - t_1} \quad \text{or} \quad a = \frac{-5 \text{ m/s} - 0 \text{ m/s}}{14 \text{ s} - 12 \text{ s}} \quad \text{or} \quad a = -\frac{5}{2} \text{ m/s}^2$$

Therefore  $a = -2.5 \text{ m/s}^2$



(b) The average acceleration can be calculated by measuring the slope from point A to E as

$$a = \frac{v_2 - v_1}{t_2 - t_1} \quad \text{or} \quad a = \frac{-5 \text{ m/s} - 0 \text{ m/s}}{14 \text{ s} - 0 \text{ s}} \quad \text{or} \quad a = -\frac{5}{14} \text{ m/s}^2$$

Therefore  $a = -0.357 \text{ m/s}^2$

(c) When these data points are plotted on acceleration time graph by connecting the points, we get the curve as

A (Joint Fighter-17) JF Thunder 17 aircraft takes off at  $70.0 \text{ m/s}$  ( $252 \text{ km/h}$ ). After accelerating uniformly at  $3.00 \text{ m/s}^2$  from rest that lasts  $6.0 \text{ s}$  during the initial phase of takeoff. The afterburner engines are then turned up to full power for an acceleration of  $7.1 \text{ m/s}^2$ . Calculate the length of runway needed and the total time of takeoff.

**Given Data:**

acceleration ' $a_1$ ' for phase 1 =  $3.0 \text{ m/s}^2$

acceleration ' $a_2$ ' for phase 2 =  $7.1 \text{ m/s}^2$

time ' $t_1$ ' for phase 1 =  $6.0 \text{ s}$

final velocity ' $v_0$ ' for phase 2 =  $70.0 \text{ m/s}$

**To Find:**

(a) length of runway ' $s$ ' = ?

(b) time of takeoff ' $t$ ' = ?

Solution:

For first phase of take-off, the distance  $s$  can be calculated by using second equation of motion

$$S = v_0 t + \frac{1}{2} a_1 t_1^2$$

Putting values  $S_1 = 0 \times 5 + \frac{1}{2} \times 3.9 \times (6.5)^2$

Or  $S_1 = \frac{1}{2} \times 3.9 \text{ m/s}^2 \times 42.25 \text{ s}$

Therefore  $S_1 = 82.3875 \text{ m}$

The final velocity at phase 1, can be calculated by using first equation of motion

$$v_1 = v_0 + a_1 t \quad \text{putting values} \quad v_1 = 0 \text{ m/s} + 3.90 \text{ m/s}^2 \times 6.5 \text{ s}$$

therefore  $v_1 = 25.35 \text{ m/s}$

For second phase of take-off, the distance  $s$  can be calculated by using third equation of motion

$$2a_2 S_2 = v_2^2 - v_1^2 \quad \text{and} \quad S_2 = \frac{v_2^2 - v_1^2}{2a_2}$$

The final velocity  $v_1$  at phase 1 which is  $25.35 \text{ m/s}$  will be initial velocity ' $v_2$ ' at phase 2, therefore

$$S_2 = \frac{v_2^2 - v_1^2}{2a_2} \quad \text{Putting values} \quad S_2 = \frac{(70.0 \text{ m/s})^2 - (25.35 \text{ m/s})^2}{2 \times 7.1 \text{ m/s}^2}$$

Or  $S_2 = \frac{4257.3775 \text{ m}^2/\text{s}^2}{14.2 \text{ m/s}^2} \quad \text{and} \quad S_2 = 299.853 \text{ m}$

Therefore  $S_2 = 299.8 \text{ m}$

For second phase of take-off, the time ' $t$ ' can be calculated by using first equation of motion

$$v_2 = v_1 + a_2 t_2 \quad \text{or} \quad v_2 - v_1 = a_2 t_2$$

Or  $t_2 = \frac{v_2 - v_1}{a_2} \quad \text{Putting values} \quad t_2 = \frac{70.0 \text{ m/s} - 25.35 \text{ m/s}}{7.1 \text{ m/s}^2}$

Hence  $t_2 = 6.3 \text{ s}$

The total distance covered is  $S = S_1 + S_2$

Putting values  $S = 82.4 \text{ m} + 299.8 \text{ m}$

$S = 382.2 \text{ m}$

Hence the minimum runway length under these conditions is 382.2 metres.

The total time taken is  $t = t_1 + t_2$

Putting values  $t = 6.5 \text{ s} + 6.3 \text{ s}$

$t = 12.8 \text{ s}$

Hence the total time for takeoff under these conditions is 12.8 seconds

**Example 2.5** Hassan and Umar are standing face to face on ice wearing ice skates. If Hassan apply a force of  $10 \text{ N [E]}$  on Umar (Assume no other opposing force exists), what are their respective accelerations? If mass of Umar is  $80 \text{ kg}$  and Hassan is  $50 \text{ kg}$

Given Data:

Hassan's Mass  $m_H = 50 \text{ kg}$

Umar's Mass  $m_U = 80 \text{ kg}$

Force  $F = 10 \text{ N [E]}$

To Find:

Hassan's acceleration  $a_H = ?$

Umar's acceleration  $a_U = ?$

Solution:

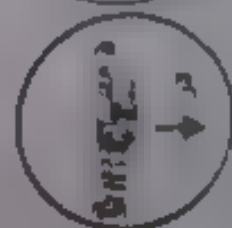
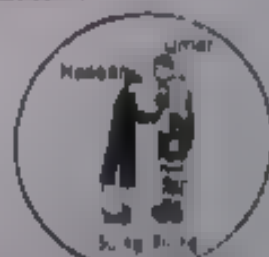
When no other opposing force exists, the action force exerted by Hassan on Umar is  $10 \text{ N}$

∴ the acceleration produced in Umar  $a_U$  by Newton's second law of motion will be

$$a_U = \frac{F}{m_U} = \frac{10 \text{ N [E]}}{80 \text{ kg}} \quad \text{or} \quad a_U = 0.125 \frac{\text{kg} \cdot \text{m/s}^2}{\text{kg}} \text{ [E]}$$

therefore  $a_U = 0.125 \text{ m/s}^2 \text{ [E]}$

The reaction force exerted by Umar on Hassan will be equal and opposite (i.e.  $-10 \text{ N [E]}$ ).



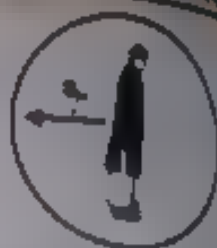
or  $10 \text{ N [W]}$ .The acceleration produced in Hassan  $a_H$  by Newton's second law of motion will be

$$a_H = \frac{F}{m_H} = \frac{-10 \text{ N [E]}}{50 \text{ kg}} \quad \text{or} \quad a_H = -0.2 \frac{\text{kg m/s}^2}{\text{kg}} [\text{E}]$$

or  $a_H = -0.2 \text{ m/s}^2 [\text{E}]$  therefore

$$\text{therefore} \quad a_H = 0.2 \text{ m/s}^2 [\text{W}]$$

Due to small mass Hassan will accelerate more than Umar



A cricket ball of mass  $0.163 \text{ kg}$  has an initial velocity of  $-36 \text{ m/s}$  as it approaches a bat. The batsman hits the ball hard and the ball moves away from the bat with velocity of  $+47 \text{ m/s}$ . (a) Determine the impulse applied to the ball by the bat. (b) Assuming that the time of contact is  $1.6 \text{ ms}$ , find the average force exerted on the ball by the bat.

Given Data:

$$\text{mass 'm'} = 0.163 \text{ kg}$$

$$\text{initial velocity 'v}_i\text{' = } -36.2 \text{ m/s}$$

$$\text{final velocity 'v}_f\text{' = } +47.0 \text{ m/s}$$

$$\text{time of contact '}\Delta t\text{' = } 1.6 \text{ ms} = 1.6 \times 10^{-3} \text{ s}$$

Required:

$$\text{Impulse applied 'J' = ?}$$

$$\text{Average force exerted } \vec{F}_{\text{ave}} = ?$$

Solution:

- (a) According to the impulse-momentum relation  $J = m\vec{v}_f - m\vec{v}_i$   
 Putting values  $J = 0.163 \text{ kg} (+47.0 \text{ m/s}) - (0.163 \text{ kg}) (-36.2 \text{ m/s})$   
 $J = 7.661 \text{ kg m/s} + 5.9006 \text{ kg m/s} = +13.5616 \text{ kg m/s}$   
 Hence  $J = +13.6 \text{ N s}$

- (b) The average force can be calculated by using equation  $J = \vec{F}_{\text{ave}} \times \Delta t$   
 $\vec{F}_{\text{ave}} = \frac{J}{\Delta t}$  putting values  $\vec{F}_{\text{ave}} = \frac{+13.6 \text{ N s}}{0.0016 \text{ s}}$   
 Hence  $\vec{F}_{\text{ave}} = +8400 \text{ N}$

In a nuclear reactor a neutron of mass  $1 \text{ u}$  ( $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$ ) moving a velocity of  $2.000 \text{ km/s}$  to the right and a heavy water molecule mass  $20.0 \text{ u}$  moving with a velocity of  $0.40 \text{ km/s}$  to the left collide head-on. What are the velocities of the neutron and water molecule after the collision?

Given Data:

$$\text{Mass of neutron } m_1 = 1 \text{ u}$$

$$\text{Mass of water molecule } m_2 = 20 \text{ u}$$

$$\text{Velocity of neutron before collision } u_1 = 2.000 \text{ km/s}$$

$$\text{Velocity of water molecule before collision } u_2 = 0.40 \text{ km/s}$$

To Find:

$$\text{Velocity of neutron after collision } v_1 = ?$$

$$\text{Velocity of water molecule after collision } v_2 = ?$$

Solution:

There is no need to convert 'u' into 'kg' as we only want to compare these values.

$$\text{For head on elastic collision} \quad v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + \frac{2m_2}{(m_1 + m_2)} u_2$$

$$\text{Putting values} \quad v_1 = \frac{(1 \text{ u} - 20 \text{ u})}{(1 \text{ u} + 20 \text{ u})} 2.000 \text{ km/s} + \frac{2 \times 20 \text{ u}}{(1 \text{ u} + 20 \text{ u})} 0.40 \text{ km/s}$$

$$\text{Or} \quad v_1 = -1.80952 \text{ km/s} + 0.76 \text{ km/s}$$

$$\text{Hence} \quad v_1 = -1.04952 \text{ km/s}$$

The negative sign shows that the neutron rebounds back after head on collision with the water molecule. Also for



head on elastic collision

$$v_2 = \frac{2m_1}{(m_1 + m_2)} u_1 - \frac{(m_1 - m_2)}{(m_1 + m_2)} u_2$$

putting values  $v_2 = \frac{2 \times 1u}{(1u + 20u)} 2000 \text{ km/s} - \frac{(1u - 20u)}{(1u + 20u)} 0.40 \text{ km/s}$

or  $v_2 = 190.48 \text{ km/s} + 0.38 \text{ km/s}$

therefore  $v_2 = 190.86 \text{ km/s}$

A cricket ball is hit and moves initially at an angle of  $35^\circ$  above the horizontal ground with a velocity of  $25.0 \text{ m/s}$ . (a) How high will the ball go? (b) How long will the ball be in the air? (c) What will be the range for this projectile?

Given Data:

angle  $\theta = 35^\circ$

initial velocity ' $v_0$ ' =  $25.0 \text{ m/s}$

Acceleration due to gravity ' $g$ ' =  $9.8 \text{ m/s}^2$

To Find:

(a) Maximum height ' $H$ ' = ?

(b) Time of flight ' $T$ ' = ?

(c) Horizontal range ' $R$ ' = ?

Solution:

(a) The maximum height  $H$  for projectile is mathematically written as

$$H = \frac{v_0^2 \sin^2 \theta}{2g} \quad \text{putting values} \quad H = \frac{(25 \text{ m/s})^2 \times (\sin 35^\circ)^2}{2(9.8 \text{ m/s}^2)}$$

Therefore  $H = 10.4 \text{ m}$

(b) Time of flight for Projectile is mathematically given as

$$T = \frac{2v_0 \sin \theta}{g} \quad \text{putting values} \quad T = \frac{2 \times (25 \text{ m/s}) \times \sin 35^\circ}{9.8 \text{ m/s}^2}$$

Therefore  $T = 2.93 \text{ s}$

(c) The Horizontal Range  $R$  for projectile is mathematically written as

$$R = \frac{v_0^2 \sin 2\theta}{g} \quad \text{putting values} \quad R = \frac{(25 \text{ m/s})^2 \times (\sin 2 \times 35^\circ)}{(9.8 \text{ m/s}^2)}$$

Therefore  $R = 59.0 \text{ m}$



## Text Book Exercises

Q.1 Select the correct answer of the following questions.

Choose the best possible answer

- A ball is thrown vertically upwards at  $19.6 \text{ m/s}$ . For its complete trip (up and back down to the starting position), its average speed is:  
A.  $19.6 \text{ m/s}$       B.  $9.8 \text{ m/s}$       C.  $6.5 \text{ m/s}$       D.  $4.9 \text{ m/s}$
- If you throw a ball downward, then its acceleration immediately after leaving your hand, assuming no air resistance, is:  
A.  $9.8 \text{ m/s}^2$       B. more than  $9.8 \text{ m/s}^2$   
C. less than  $9.8 \text{ m/s}^2$       D. Speed of throw is required for answer
- The time rate of change of momentum gives:  
A. Force      B. Impulse      C. Acceleration      D. Power

4. The area between the velocity-time graph is numerically equal to:  
 A Velocity B Displacement C Acceleration D Time
5. If the slope of velocity-time graph gradually decreases, then the body is said to be moving with.  
 A Positive acceleration B Negative acceleration C Uniform velocity D Zero acceleration
6. A 7.0-kg bowling ball experiences a net force of 50 N. What will be its acceleration?  
 A  $35 \text{ m/s}^2$  B  $7.0 \text{ m/s}^2$  C  $5.0 \text{ m/s}^2$  D  $0.7 \text{ m/s}^2$
7. SI unit of impulse is:  
 A  $\text{kg m/s}^2$  B  $\text{N s}$  C  $\text{N s}^2$  D  $\text{N s}^{-1}$
8. A ball with original momentum  $+4.0 \text{ kg} \times \text{m/s}$  hits a wall and bounces straight back without losing its kinetic energy. The change in momentum of the ball is  
 A  $+4 \text{ N s}$  B  $-4 \text{ N s}$  C  $+8 \text{ N s}$  D  $-8 \text{ N s}$
9. A body is traveling with a constant acceleration of  $10 \text{ m/s}^2$ . If  $S_1$  is the distance traveled in 1<sup>st</sup> second and  $S_2$  is the distance traveled in 2<sup>nd</sup> second, which of the following shows a correct relation between  $S_1$  and  $S_2$ ?  
 A  $S_1 = S_2$  B  $S_1 = 3 S_2$  C  $S_2 = 3 S_1$  D  $2 S_1 = 3 S_2$
10. During projectile motion, the horizontal component of velocity:  
 A Changes with time B Becomes zero C Remains constant D Increases with time
11. A projectile is thrown horizontally from a 490m high cliff with a velocity of  $100 \text{ m/s}$ . The time taken for projectile to reach the ground is  
 A 2.5 s B 5 s C 7.5 s D 10 s
12. A projectile is launched at  $45^\circ$  to the horizontal with an initial kinetic energy  $E$ . Assuming air resistance is negligible what will be the kinetic energy of the projectile when it reaches its highest point?  
 A 0.50 E B 0.71 E C 0.70 E D E
13. To improve the jumping record the long jumper should jump at an angle of  
 A  $30^\circ$  B  $45^\circ$  C  $60^\circ$  D  $90^\circ$
14. Range of a projectile on a horizontal plane is same for the following pair of angles  
 A  $15^\circ$  and  $18^\circ$  B  $43^\circ$  and  $47^\circ$  C  $20^\circ$  and  $80^\circ$  D  $52^\circ$  and  $62^\circ$

| Note: Options are given in the margin |   |                     | EXPLANATION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|---------------------------------------|---|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1.                                    | B | $9.8 \text{ m/s}$   | $v_f = v_i + gt$ $\Rightarrow 0 = 19.6 - 9.8t$ $\Rightarrow -\frac{19.6}{-9.8} = t$ $\Rightarrow t = 2 \text{ s}$ <p>So, total time of travel on both sides<br/> <math>T = 2 \times 2 = 4 \text{ s}</math></p> <p>Distance to reach maximum height</p> $S = v_i t + \frac{1}{2} g t^2$ $\Rightarrow S = (19.6)(2) + \frac{1}{2}(-9.8)(2)^2$ $\Rightarrow S = 39.2 - 19.6 = 19.6 \text{ m}$ <p>So total distance covered on both sides<br/> <math>S = 2 \times 19.6 = 39.2 \text{ m}</math></p> <p>Average speed is</p> $v_{av} = \frac{\text{Total distance}}{\text{Total time}} = \frac{39.2}{4} = 9.8 \text{ m/s}$ |
| 2.                                    | A | $9.8 \text{ m/s}^2$ | Because after leaving hand, only force acting on ball is gravity or weight of the ball which produces gravitational acceleration.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| 3.                                    | A | Force               | $F = \frac{\Delta p}{\Delta t}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |

|     |                           |                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-----|---------------------------|---------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4.  | B                         | Displacement              | Area under velocity time graph is equal to displacement covered by body                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| 5.  | Beth A & B can be answers |                           | Slope of v-t graph gives acceleration. If slope decreases, then acceleration will decrease.<br>If slope is positive and decreasing then it has decreasing positive acceleration and if slope is negative and slope is decreasing then it has decreasing negative acceleration.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| 6.  | D                         | $0.71 \text{ m/s}^2$      | $a = \frac{F}{m} = \frac{5}{7} = 0.71 \text{ m/s}^2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| 7.  | B                         | $4 \text{ Ns}$            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| 8.  | D                         | $-8 \text{ Ns}$           | $P_i = 4 \text{ kg m/s}$<br>$P_f = -4 \text{ kg m/s}$<br>(as no energy is lost i.e. ball bounces back elastically)<br>$\Delta P = P_f - P_i = -4 - 4 = -8 \text{ kg m/s}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 9.  | C                         | $S_2 = 3 S_1$             | $S_1 = \frac{1}{2} a t^2 = \frac{1}{2} \times 10 \times 1^2 = 5 \text{ m}$ &<br>$S_{nth} = \frac{1}{2} a (t_{nth}^2 - t_{nth-1}^2) \rightarrow S_2 = \frac{1}{2} (10) (2^2 - 1^2)$<br>$S_2 = 5 (4 - 1) = 5 \times 3 = 3 \times 5 = 15$<br>Alternate Method<br>Let $v = 0$ (use $S = vt + \frac{1}{2} at^2$ )<br>$s = \frac{1}{2} at^2$<br>$\Rightarrow s_1 = \frac{1}{2} a(1)^2$<br>$\Rightarrow s_1 = \frac{1}{2} a(1)$<br>$\Rightarrow s_1 = \frac{1}{2} a$<br>and distance after $n$ seconds<br>$s = \frac{1}{2} at^2$<br>$s = \frac{1}{2} a(2)$<br>$s = \frac{1}{2} a(4)$<br>Distance in 2 <sup>nd</sup> second is<br>$S_2 = s - s_1 = \left(\frac{1}{2} a(4)\right) - \frac{1}{2} a$<br>$S_2 = \frac{1}{2} a(4 - 1)$<br>$S_2 = \left(\frac{1}{2} a(3)\right)$<br>$S_2 = S_1 \times 3$<br>$S_2 = 3S_1$ |
| 10. | C                         | Remains constant          | As no force act on the projectile along horizontal.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 11. | D                         | $10 \text{ s}$            | Use 2 <sup>nd</sup> equation of motion<br>$S = ut + \frac{1}{2} at^2$<br>For y axis, equation becomes<br>$S = ut + \frac{1}{2} at^2$<br>$\Rightarrow S = (0)t + \frac{1}{2} gt^2 = 0 + \frac{1}{2} gt^2$<br>$\Rightarrow S = \frac{1}{2} gt^2$<br>$\Rightarrow t = \sqrt{\frac{2S}{g}} = \sqrt{\frac{2 \times 490}{9.8}} = \sqrt{100}$<br>$\Rightarrow t = \sqrt{100} = 10 \text{ s}$                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 12. | A                         | $0.5 E$                   | $K E_{\text{rem}} = K E_i \cos^2 \theta$ (at highest point)<br>$\Rightarrow E' = E \cos^2(45^\circ) = E \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{E}{2} = 0.5 E$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| 13. | B                         | $45^\circ$                | Range is maximum when angle of projection is $45^\circ$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| 14. | B                         | $43^\circ$ and $47^\circ$ | Ranges are equal for angles of projection if their sum is $90^\circ$ i.e. they are complementary angles.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |



## Short Answers of the Exercise

**Q 2** Give a short response to the following questions.

**Q 1** If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward. Why?

**Ans** This is due to relative motion between the two trains.

**Explanation:**

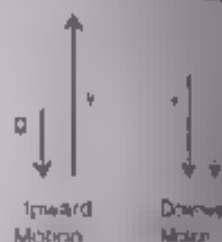
When you are sitting in faster train then you are in rest w.r.t. your train. So, when you look outside the window then poles, trees etc. will appear to you moving in the backward direction (as they are coming closer to you). Similarly, when you cross the slower train then that train will also appear to you as moving in the backward direction, which will indicate that other train is slower than yours.

**Q 2** Can the velocity of a body reverse the direction when acceleration is constant? If you think so, give an example.

**Ans** Yes, it can be possible.

**Example:**

When a body is thrown vertically upward, its velocity goes on decreasing due to gravity and becomes zero at the maximum height. After that, it will reverse its direction of velocity, but the magnitude of acceleration remains constant during whole flight (i.e.  $9.8 \text{ m/s}^2$ ).



**Q 3** When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?

**Ans** When we stand on the ground then we exert force on ground equal to our weight

$$F = W = mg$$

This force acts as action on ground. According to Newton's 3<sup>rd</sup> law, ground exerts equal but opposite reaction. This reaction is just sufficient to keep us standing on ground.

This reaction force can not make us to rise in air. For that purpose, upward force must be greater than our weight which is acting downward.

**Q 4** A man standing on the top of a tower throws a ball vertically up with certain velocity. He also throws another ball vertically down with the same speed. Neglecting air resistance, which ball will hit the ground with higher speed?

**Ans** Both the balls hit the ground with same speed.

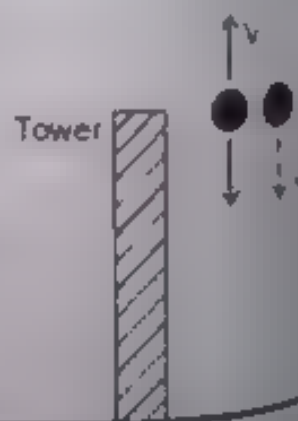
**Explanation:**

The ball which is thrown vertically up with velocity  $v_i$  will have same velocity  $v_i$  when it reaches back to the top of tower. So the two balls have same downward velocity at top of tower. Hence they hit the ground with same final velocity.

$$v_f = v_i + 2gh$$

$$v_f = \sqrt{v_i^2 + 2gh}$$

As  $v_i$ ,  $h$  and  $g$  are same for two balls therefore  $v_f$  is also same.



**Q 5** The cricket coach explains that the follow-through with the shot will make the ball travel greater distance. Explain the reasoning in terms of the impulse-momentum theorem.

**Ans** Follow through is advised to increase the momentum and hence increases velocity of the ball to travel a greater distance.

**Explanation:**

During the follow through, the bat is in contact with the ball for a longer time. According to impulse-momentum theorem, the change in momentum is equal to the impulse.

$$\vec{J} = \vec{F} \times \Delta t \Rightarrow J = \Delta p$$

In order to impart more momentum into the ball so that the ball travels a greater distance, a greater impulse is needed which is achieved by a longer time of contact between bat and ball.

**Q.6** When you release an inflated but untied balloon, why does it fly across the room?

**Ans.** When you release an inflated but untied balloon, it flies across the room to conserve the linear momentum.

**Explanation.**

In an inflated balloon, air is at higher pressure than atmospheric pressure. When it is released, air escapes from the balloon that carries momentum. To conserve the momentum, the balloon acquires momentum which is exactly opposite to the momentum of the escaping air under the effect of reaction force i.e.  $P_{\text{air}} = P_{\text{balloon}}$ .

**Q.7** Modern cars are not rigid but are designed to have 'crumple zones' (irregular fold) that collapse upon impact. What is the advantage of this new design?

**Ans.** Crumple zones like bumpers are made in the cars to reduce the force during an accident.

**Explanation.**

We know that

$$J = \vec{F} \times \Delta t$$

$$\Rightarrow F = \frac{J}{\Delta t}$$

During an accident, crumple zones like bumpers collapse, that will increase the collision time ( $\Delta t$ ) and hence reduce the impact force. This will make passengers safer.

We can also say that crumple zone absorbs the energy during head-on collision, passenger will feel lesser jerk during collision and remain safe.

**Q.8** Why we can hit a long sixer in a cricket match rather than if we toss a ball for our selves?

**Ans.** It may be due to

- i. In Cricket match, the time of collision between ball and bat decrease which increase the striking force. As a result a ball go larger distance.

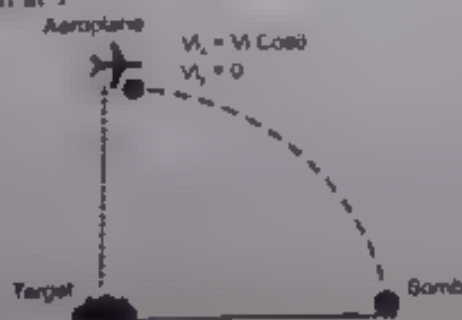
$$F = \frac{J}{\Delta t}$$

So, when batsman hit pitched ball,  $\Delta t$  is small so force will be greater.

- ii. The motion of the ball when hit for six is like a projectile motion. A projectile has maximum range when it is hit at angle of  $45^\circ$  with the ground. In a cricket match, this angle is possible easily to achieve but it is difficult to achieve when we toss the ball for ourselves.
- iii. When opponent bowler throws the ball usually with a greater force and we strike it with proper follow through then it will increase the collision time and hence increases the momentum imparted to ball. This will increase the speed of the ball sufficiently to go for longer distances.

**Q.9** An aeroplane while travelling horizontally, dropped a bomb when it was exactly above the target, the bomb missed the target. Explain.

**Ans.** The bomb has the same velocity as that of the aeroplane when it is dropped. If the bomb is dropped when the aeroplane is vertically above the target, it will strike a point ahead of the target due to constant horizontal velocity component and inertia, the bomb misses the target. The bomb moves like a projectile as shown in fig. It will not hit the target but it will hit at T.



**Q 10** Calculate the angle of projection for which kinetic energy at the summit is equal to one-fourth of its kinetic energy at point of projection.

**Sol** K.E.<sub>sum</sub> of the projectile at highest point can be written as

$$K.E._{sum} = \frac{1}{2} m v_1^2 \cos^2 \theta$$

$$\therefore K.E._{sum} = K.E. \cos^2 \theta \quad \left[ \because \frac{1}{2} m v^2 = K.E. \right]$$



Given condition:

$$K.E._{sum} = \frac{1}{4} K.E.$$

Put value of K.E.<sub>sum</sub>, so

$$K.E. \cos^2 \theta = \frac{1}{4} (K.E.)$$

$$\Rightarrow \cos^2 \theta = \frac{1}{4}$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1} \left( \frac{1}{2} \right)$$

$$\theta = 60^\circ$$

**Q 11** For any specific velocity of projection, the maximum range is equal to four times of the corresponding height. Discuss.

**Sol** Maximum range of projectile is at  $\theta = 45^\circ$  and it is given by

$$R_{max} = \frac{v_1^2}{g}$$

Height of projectile is

$$H = \frac{1}{4} R_{max}$$

At angle of projection  $\theta = 45^\circ$

$$H = \frac{v_1^2 \sin^2 (45^\circ)}{2g}$$

$$\Rightarrow H = \frac{v_1^2}{2g} \left( \frac{1}{\sqrt{2}} \right)^2$$

$$\Rightarrow H = \frac{v_1^2}{2g} \cdot \frac{1}{2} = \frac{v_1^2}{4g}$$

$$H = \frac{1}{4} \left( \frac{v_1^2}{g} \right)$$

$$\left[ R_{max} = \frac{v_1^2}{g} \right]$$

$$\Rightarrow H = \frac{1}{4} R_{max}$$

$$\Rightarrow R_{max} = 4 \times H$$

**Q 12** What is the angle for which the maximum height reached and corresponding range are equal.

**Sol** Given Data,

Maximum height = horizontal range

To Find

Angle of projection =  $\theta = ?$

## Calculations

- Ans Maximum height  $H = \frac{v^2 \sin^2 \theta}{2g}$
- Range of projectile  $R = \frac{v^2 \sin 2\theta}{g}$
- According to given condition,
- $$H = R$$
- $$\frac{v^2 \sin^2 \theta}{2g} = \frac{v^2 \sin 2\theta}{g}$$
- or  $\sin^2 \theta = 2 \sin 2\theta$
- or  $\sin \theta = 4 \cos \theta$
- or  $\tan \theta = 4$
- or  $H = \tan^2 \theta$



### Topic-wise Questions

**Q3** Give a short response to the following questions

1. Explain displacement - time graph and velocity - time graph. In each type give brief details along with appropriate diagram for illustration.

**Ans** See Q # 8, 9 and 10

2. Apply Newton's Laws to explain the motion of objects in a variety of context.

**Ans** See Q # 12

3. What is linear momentum? Derive and state Newton's second law in terms of linear momentum

**Ans** See Q # 3 and 15

4. State and explain law of conservation of linear momentum for an isolated system of bodies

**Ans** See Q # 7 and 8

5. Define elastic and inelastic collisions. Give examples in each case. Derive mathematical equations for calculating the final velocities of the elastically colliding bodies in one dimension

**Ans** See Q # 9 and 10

6. What is projectile motion? Give examples. Find out the expression of instantaneous velocity for a projectile.

**Ans** See Q # 3

7. What is maximum height and time of flight for projectile? Derive mathematical equations for Maximum height attained and time of flight

**Ans** See Q # 2

8. What is range of a projectile. State in which condition the range will be maximum if speed of projection is kept constant in a uniform gravitational field. Also show that there are two projection angles for the same range.

**Ans:** See Q # 24 and 25



## NUMERICAL QUESTIONS

1. An object is falling freely under gravity. How much distance will it travel in 2nd and 3rd second of its journey?

**Given data:**

$$s_{1st} = ?$$

$$s_{2nd} = ?$$

**To Find:**

To find the distance in a specific second for a freely falling body we have

$$s_{nth} = \frac{1}{2}g(t_{nth}^2 - (t_{(n-1)th}^2))$$

Distance covered in 2<sup>nd</sup> second is

$$s_{2nd} = \frac{1}{2} \times 9.8(2^2 - 1^2)$$

$$s_{2nd} = 4.9(4 - 1) = 4.9(3) = 14.7 \text{ m} = 15 \text{ m}$$

Distance covered in 3<sup>rd</sup> second is:

$$s_{3rd} = \frac{1}{2} \times 9.8(3^2 - 2^2)$$

$$s_{3rd} = 4.9(9 - 4) = 4.9(5) = 24.5 \text{ m} = 25 \text{ m}$$

2. A helicopter is ascending vertically at a speed of  $19.6 \text{ m s}^{-1}$ . When it is at a height of  $156.8 \text{ m}$  above the ground a stone is dropped. How long does the stone take to reach the ground?

**Given data:**

Initial velocity of the helicopter =  $v_i = 19.6 \text{ m s}^{-1}$  (upward)

Net vertical distance covered by the stone =  $s = -156.8 \text{ m}$  (downward)

(There is negative sign, being displacement opposite to initial velocity)

Acceleration due to gravity =  $g = 9.8 \text{ m s}^{-2}$

**To Find**

Time =  $t = ?$

Using the following equation of motion

$s = v_i t + \frac{1}{2} g t^2$ , putting the values, we get

$$-156.8 = 19.6t + \frac{1}{2}(-9.8)t^2$$

$$-156.8 = 19.6t - 4.9t^2$$

$$4.9t^2 - 19.6t - 156.8 = 0$$

$$4.9(t^2 - 4t - 32) = 0$$

$$t^2 - 4t - 32 = 0, \text{ making factors, we get}$$

$$t^2 - 8t + 4t - 32 = 0$$

$$(t - 8) + 4(t - 8) = 0$$

$$(t - 8)(t + 4) = 0$$

$$\text{or } t - 8 = 0, t = 8 \text{ sec}$$

But time cannot be negative, thus

**8 sec** (i.e. time taken by stone)

$$\text{OR } t + 4 = 0, t = -4 \text{ sec}$$

3. A car moving at 20.0 m/s (72.0 km/h) crashes into a tree. Find the magnitude of the average force acting on a passenger of mass 70 kg in each of the following cases. (a) The passenger is not wearing a seat belt. He is brought to rest by a collision with the windshield and dashboard that lasts 2.0 ms. (b) The car is equipped with a passenger-side air bag. The force due to the air bag acts for 45 ms, bringing the passenger to rest.

**Solution** Initial velocity =  $v = 0$   
Mass of passenger =  $m = 70$  kg

**To Find**

- (a) Average force on passenger for  
 $\Delta t = 2 \text{ ms} = 2 \times 10^{-3} \text{ sec}$   
(b) Average force on passenger for  
 $\Delta t = 45 \text{ ms} = 45 \times 10^{-3} \text{ sec}$

**Calculation**

(a)  $J = F_{\text{ave}} \Delta t$

$\Rightarrow F_{\text{ave}} = \frac{J}{\Delta t} \quad (1)$

Also,  $\Delta v = v_f - v_i = 0 - 20 = -20 \text{ m/s}$

$\Rightarrow J = -1400 \text{ N s}$

Now putting  $J = -1400 \text{ N s}$  and  $\Delta t = \Delta t = 2 \times 10^{-3} \text{ sec}$  in equation (1), we get

$$F_{\text{ave}} = \frac{-1400}{2 \times 10^{-3}} \text{ N} = -700 \times 10^3 \text{ N} = -7 \times 10^5 \text{ N} \quad \text{in magnitude } F_{\text{ave}} = 7 \times 10^5 \text{ N}$$

(b) Putting  $J = -1400 \text{ N s}$  and  $\Delta t = \Delta t = 45 \times 10^{-3} \text{ sec}$  in equation (1), we get,

$$F_{\text{ave}} = \frac{-1400}{45 \times 10^{-3}} \text{ N} = -31.1 \times 10^3 \text{ N}$$

$\Rightarrow \overline{F_{\text{ave}}} = 3.11 \times 10^4 \text{ N} \quad \text{in magnitude } F_{\text{ave}} = 3.11 \times 10^4 \text{ N}$

4. A 0.4 kg ball traveling with the speed of 15 m/s strikes a rigid wall and rebounds elastically. If the ball is in contact with the wall for 0.045 s, what is (a) the momentum imparted to the wall and (b) the average force exerted on the wall? (12 kg m/s, 266.7 N)

**Solution** Mass of ball =  $m = 0.4$  kg  
Velocity of ball =  $v = 15$  m/s  
Time interval =  $\Delta t = 0.045$  s

(a) Momentum imparted to the wall =  $\Delta P = ?$

(b) Average force exerted on the wall =  $F_{\text{ave}} = ?$

- (a) For elastic collision, the change in momentum of the ball due to force applied by wall is given by

$$\begin{aligned} \Delta P &= P_f - P_i = mv_f - mv_i \\ &= m(v_f - v_i) \\ &= -2mv \end{aligned}$$

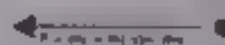
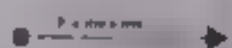
So, the momentum imparted to wall by ball is

$$\Delta P = +2mv$$

$\Rightarrow \Delta P = 2 \times 0.4 \times 15 = 12 \text{ kg m/s}$

(b)  $F_{\text{ave}} = \frac{\Delta P}{\Delta t}$

$\Rightarrow F = \frac{12}{0.045} = 266.7 \text{ N}$



- 5 One ball of mass 0.500 kg traveling 9.00 m/s to the right collides head on elastically with a second ball of mass 0.300 kg traveling 8.00 m/s to the left. After the collision, what are the velocities after collision? (2.33 m/s (2.33 m/s to right) and 14.67 m/s (14.67 m/s to left))

### Solution

Mass of 1<sup>st</sup> ball =  $m_1 = 0.500 \text{ kg}$

$u_1$  = Speed of 1<sup>st</sup> ball = 9.00 m/sec

Mass of 2<sup>nd</sup> ball =  $m_2 = 0.300 \text{ kg}$

$u_2$  = Speed of 2<sup>nd</sup> ball = 8 m/sec

### Given Data

a. Velocity of 1<sup>st</sup> ball after collision =  $v_1 = ?$

b. Velocity of 2<sup>nd</sup> ball after collision =  $v_2 = ?$

### Calculation

a. We know that,

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2 u_2}{m_1 + m_2} \quad \text{--- (1)}$$

$$\text{Substituting } v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2 u_2}{m_1 + m_2} \quad \text{--- (2)}$$

Putting the values in eq. (1) we get

$$v_1 = \frac{0.600 - 0.300}{0.600 + 0.300} (9.00) + \frac{2 \times 0.300 \times 8}{0.600 + 0.300} \text{ m/sec}$$

$$v_1 = \frac{0.3 \times 9}{0.9} + \frac{4.8}{0.9} \text{ m/sec} = 3.534 \text{ m/sec}$$

$$v_1 = 2.33 \text{ m/sec}$$

b. Putting the value in eq. (2) we get

$$v_2 = \left[ \frac{0.300 - 0.600}{0.600 + 0.300} (9) + \frac{2 \times 0.600 \times 9}{0.600 + 0.300} \right] \text{ m/sec}$$

$$v_2 = \left[ \frac{0.3 \times 9}{0.9} + \frac{10.8}{0.9} \right] \text{ m/sec} = (2.666 + 12) \text{ m/sec}$$

$$v_2 = 2.67 + 12 \text{ m/sec} = 14.67 \text{ m/sec}$$

$$v_2 = 14.67 \text{ m/sec}$$

- 6 In a wedding a bullet is fired in air at a speed of 500 m/s making an angle of  $60^\circ$  with horizontal from an AK 47 rifle. (a) How high will the bullet rise? (b) What time would it take to reach ground? (c) How far would it go?

(Ignore air resistance) (a) 9.560 m (b) 88.3 s (c) 22.078 m)

### Solution

### Given Data

Initial speed of bullet in air =  $u = 500 \text{ m/sec}$

Angle of projection =  $60^\circ$

### Required

a. Max. height =  $h$  (m)

b. Time taken to reach ground =  $t$  (s)

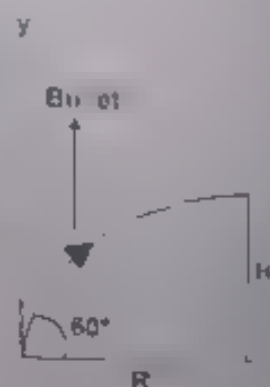
c. Horizontal range =  $R$  (m)

### Calculation

a. We know that the max. height is given by

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

The negative sign is used because both bodies move in opposite direction.



Putting the values in equation (1), we get

$$H = \left[ \frac{(20)^2 \sin^2 60^\circ}{2 \times 9.8} \right] \text{ m} = \frac{250000 \times 0.75}{9.8} \text{ m}$$

$$\Rightarrow H = 9566.3 \text{ m}$$

We know that the time of flight is given by

$$T = \frac{2v \sin \theta}{g} \quad (2)$$

Putting the values in equation (2), we get

$$T = \frac{2 \times 200 \times \sin 60^\circ}{9.8} \text{ sec} = \frac{3000 \times 0.866}{9.8} \text{ sec}$$

$$\Rightarrow T = \frac{866}{4.9} \text{ sec} = 88.20 \text{ sec} = 88 + \text{sec}$$

$$\Rightarrow T = 88.4 \text{ sec}$$

We know that the horizontal range is given by

$$R = \frac{v^2 \sin 2\theta}{g}$$

Putting the values in equation (3), we get

$$R = \frac{(200)^2 \sin 120^\circ}{9.8} = \frac{25000 \times 0.866}{9.8}$$

$$\Rightarrow R = \frac{216500}{9.8} \text{ m} = 22091.8 \text{ m}$$

$$\Rightarrow R = 22091.8 \text{ m}$$

7. The catapult hurls a stone of mass 32.0 g with a velocity of 50.0 m/s at a  $33.0^\circ$  angle of elevation. (a) What is the maximum height reached by the stone? (b) What is its range? (c) How long has the stone been in the air when it returns to its original height? a) 11.37 m b) 51.88 m c) 2.08 s

#### Given Data

Mass of stone =  $m = 32 \text{ g}$

Initial velocity =  $v = 50 \text{ m/s}$

Angle of launch =  $\theta = 33^\circ$

#### Required

- Maximum height =  $H = ?$
- Horizontal range =  $R = ?$
- Time of flight =  $t = ?$

#### Calculation:

- (a) We know that maximum height is given by

$$H = \frac{v^2 \sin^2 \theta}{2g} \quad (1)$$

Putting the values in equation (1), we get

$$H = \left[ \frac{(50)^2 \sin^2 33^\circ}{2 \times 9.8} \right] \text{ m} = \frac{2500 \times 0.35}{19.6}$$

$$\Rightarrow H = \left[ \frac{2500 \times 0.35}{19.6} \right] \text{ m} = 11.38 \text{ m}$$

$$\Rightarrow H = 11.38 \text{ m}$$

We know that horizontal range is given by

$$R = \frac{v^2 \sin 2\theta}{g} \quad (2)$$

Putting the values in equation (2), we get

$$R = \frac{(50)^2 \sin 66^\circ}{9.8} = \frac{2500 \times 0.9135}{9.8}$$

$$\Rightarrow R = \left[ \frac{2500 \times 0.866}{9.8} \right] \text{m} = 220.9 \text{m}$$

$$\Rightarrow R = 220.9 \text{m}$$

(c) We know that time of flight is given by,

$$T = \frac{2v \sin \theta}{g} \quad (3)$$

Putting the values in equation (3), we get,

$$T = \frac{2 \times 50 \times \sin(30^\circ)}{9.8} = \left( \frac{100 \times 0.5}{9.8} \right) \text{sec}$$

$$\Rightarrow T = 5.10 \text{ sec}$$



## Additional Conceptual Short Questions With Answers

1 Why does a cricket player retrace his hands backward while catching the ball?

**Ans:** Player does so because it will allow him to have greater time for change in momentum.

$$\text{Since } F = \frac{\Delta p}{\Delta t} \text{ (}\Delta t \text{ becomes greater)} \Rightarrow F \propto \frac{1}{\Delta t}$$

$\therefore$  So ball will exert a lesser force on his hands.

2 Show that relation between maximum height and time of flight of projectile is  $H = \frac{gT^2}{8}$ .

**Ans:** Maximum height of projectile is

$$H = \frac{v^2 \sin^2 \theta}{2g} \quad (1)$$

And its time of flight is

$$T = \frac{2v \sin \theta}{g}$$

$$\Rightarrow (T)^2 = \left( \frac{2v \sin \theta}{g} \right)^2$$

$$T^2 = 4 \left( \frac{v^2 \sin^2 \theta}{g^2} \right)$$

$$T^2 = \frac{4}{g} \times \left( \frac{v^2 \sin^2 \theta}{2g} \right)$$

$$T^2 = \frac{4}{g} (H) \quad [\text{Put value from eq. (1)}]$$

$$\Rightarrow H = \frac{gT^2}{8}$$

3 Show that range of projectile on moon is 8 times greater than its range on the earth for same velocity and angle of projection  $\theta$ .

**Ans:** Let range of projectile on earth is

$$R = \frac{v^2}{g} \sin 2\theta$$

Let  $R$  be the range of projectile on the moon,

$$R' = \frac{v^2}{g'} \sin 2\theta$$

Since value of gravitational acceleration on the moon is

$$g' = \frac{g}{6}$$

$$\Rightarrow R = \frac{v^2}{\left(\frac{g}{6}\right)} (\sin 2\theta)$$

$$R = 6 \left[ \frac{v^2}{g} \sin 2\theta \right]$$

$$R = 6R'$$

So range of projectile on the moon is 6 times greater than its value on the earth

4. The horizontal range of projectile is 4 times of its maximum height. What is its angle of projection?

Ans. Condition  $\rightarrow R = 4H$

$$\Rightarrow \frac{v^2}{g} \sin 2\theta = 4 \frac{v^2 \sin^2 \theta}{2g}$$

$$\Rightarrow \sin 2\theta = 2 \sin^2 \theta$$

$$2 \sin \theta \cos \theta = 2 \sin^2 \theta$$

$$\Rightarrow \cos \theta = \sin \theta$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = 1$$

$$\Rightarrow \tan \theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45^\circ$$

6. In long jump, what factors determine the span of the jump?

Ans. Long jump is the example of projectile motion. So span of the jump is like the range of projectile.

$$R = \frac{v^2}{g} \sin 2\theta$$

So span of jump depends upon angle of projection (i.e. angle with horizontal) and initial velocity of the athlete. This is why an athlete starts running before he takes the jump.

8. The angle of projection of a projectile is  $30^\circ$ , then find the ratio between its maximum height and range of projectile?

Ans. If  $\theta = 30^\circ$

Then

$$R = \frac{v^2}{g} \sin 2(30^\circ)$$

$$R = \frac{v^2}{g} \sin 60^\circ$$

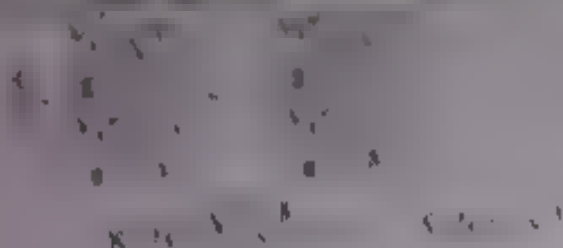
$$R = \frac{v^2}{g} \left( \frac{\sqrt{3}}{2} \right)$$

$$H = \frac{v^2 \sin^2 30^\circ}{2g}$$

$$H = \frac{v^2 \left( \frac{1}{2} \right)^2}{2g}$$

$$H = \frac{v^2}{8g}$$

So ratio between horizontal range and height is.



Two points are plotted in the plane at  $70^\circ$  and  $80^\circ$  with the horizontal. The horizontal distance between the two points is 10 units. The vertical distance between the two points is 10 units. The horizontal distance between the two points is 10 units. The vertical distance between the two points is 10 units.

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Can there be a y acceleration when a body is moving with a constant speed?

Yes, no acceleration when a body is moving with a constant speed. Example: when a car is moving along a circular path with a constant speed, it will have an acceleration in the direction of the center of the circle.

Newton's first law of motion states that a body at rest or moving with a constant velocity will remain in that state unless acted upon by an external force.

A body at rest or moving with a constant velocity will remain in that state unless acted upon by an external force.

If  $v = 0$ , then  $a = 0$ . If  $v \neq 0$ , then  $a \neq 0$ . If  $v = 0$ , then  $a = 0$ . If  $v \neq 0$ , then  $a \neq 0$ . If  $v = 0$ , then  $a = 0$ . If  $v \neq 0$ , then  $a \neq 0$ .

Newton's first law of motion states that a body at rest or moving with a constant velocity will remain in that state unless acted upon by an external force.

- 10 At which angle of projection, the range of projectile is half of its maximum range?

**Ans:** The range of projectile is given by

$$R = \frac{1}{2} (R_{\max})$$

$$\frac{V \sin 2\theta}{g} = \frac{1}{2} \left( \frac{V^2}{g} \right)$$

$$\sin 2\theta = \frac{1}{2}$$

$$2\theta = \sin^{-1} \left( \frac{1}{2} \right)$$

$$2\theta = 30^\circ$$

$$\theta = 15^\circ$$

Hence the range of projectile is half than its maximum range at an angle of  $15^\circ$ .

- 11 Two blocks of masses  $m_1$  and  $m_2$  are resting on a frictionless horizontal surface and are in good contact with each other. A force  $F$  acts on  $m_1$ , as shown in fig 3.17. Derive equation to find the magnitudes of effective forces acting on  $m_1$  and  $m_2$  respectively.

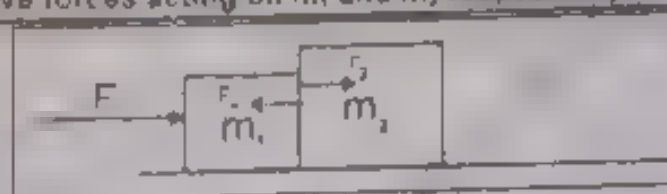


Fig 3.17

**Ans:** Let  $a$  be the acceleration produced in the whole system due to external applied force  $F$  and is given by

$$F = (m_1 + m_2)a \Rightarrow a = \frac{F}{(m_1 + m_2)} \quad (i)$$

Since the masses move together, acceleration is same for both of them. Let  $F_2$  be the effective force acting on  $m_2$  due to  $m_1$  and is given by

$$F_2 = m_2 a = m_2 \left[ \frac{F}{(m_1 + m_2)} \right]$$

$$\text{or } F_2 = \left[ \frac{m_2}{(m_1 + m_2)} \right] F \quad (ii)$$

According to Newton's 3rd law,  $m_1$  and  $m_2$  exert the same forces in magnitude but opposite in direction as shown. Thus on  $m_1$ , now two forces are acting. Let  $F_1$  be the net force acting on  $m_1$  and is given by

$$F_1 = F - F_2 = F - \left[ \frac{m_2}{(m_1 + m_2)} \right] F = \left[ \frac{m_1}{(m_1 + m_2)} \right] F \quad (iii)$$

Eq (ii) and Eq (iii) can be used to find the magnitudes of effective forces acting on  $m_1$  and  $m_2$  in such cases.



# MCQ's From Past FRISE PAPER (FEDERAL BOARD)

1. When body is in motion its \_\_\_\_\_ always changes  
 A. Velocity B. Acceleration C. Displacement D. Motion
2. Instantaneous and average velocities become equal when body \_\_\_\_\_  
 A. has zero acceleration B. has uniform acceleration  
 C. has variable acceleration D. moves in a circle
3. Which of the following pair has same direction always?  
 A. Force displacement B. Force velocity  
 C. Force acceleration D. Force momentum
4. Area under V-t graph gives \_\_\_\_\_  
 A. Speed B. Distance C. Acceleration D. Momentum
5. Rate of change of momentum of a body is equal to \_\_\_\_\_  
 A. Force B. Impulse C. Acceleration D. Momentum
6. Angle between the velocity and acceleration at the maximum height of the projectile is  
 A. 0 B.  $45^\circ$  C.  $90^\circ$  D.  $135^\circ$
7. At angle of projection of  $45^\circ$ , height of projectile will equal to \_\_\_\_\_ times of range  
 A. 1 B.  $1/4$  C.  $2/\sqrt{2}$  D.  $1/\sqrt{2}$
8. If velocity of moving body is doubled then stopping distance will become \_\_\_\_\_  
 A. Twice B. Four times C. Three times D. Remain same
9. When velocity of moving object is doubled then which of following quantity becomes double  
 A. Acceleration B. Kinetic energy C. Momentum D. Displacement
10. If range of projectile is half of its maximum range then angle of projection is \_\_\_\_\_  
 A.  $30^\circ$  B.  $22.5^\circ$  C.  $45^\circ$  D.  $60^\circ$
11. The unit of impulse is \_\_\_\_\_  
 A. N B.  $\text{N s}$  C.  $\text{N m}$  D.  $\text{N s}^2$
12. One dyne is equal to \_\_\_\_\_  
 A.  $10^{-5}$  N B.  $10^{-8}$  N C.  $10^{-7}$  N D.  $10^{-6}$  N
13. If projectile is projected at  $\theta = 45^\circ$  with initial velocity  $V$  then velocity at highest point is \_\_\_\_\_  
 A. 0 B.  $\frac{V}{2}$  C.  $\frac{V}{\sqrt{2}}$  D.  $\frac{V}{4}$
14. The direction of acceleration is always along the direction of \_\_\_\_\_  
 A. velocity B. Momentum C. Force D. None of these
15. Distance covered by a freely falling body in 2 seconds will be \_\_\_\_\_  
 A. 2 m B. 10.2 m C. 9.4 m D. 19.6 m
16. A car starts from rest and covers a distance of 100 m in one second with uniform acceleration is \_\_\_\_\_  
 A.  $100 \text{ ms}^{-2}$  B.  $200 \text{ ms}^{-2}$  C.  $100 \text{ ms}^{-1}$  D.  $200 \text{ ms}^{-1}$
17. A ball rolls off the edge of a table. The horizontal component of the ball's velocity remains constant during its entire trajectory because \_\_\_\_\_  
 A. The force acting on the ball is zero  
 B. The ball is not acted upon by a force in the horizontal direction  
 C. The ball is not acted upon by a force in the horizontal direction  
 D. The ball is not acted upon by a force in the vertical direction

18. Motorcycle safety helmet extends the time of collision and decreaseser \_\_\_\_\_ (ANNUAL 2017)  
 A Impulse B Change of collision C Force acting D Velocity of Vehicle
19. A brick of mass 2 kg is dropped from a rest position 5 m above the ground. What is its velocity at a height of 3 m above the ground? (ANNUAL 2017)  
 A 2.4 m/s B 6.3 m/s C 7 m/s D 12 m/s
20. The motion and rest are. (ANNUAL 2018)  
 A Discrete B Random C Absolute D Relative
21. The change in position of a body from initial position to final position is called (ANNUAL 2018)  
 A Displacement B Acceleration C Position vector D Velocity
22. The notation delta ( $\Delta$ ) is used to represent a. (ANNUAL 2018)  
 A Small change B Big change C Zero change D Very small change
23. When a block of wood of mass 2 kg is pushed along a horizontal flat surface of a bench, the force of friction is 4N. When the block is pushed along the bench with a force of 50N, it moves with a constant. (ANNUAL 2018)  
 A Speed of 5ms<sup>-1</sup> B Acceleration of 5ms<sup>-2</sup> C Acceleration of 5ms<sup>-1</sup> D Speed of 5ms<sup>-1</sup>
24. A projectile is thrown so that it travels a maximum range of 1000m. How high will it rise? (ANNUAL 2018)  
 A 400m B 500m C 600m D 250m
25. A car takes 1 hour to travel 100 km along a main road and then ¼ hour to travel 20 km along a side road. What is the average speed of the car for the Whole Journey? (ANNUAL 2018)  
 A 60kmh<sup>-1</sup> B 70kmh<sup>-1</sup> C 80kmh<sup>-1</sup> D 100kmh<sup>-1</sup>
26. Rate of change of velocity is called \_\_\_\_\_ (ANNUAL 2018)  
 A Speed B Distance C Acceleration D Displacement

## Answers Key

|    |   |    |   |    |   |    |   |    |   |    |   |    |   |    |   |    |   |
|----|---|----|---|----|---|----|---|----|---|----|---|----|---|----|---|----|---|
| 18 | A | 19 | B | 20 | D | 21 | A | 22 | A | 23 | B | 24 | D | 25 | C | 26 | C |
|----|---|----|---|----|---|----|---|----|---|----|---|----|---|----|---|----|---|



## SELF ASSESSMENT PAGE

Total Mark 40

17 + 8 = 25

Question No 1

Choose the correct answer from the given options

## SECTION - A

- A ball is thrown vertically upwards at  $19.6 \text{ m/s}$ . For its complete trip (up and back down to the starting position), its average speed is  
(A)  $19.6 \text{ m/s}$  (B)  $9.8 \text{ m/s}$  (C)  $6.5 \text{ m/s}$  (D)  $4 \text{ m/s}$
- A projectile is thrown horizontally from a  $490 \text{ m}$  high cliff with a velocity of  $100 \text{ m/s}$ . The time taken by projectile to reach the ground is  
(A)  $2.5 \text{ s}$  (B)  $5 \text{ s}$  (C)  $7.5 \text{ s}$  (D)  $10 \text{ s}$
- If range of projectile is half of its maximum range then angle of projection is  
(A)  $30^\circ$  (B)  $22.5^\circ$  (C)  $15^\circ$  (D)  $10^\circ$
- If the slope of the Velocity Time Graph remains constant then body is moving with  
(A) Uniform Velocity (B) Negative Variable Acceleration  
(C) Variable Acceleration (D) Uniform Acceleration
- If a force of  $10 \text{ N}$  acts on a body of mass  $5 \text{ kg}$  for one second, what is its rate of change of momentum?  
(A)  $10 \text{ kg m/s}$  (B)  $50 \text{ kg m/s}$  (C)  $5 \text{ kg m/s}$  (D)  $2 \text{ kg m/s}$
- If the body of mass  $2 \text{ kg}$  moving with  $15 \text{ m/s}$  collides with stationary body of same mass then after elastic collision the 2nd body will move with the velocity of  
(A)  $15 \text{ m/s}$  (B)  $30 \text{ m/s}$  (C) Zero  $\text{m/s}$  (D)  $7.5 \text{ m/s}$

Question No 2

Give short answers of following

17

## SECTION - B

- Show that final velocity of two bodies after collision is given by  $v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$
- On a highway a car of mass  $1500 \text{ kg}$  stopped at traffic signal. A truck of mass  $3000 \text{ kg}$  moving from behind and hits the stopped car. Assuming the collision is elastic, the truck comes to rest and just the car starts moving on the highway at  $10.0 \text{ m/s}$ . How fast was the truck before collision?
- Define impulse and its relation to momentum?
- Modern cars are not rigid but are designed to have crumple zones. Explain the advantage of this new design?
- A bomb is dropped from a plane traveling horizontally. Explain the path of the bomb.
- A  $0.4 \text{ kg}$  ball moving with the speed of  $15 \text{ m/s}$  strikes a rigid wall and rebounds with the same speed. What is (a) the momentum imparted to the wall and (b) the force exerted on the wall?
- Differentiate between elastic and inelastic collision. Give example of each.

Question No 3

Exclusive Questions

## SECTION - C

- For a collision of two bodies, show that speed of approach of two bodies is equal to speed of separation.
- A projectile is thrown with initial velocity  $v$ , making an angle  $\theta$  with the horizontal. Find  
(a) Maximum height (b) Time of flight (c) Range

\*\*\* The End \*\*\*

# CHAPTER

# 4

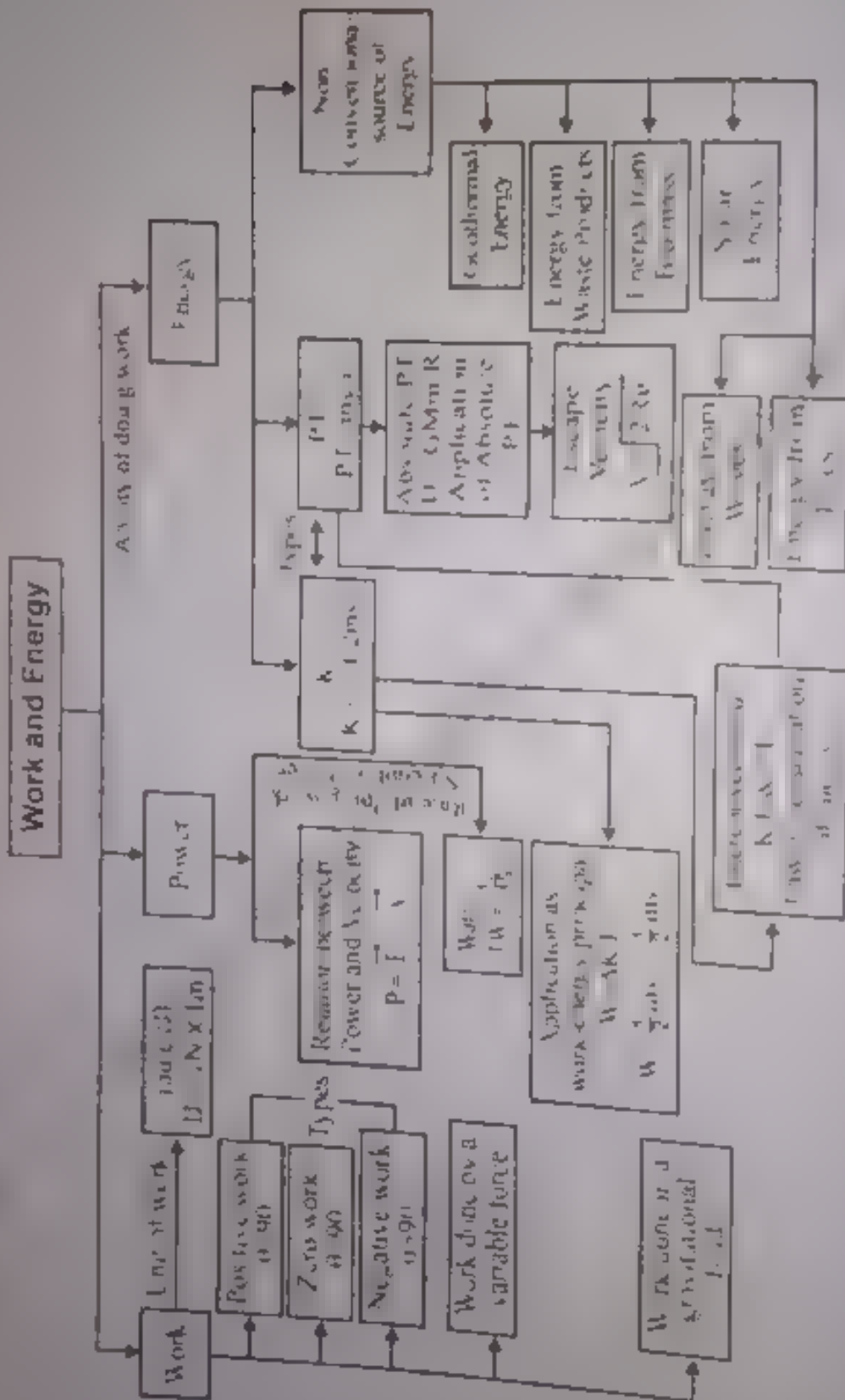
## WORK & ENERGY

### Learning Objectives

- ❖ Describe the concept of work in terms of the product of the force  $\vec{F}$  and displacement  $\vec{d}$  in the direction of force.
- ❖ Distinguish between positive, negative and zero work with suitable examples.
- ❖ Plot the work done by a constant force as a function of the displacement  $g$  graph.
- ❖ Explain gravitational field as an example of field of force and define gravitational field strength as force per unit mass at a given point.
- ❖ Prove that gravitational field is a conservative field.
- ❖ Compute and show that the work done by gravity is when a mass  $m$  is moved from one given point to another point in a gravitational field.
- ❖ Describe that the gravitational potential energy is measured from a reference level and can be positive or negative depending on the location from the reference level.
- ❖ Define the gravitational potential as work done in bringing unit mass from infinity to that point.
- ❖ Express the concept of escape velocity in terms of gravitational constant  $G$ , mass  $m$  and radius of the planet  $r$ .
- ❖ Differentiate conservative and non-conservative forces giving examples of each.
- ❖ Express power as a scalar product of force and velocity.
- ❖ Explain that work done against friction is dissipated as heat in the environment.
- ❖ State the implications of energy losses in practical devices and the concept of efficiency.
- ❖ Utilize work-energy theorem in a resistive medium to solve problems.
- ❖ Discuss and make a list of simulations of some conventional sources of energy.
- ❖ Discuss potentials of some non-conventional sources of energy.

# Chapter No. 4

## CONCEPT MAP



## Introduction

The energy of an object changes if an exchange of energy occurs between the object and its environment. Such a transfer can be due to a force or due to an exchange of heat.

The transfer of energy via force is a process called doing work. Doing work is being **force** during the energy. Work, then, is **transferred energy**. Energy transferred to the object is **positive work**. Energy transferred from the object is **negative work**.

Work is often thought in terms of physical or mental effort. In Physics, however, the term involves two things: **force** and **displacement**.

**Q 1** What do you understand by the term work? Explain.

### Work Done by Constant Force

#### Definition

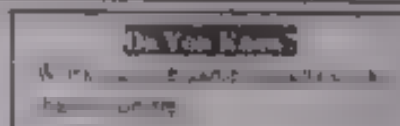
Work is the dot product of force and displacement.

$$W = \vec{F} \cdot \vec{d} \quad (\text{or})$$

Work done on a body by a constant force is defined as the product of magnitude of displacement and the component of force in the direction of displacement.

$$W = (F \cos \theta) d$$

When a constant force is applied on a body and it covers displacement in the direction of force then work is done.



#### Definition

Consider a constant force  $\vec{F}$  is applied on a body at an angle  $\theta$  with displacement  $\vec{d}$  is in the direction of force.

- Resolving the force  $\vec{F}$  into two components  $F \cos \theta$  and  $F \sin \theta$ .
- The component  $F \cos \theta$  is the component of force in the direction of displacement  $\vec{d}$  and is responsible for work.

Work done by the force  $\vec{F}$  is the component of force in the direction of displacement.

$$W = F \cos \theta \cdot d$$

$$W = F d \cos \theta$$

$$W = F d \cos \theta$$

That is the formula for the calculation of work done. Work done depends on three factors:

- $F$  = force applied on body
- $d$  = displacement covered by the body
- $\theta$  = angle between force and displacement



Fig. 4.1

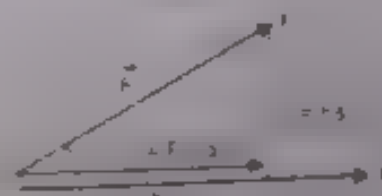
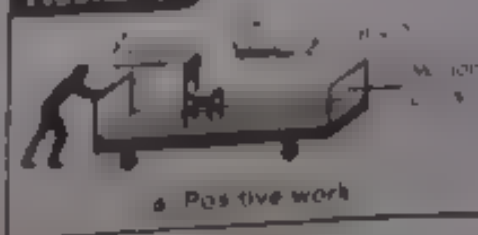


Fig. 4.2

FIGURE 4.2



a. Positive work

#### Special Cases

- Maximum Positive Work**  
If applied force  $\vec{F}$  and displacement  $\vec{d}$  are in the same direction.

$$\text{i.e. } \theta = 0^\circ$$

$$W = F d \cos 0^\circ$$

$$W = F d (1)$$

$$W = F d$$

(This is maximum positive work done)

**Example** A person is pushing the cart and applied force is in the direction of displacement so it is performing maximum positive work. This positive work will increase its kinetic energy.

## ii Maximum Negative Work

Suppose force  $F$  and displacement  $d$  are in opposite direction i.e.  $\theta = 180^\circ$

$$W = Fd \cos 180^\circ$$

$$W = -Fd \text{ (J)}$$

$W = -Fd$  (work done is negative)

### Example of negative work

a. Consider the cart is moving forward but the person applies force opposite to its motion to stop it. So work done will be negative. This negative work slows it down and reduces its kinetic energy.

b. Work done by frictional force is negative because direction of frictional force is opposite to displacement.

c. When a body is lifted then work done by gravitational force is negative because gravitational force is downward and its direction is opposite to displacement.

## iii Zero Work

i. If applied force  $F$  and displacement  $d$  are in the perpendicular

$$\theta = 90^\circ \text{ or } 270^\circ$$

$$W = Fd \cos 90^\circ$$

$$W = Fd \cos 270^\circ = 0 \text{ (No work is done)}$$

### Examples

a. A person is sitting on the cart exerting a downward force i.e. weight while cart has horizontal motion. So applied force and displacement are perpendicular. Therefore no work is done.

b. Work done by centripetal force is zero because centripetal force is perpendicular to direction of motion of the body.

c. If a person is applying force on a body but body is not covering any displacement.

$$W = Fd \cos 0^\circ = F(0) \cos 0^\circ = 0$$

### Examples

a. Person is applying the force on cart but cart is not moving. So, no work is done.

b. A person at rest holding a heavy weight is doing no work.

• Work is a scalar quantity.

• SI unit of work is **joule** ( $J = Nm = kg m^2 s^{-2}$ )

• The dimension of work is  $[ML^2T^{-2}]$

### Definition of joule (J)

$$1 J = 1 N \times 1 m$$

When one newton force acts on the body and the body covers a distance of one meter in the direction of force, the work done is said to be one joule.

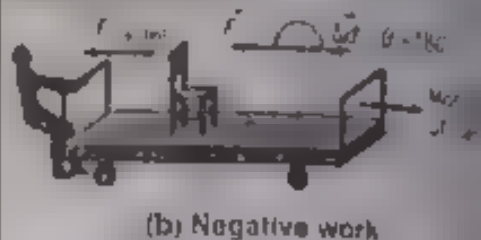
### Graphical representation of Work

Graphically, the area under the force displacement curve represents the work done by force.

If we plot graph between force and displacement then,

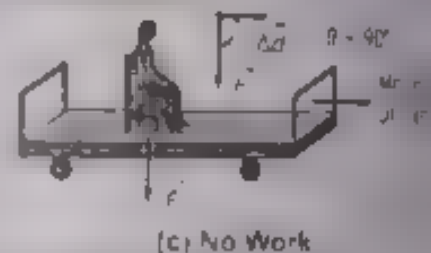
Area under  $F$  vs  $d$  the graph = (OPKOR)

FIGURE 4.2



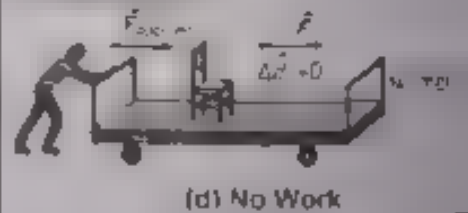
(b) Negative work

FIGURE 4.2



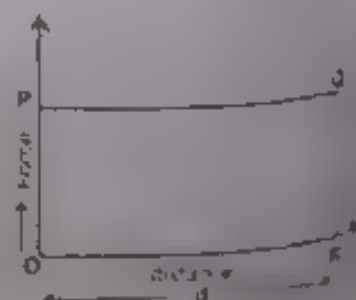
(c) No Work

FIGURE 4.2



(d) No Work

egs unit of work is erg  
 $1 J = 10^7 \text{ erg}$



Area under  $F$ - $d$  the graph  $= F \cdot d$   
 $= W$

If force  $F$  makes an angle  $\theta$  with horizontal. Then the graph is plotted between  $F \cos \theta$  and  $d$

**Q 2** How much work is being done by upward force when a person is holding the bucket and moving forward?

**Ans** In this case no work is being done.  
 Reason

Because the angle between  $\vec{F}$  and  $\vec{d}$  is  $90^\circ$   
 $\therefore W = Fd \cos 90^\circ$   
 $W = 0$



**Q 3** Is any work done by centripetal force?

**Ans** No work is being done.  
 Reason

because the force is always perpendicular to the instantaneous displacement of the body in the circular motion.

So  $W = Fd \cos 90^\circ$   
 $W = 0$

**Q 4** Give examples of variable force?

**Ans** In many cases force is not constant. Consider the process of doing work. For example

- As the rocket moves away from earth, work is done against the force of gravity which decreases as inversely proportional to the square of distance as  $F \propto \frac{1}{r^2}$  and  $W$  does not remain constant.
- Force exerted by spring increases with the displacement  $x$  because  
 $\text{since } F = kx \Rightarrow F \propto x$

So spring force is variable force.

**Q 5** How can we calculate the work done by variable force?

**Ans** **Work Done by a Variable Force (Analytical Approach)**

We follow the following steps

- Let us consider the path of particle in xy plane from point a to b as shown in figure.
- Divide the path into small intervals of displacements  $\Delta d_1, \Delta d_2, \dots, \Delta d_n$ .
- The forces acting during these intervals are  $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n$  respectively.

The force is considered to be approximately constant for each interval of displacement.

- Then we calculate the work done for the first interval is

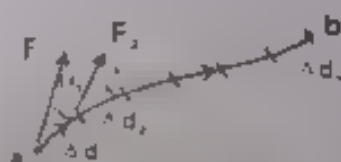
$$\Delta W = \vec{F}_1 \cdot \Delta \vec{d}_1 = F_1 \cos \theta_1 \Delta d_1$$

Similarly

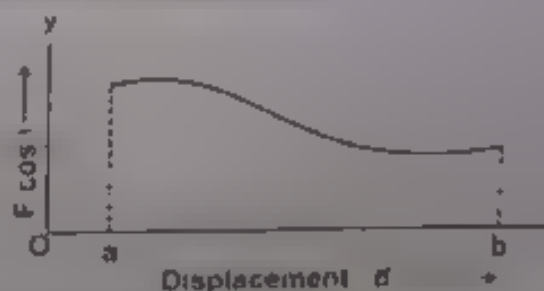
$$\Delta W = \vec{F}_2 \cdot \Delta \vec{d}_2 = F_2 \cos \theta_2 \Delta d_2$$

and up to  $n$ th interval

$$\Delta W_n = \vec{F}_n \cdot \Delta \vec{d}_n = F_n \cos \theta_n \Delta d_n$$



A particle acted upon by a variable force moves along the path shown from point a to point b.



Now the total work done in moving the body from point a to b is

$$W = \Delta W_1 + \Delta W_2 + \dots + \Delta W_n$$

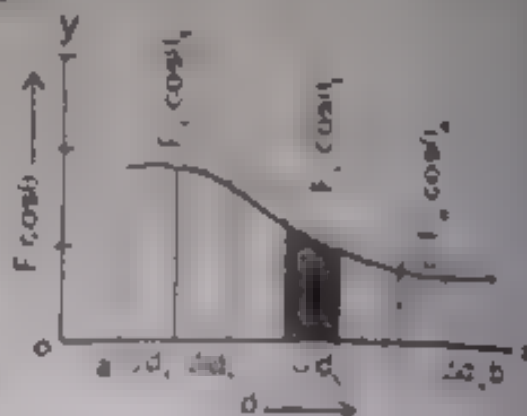
OR  $W = F_1 \cos \theta_1 \Delta d_1 + F_2 \cos \theta_2 \Delta d_2 + \dots + F_n \cos \theta_n \Delta d_n$

OR  $W = \sum F \cos \theta \Delta d$

### How to calculate work graphically

To calculate the work done graphically follow these steps

- Plot  $F \cos \theta$  versus  $d$
- Area under the graph is divided into  $n$  rectangles of width  $\Delta d$
- Area of each rectangle represents the work done during that interval
- The total work done is equal to sum of areas of all the rectangles



### For more accurate calculation of work

The work done can be calculated more accurately by taking a large number of steps so that each  $\Delta d$  becomes very small, i.e.  $\Delta d \rightarrow 0$

So  $W = \lim_{\Delta d \rightarrow 0} \sum F \cos \theta \Delta d$

here we find the total area of rectangles as very small, the area under curve versus  $d$  is  $W$



The Moon revolves around the Earth in a nearly circular orbit kept there by the gravitational force exerted by the Earth. Does gravity do (a) positive work, (b) negative work, or (c) no work on the Moon?

**Ans.** Gravitational force exerted by earth on moon tries to pull it towards earth.  
**Reason.** Gravitational force exerted by earth on moon acts as centripetal force along the radius and its instantaneous displacement is tangential to the path. So both force and displacement are perpendicular to each other, i.e.  $\theta = 90^\circ$

$$W = F d \cos 90^\circ$$

$$W = F d \cdot 0 = 0$$

That's why moon can rotate without the expenditure of any energy.

### MCQ's From Past Board Papers

- Which of the following have same dimension as that of work?  
 (A) Torque (B) Momentum (C) Velocity (D) Power
- The work done is negative when the angle between the force and displacement is  
 (A)  $45^\circ$  (B)  $90^\circ$  (C)  $0^\circ$  (D)  $60^\circ$
- The area under the curve of force-displacement graph represents  
 (A) Force (B) Work (C) Power (D) displacement
- Work done is maximum when the angle between force and displacement is  
 (A)  $90^\circ$  (B)  $0^\circ$  (C)  $180^\circ$  (D)  $30^\circ$
- Mass is considered as highly concentrated form of energy. According to this 1 kg mass has energy  
 (A)  $9 \times 10^7 \text{ J}$  (B)  $9 \times 10^8 \text{ J}$  (C)  $9 \times 10^9 \text{ J}$  (D)  $9 \times 10^{10} \text{ J}$
- The work is zero then what is the angle between the force and displacement?  
 (A)  $45^\circ$  (B)  $90^\circ$  (C)  $0^\circ$  (D)  $180^\circ$
- Which of the following are the dimension of work?  
 (A)  $[MLT^{-2}]$  (B)  $[MLT^{-1}]$  (C)  $[MLT^{-1}]$  (D)  $[MLT^{-2}]$

(iii)  $\frac{1}{2}$

What is gravitational field and gravitational field strength? Show that gravitational field is a conservative field.

Show that in gravitational field

- Work done in gravitational field is independent of path followed.
- Work done along a closed path in a gravitational field is zero.

**Gravitational field**

The space around the earth in which a gravitational force acts on a body is called the gravitational field.

At any point in the field, the force acting on a unit mass is called the gravitational field strength. It is denoted by  $g$ . The direction of the field is the direction of the force acting on a unit mass.

**Gravitational Field Strength**

The gravitational force per unit mass on a body is called the gravitational field strength.

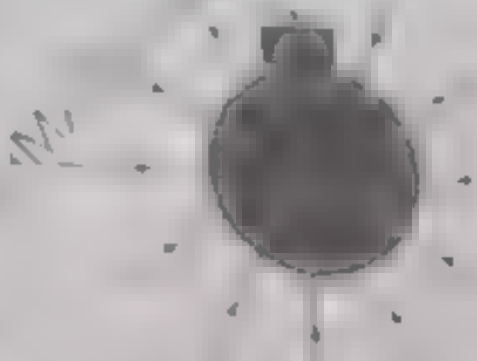


Fig 4.5 Gravitational field around the earth

**Conservative Field**

The field in which work done is independent of the path followed is called a conservative field.

The field in which work done along a closed path is zero is called a conservative field.

**Examples**

- Gravitational field
- Electric field

Work done along a closed path by Gravitational Field is zero.

Consider a closed path with A to B in a gravitational field as shown in the figure. The work done along the path from A to B is  $W_{AB}$  and the work done from B to A is  $W_{BA}$ . The total work done along the closed path is zero.

The work done from A and B will be

$$W_{AB} = mgh$$

$$W_{BA} = -mgh$$

$$W_{AB} + W_{BA} = 0$$

$$W_{AB} = -W_{BA}$$

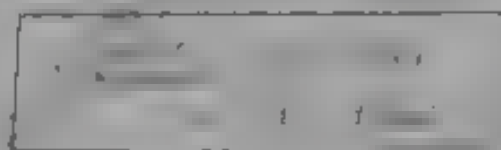
That total work done in a closed path ABCA in gravitational field is zero. Gravitational field is a conservative field.

Work Done by Gravitational Field is independent of the path followed.

Let us consider a closed path ABCA.

Let the work done by the gravitational field in moving a mass  $m$  from A to B is  $W_{AB}$ .

(i) Direct path from A to B



Let the work done by the gravitational field in moving a mass  $m$  from A to B is  $W_{AB}$ .

Let the work done by the gravitational field in moving a mass  $m$  from B to C is  $W_{BC}$ .

(ii) For an indirect path ABC

Let the work done by the gravitational field in moving a mass  $m$  from A to B is  $W_{AB}$ .

Let the work done by the gravitational field in moving a mass  $m$  from B to C is  $W_{BC}$ .

Let the work done by the gravitational field in moving a mass  $m$  from C to A is  $W_{CA}$ .

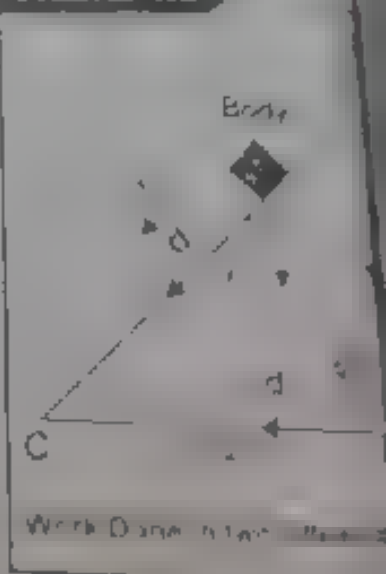
Let the work done by the gravitational field in moving a mass  $m$  from A to C is  $W_{AC}$ .

Let the work done by the gravitational field in moving a mass  $m$  from C to B is  $W_{CB}$ .

Let the work done by the gravitational field in moving a mass  $m$  from B to A is  $W_{BA}$ .

Now, if we consider the work done by the gravitational field in moving a mass  $m$  from A to B, then we see that the work done by the gravitational field in moving a mass  $m$  from A to B is independent of the path followed by the mass.

FIGURE 4.10



Note: Strictly speaking magnetic field is not a conservative field because it is not irrotational.

However, in the case of a uniform magnetic field, the work done by the magnetic field in moving a charge  $q$  from A to B is independent of the path followed by the charge.

Q 7 Is frictional force conservative force? Explain

**Ans** The frictional force is a non conservative force because if an object is moved over a rough surface between two points along different paths the work done along different paths cannot be same i.e. the work done against the frictional force certainly depends on the path followed.  
Also note that non conservative forces like friction do not store energy but dissipate energy in the form of heat, sound etc. that's why energy work done by such forces depends upon path. Hence total work done can be very

Q 8 Define power and instantaneous power Give its unit

**Ans** Power

Power is the rate at which work done per unit time  
 $P = \frac{W}{t}$

Power is a scalar quantity and it is a rate of doing work  
(OR)

Rate of doing work is called power

If  $\vec{F}$  is the force exerted on a body and  $\vec{v}$  is the velocity of the body then  
 $P = \vec{F} \cdot \vec{v}$

Average Power

If  $\Delta W$  is the total work done divided the total time taken

Mathematically

$$P_{avg} = \frac{\Delta W}{\Delta t}$$

Where

$\Delta W$  = total work done  
 $\Delta t$  = time taken

Instantaneous Power

It is the power at any instant of time called instantaneous power  
 $P = \frac{dW}{dt}$

Instantaneous power is the limiting value of  $\frac{\Delta W}{\Delta t}$  as  $\Delta t \rightarrow 0$  i.e. when  $\Delta t$  becomes infinitesimally small  
So  $P = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$

Unit of Power

Power is a scalar quantity. SI unit of power is joule/second, called watt (W)

Definition of watt

The power is said to be one watt if one joule of work is done in one second

$$1 \text{ Joule} = 1 \text{ J} = 1 \text{ W} \cdot \text{s}$$

- In British Engineering System of units, unit of power is horse power (hp)  
 $1 \text{ hp} = 746 \text{ W}$
- Dimensions of power are  $[M L^2 T^{-2}]$

**GO SOLVE THESE**  
Gravitational force  
Tension in a string  
Friction force  
**Not Conservative Forces**  
Magnetic field  
A spring force  
Tension in a string  
Normal force  
Propulsion force of a rocket  
Propulsion force of a motor

**Informations**  
Electric fans and  
in the form of bulbs  
electric fans and other for  
the home appliances  
The power of a  
electric fan is power  
possessing electric fan  
given in mega watts  
(MW)  
3. Power of TV set is 120  
watts 4. Power of  
calculator is  
 $P = 10^{-4}$  watts

Q 9 Show that instantaneous power  $P = \vec{F} \cdot \vec{v}$

**Ans** Proof

Let  $\vec{F}$  is the constant force acting on a moving body and  $\vec{v}$  is constant velocity of the body. Then the power delivered to a body at any instant is given by

$$P = \frac{\Delta W}{\Delta t}$$

$$\Rightarrow P = \frac{\vec{F} \cdot \Delta \vec{d}}{\Delta t}$$

$$\Rightarrow P = F^2 \left( \frac{\Delta d}{\Delta t} \right)$$

$$\Rightarrow P = F^2 v$$

$$\Rightarrow P = F v \cos \theta$$

### Another Definition of power

Power can also be defined as the scalar product of force applied on the body and its velocity.

**Q 10** Define commercial unit of electrical energy

**Ans** Commercial Unit of electrical energy

The commercial unit of electrical energy is kilowatt-hour

**Kilowatt-hour**

Kilowatt hour is the work done in one hour by an agency whose power is one kilowatt

**OR** If kilowatt power is maintained by an agency for a hour then energy consumed will be 1kwh

$$\begin{aligned} \text{So, } 1 \text{ kWh} &= 1000 \text{ W} \times 3600 \text{ sec} \\ &= 1000 \text{ J/sec} \times 3600 \text{ sec} \\ &= 3600000 \text{ J} \\ &= 3.6 \times 10^6 \text{ J} \\ 1 \text{ kWh} &= 3.6 \text{ MJ} \end{aligned}$$

### For your information

#### Approximate Power

| Device                  | Power (W) |
|-------------------------|-----------|
| Jet engine              | 3000      |
| Car at 90 km/h          | 100       |
| Electric motor          | 200       |
| Electric fan            | 20        |
| Incandescent light bulb | 100       |
| Hand calculator         | 0.001     |

### Example 10

**Q 10** A car of mass 1000 kg is moving with a velocity of 20 m/s. Calculate the kinetic energy of the car.

**Sol** We know that the kinetic energy of a body is given by the formula:

$$K.E. = \frac{1}{2} m v^2$$

Where,  $m$  = mass of the body = 1000 kg  
 $v$  = velocity of the body = 20 m/s

$$K.E. = \frac{1}{2} \times 1000 \times (20)^2$$

$$K.E. = 200000 \text{ J}$$

$\therefore$  The kinetic energy of the car is 200000 J.

### Assignment 4.7

What is the power of an airplane of mass 3000 kg if when on the runway it is capable of reaching a speed of 80 m/s from rest in 40 s?

**Solution**

Power =  $\frac{W}{t}$

Mass =  $m = 3000 \text{ kg}$

Final velocity =  $v_f = 80 \text{ m/s}$

Initial velocity =  $v_i = 0 \text{ m/s}$

Time =  $t = 40 \text{ s}$

Its acceleration is

$$a = \frac{v_f - v_i}{t}$$

$$\Rightarrow a = \frac{80 - 0}{40} = \frac{80}{40} = 20$$

Power of a plane can be calculated by

$$P = F \cdot v$$

$$P = (ma) \cdot v$$

$$P = 3000 \times 20 \times 80$$

$$P = 4800000 \text{ W} = 4.8 \times 10^6 \text{ W} = 4.8 \text{ MW}$$

### MCQ's

- What is the ratio of dimensions of K.E. and power?  
 (A) 1:1 (B) [T] (C) 1: [T] (D) [M] [T]
- What are the dimensions of power?  
 (A) [ML<sup>2</sup>T] (B) [ML<sup>2</sup>T<sup>2</sup>] (C) [ML<sup>2</sup>T<sup>-1</sup>] (D) [ML<sup>2</sup>T<sup>-2</sup>]
- Power is equal to the dot product of force and \_\_\_\_\_.  
 (A) Displacement (B) Acceleration (C) Velocity (D) Position vector
- 8 joule of work is done in 2 seconds then find its power?  
 (A) 8 watt (B) 2 watt (C) 16 watt (D) 2 watt

6. Which one is non-conservative force?  
(A) Electric force (B) Magnetic force (C) Gravitational force (D) Frictional force
7. The ratio of dimensions of power to work is  
(A)  $1/T$  (B)  $T/1$  (C)  $1/T^2$  (D)  $T^2/1$
8. The scalar product of force and velocity is \_\_\_\_\_  
(A) Work (B) Power (C) Momentum (D) Energy
9. Which one is a conservative force?  
(A) Electric field force (B) Frictional force (C) Air Resistance (D) Tension in the string
10. Kilowatt hour is the unit of \_\_\_\_\_  
(A) Work (B) Force (C) Power (D) Momentum
11. Which of the following is the example of conservative force?  
(A) Restoring force in compressed spring (B) Tension in the string (C) Frictional force in a road (D) Gravitational field

**Answers Key:**

|     |     |     |     |     |     |     |     |     |      |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| 1 B | 2 C | 3 C | 4 D | 5 D | 6 A | 7 B | 8 A | 9 A | 10 D |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|

**Q 11 Define energy. Give the two types of mechanical energy.**

**Ans:** Energy

The ability of a body to do work is called Energy.

(1P)

Energy is the agent which causes some changes in the state of the system.

**Types of Mechanical energy**

1 Kinetic Energy

2 Potential Energy

**Kinetic Energy**

Energy possessed by a body due to its motion is called kinetic energy.

**Mathematical**

$$K.E = \frac{1}{2}mv^2$$

Where  $m$  is the mass of the body moving with velocity  $v$ .

Kinetic energy is a scalar quantity.

**Relation between K.E. and Momentum of the body**

$$K.E = \frac{1}{2}mv^2$$

Multiplying and dividing K.E. by  $m$

$$K.E = \frac{mv^2}{2m}$$

$$K.E = \frac{p^2}{2m}$$

**Potential Energy**

The energy possessed by a body because of its position or a force field or because of a physical condition is called potential energy.

**Gravitational Potential Energy**

The potential energy due to gravitational field near the surface of the earth at a height  $h$  is

$$P.E = mgh$$

**Elastic Potential Energy**

The energy stored in a compressed or stretched spring is called elastic potential energy.

**Do You Know**

It takes about  $10^8$  J to make a car and the car then uses about  $1 \times 10^7$  J of energy from petrol in its life time.

**For Your Information**

| Unit         | Symbol                                       |
|--------------|----------------------------------------------|
| Energy       | Joule (J)                                    |
| Work         | Joule (J)                                    |
| Power        | Watt (W)                                     |
| Force        | Newton (N)                                   |
| Displacement | metre (m)                                    |
| Velocity     | metre per second (m/s)                       |
| Acceleration | metre per second squared (m/s <sup>2</sup> ) |

**For Your Information**

If  $p$  is the momentum and  $E$  is the energy of mass  $m$  then

$$E = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

## POINT TO Ponder

The pyramids in Egypt are thought to have been built by slaves dragging loads to height by inclined planes. It was the first use of inclined plane in construction happened around 2600BC. A pyramid at Egypt believed to have been built by dragging stones up inclined planes.



$$\text{Elastic potential energy} = \frac{1}{2} kx$$

Where  $k$  spring constant and  $x$  is the extension

> The  $\Delta U$  &  $\Delta W$  are the same as in case of work

**Q 12** Define and explain the terms Input, Output, efficiency for a machine. What is implication of energy losses in practical devices?

**Ans** Machine

A machine is a device for multiplying forces or simply changing the direction of forces.

A machine helps us do different types of work easily and reduces the human effort.

The principle underlying every machine is the conservation of energy.

(i) **Input of Mechanical Machines:**

Amount of work done on a machine  $P_{in}$  is called input.

Input on machine

If an effort force  $F_{in}$  moves through a distance ' $D_{in}$ ' then work done on the machine is called input.

Input = effort  $\times$  distance through which effort acts

$$\text{Or } P_{in} \times D_{in}$$

(ii) **Output of Mechanical Machines**

Amount of work done by a machine on the load (weight) is called output of the machine.

If the machine moves, it's the load  $W$  through the distance ' $h$ ', then work done by the machine is called output.

Output = load (weight)  $\times$  distance covered by the load

Or

$$\text{Output} = F_{out} \times U_{out}$$

$$\text{OR } \text{Output} = W \times h$$

(iii) **Efficiency**

The ratio of output of a machine to the input applied on machine is called its efficiency.

$$\eta = \frac{\text{Output}}{\text{Input}}$$

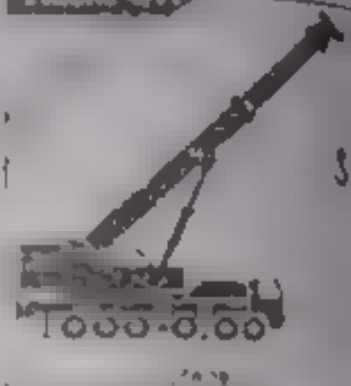
$$\eta = \frac{P_{out} \times D_{out}}{P_{in} \times D_{in}}$$

$$\text{Efficiency in percentage} = \frac{P_{out} \times D_{out}}{P_{in} \times D_{in}} \times 100\%$$

It has no unit

In physics, mechanical efficiency is the effectiveness of a machine.

## FIGURE 4.31



The crane is a machine that helps in lifting heavy loads. It uses a pulley system to reduce the effort required to lift the load.

1. Effort
2. Load
3. Distance

## EXERCISES

1. A crane is used to lift a load of 1000 N through a height of 5 m. The crane has a pulley system with 4 pulleys. Calculate the effort required to lift the load.

2. A machine is used to lift a load of 200 N through a height of 3 m. The machine has a pulley system with 3 pulleys. Calculate the effort required to lift the load.

3. A machine is used to lift a load of 500 N through a height of 4 m. The machine has a pulley system with 2 pulleys. Calculate the effort required to lift the load.

4. A machine is used to lift a load of 1500 N through a height of 6 m. The machine has a pulley system with 5 pulleys. Calculate the effort required to lift the load.

5. A machine is used to lift a load of 300 N through a height of 2 m. The machine has a pulley system with 1 pulley. Calculate the effort required to lift the load.

6. A machine is used to lift a load of 400 N through a height of 3 m. The machine has a pulley system with 2 pulleys. Calculate the effort required to lift the load.

7. A machine is used to lift a load of 600 N through a height of 4 m. The machine has a pulley system with 3 pulleys. Calculate the effort required to lift the load.

8. A machine is used to lift a load of 800 N through a height of 5 m. The machine has a pulley system with 4 pulleys. Calculate the effort required to lift the load.

9. A machine is used to lift a load of 1000 N through a height of 6 m. The machine has a pulley system with 5 pulleys. Calculate the effort required to lift the load.

10. A machine is used to lift a load of 1200 N through a height of 7 m. The machine has a pulley system with 6 pulleys. Calculate the effort required to lift the load.

## Application of Energy Losses in Practical Devices

The efficiency of an ideal machine is 100% but an actual machine's efficiency will always be less than 100%. This means that some of the work put into the system is dissipated in the form of thermal energy (heat). In a mechanical system, friction is the most common cause of the energy lost to heat.

The actual Mechanical advantage of a system is always less than the ideal mechanical advantage due to these losses.



**Q 13** Explain the work-energy principle in case of change in K.E. of the body

**Ans** Work-Energy Principle

**Statement**

Work done on a body is equal to the change in its kinetic energy.

**Proof**

Let

- (i) mass of body
- (ii) initial velocity of the body
- (iii) final velocity of the body
- (iv) acceleration of the body
- (v) distance travelled by the body
- (vi) time taken by the body

The A.S. is given by

$$W = F \cdot s \quad (1)$$

Now we know that  $F = ma$

$$W = mas \quad (2)$$

$$W = m \cdot \frac{v_f^2 - v_i^2}{2a} \cdot a \quad (3)$$

$$\text{And we know that } K.E. = \frac{1}{2}mv^2$$

$$\therefore \text{Work done} = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$\text{Work done} = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$\text{OR } \text{Work done} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\text{OR } \text{Work done} = \text{final K.E.} - \text{initial K.E.} = \text{change in K.E.}$$

**Note**

- If a body is raised from the surface of earth, the work done changes its gravitational P.E.
- If a spring is compressed, the work done on it is equal to the increase in elastic potential energy.

### Work-Energy Principle

When a force acts on a body and displaces it, the work done is equal to the change in kinetic energy of the body.

### Table 4.1: Work-Energy Principle

| Force                                                                                                                                     | Displacement | Work Done                               | Change in K.E.                                        |
|-------------------------------------------------------------------------------------------------------------------------------------------|--------------|-----------------------------------------|-------------------------------------------------------|
| 1. A constant force $F$ acts on a body of mass $m$ and displaces it by a distance $s$ in the direction of the force.                      | $s$          | $W = F \cdot s$                         | $\Delta K.E. = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ |
| 2. A constant force $F$ acts on a body of mass $m$ and displaces it by a distance $s$ perpendicular to the direction of the force.        | $s$          | $W = 0$                                 | $\Delta K.E. = 0$                                     |
| 3. A constant force $F$ acts on a body of mass $m$ and displaces it by a distance $s$ at an angle $\theta$ to the direction of the force. | $s$          | $W = F \cdot s \cdot \cos \theta$       | $\Delta K.E. = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ |
| 4. A variable force $F$ acts on a body of mass $m$ and displaces it by a distance $s$ in the direction of the force.                      | $s$          | $W = \int F \cdot ds$                   | $\Delta K.E. = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ |
| 5. A variable force $F$ acts on a body of mass $m$ and displaces it by a distance $s$ perpendicular to the direction of the force.        | $s$          | $W = 0$                                 | $\Delta K.E. = 0$                                     |
| 6. A variable force $F$ acts on a body of mass $m$ and displaces it by a distance $s$ at an angle $\theta$ to the direction of the force. | $s$          | $W = \int F \cdot ds \cdot \cos \theta$ | $\Delta K.E. = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ |

## MCQ's From Past Board Papers

- If velocity  $v$  of an object is double, then K.E. becomes
  - (A) double
  - (B) remains same
  - (C) four times
  - (D) sixteen times
- What is the K.E. of bullet of mass 800 gm moving at a speed of  $200 \text{ ms}^{-1}$ ?
  - (A) 250J
  - (B) 125J
  - (C) 1600J
  - (D) 16000J
- Kilowatt hour is a unit of
  - (A) Energy
  - (B) Power
  - (C) Pressure
  - (D) Force
- K.E. can be defined as the dot product of
  - (A) Momentum and force
  - (B) Force and velocity
  - (C) Average momentum and velocity
  - (D) none

- 5 Which one is the biggest unit of energy  
 (A) erg (B) joule (C) watt hour (D) kilowatt hour
- 6 Earth receives larger amount of energy directly from  
 (A) wind (B) water (C) Sun (D) moon
- 7 A body of mass 1kg drops from the top of tower of height 50m. what will be its K.E. 10m below the top  
 (A) 40 J (B) 48 J (C) 40 J (D) 48 J

**Answers Key**

|     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|
| 1 C | 2 D | 3 A | 4 C | 5 D | 6 C | 7 C |
|-----|-----|-----|-----|-----|-----|-----|

**Q 14** Define absolute potential energy. Derive relation for absolute P.E. of body of mass  $m$  at distance  $r$  from the center of earth.

**Ans** Absolute Potential Energy

The absolute potential energy of an object at a point is the work done by gravitational force in displacing the object from that point to infinity where the force becomes zero.

**OR**

The amount of work done in moving the body from a reference point to infinity is called absolute potential energy. The reference point is taken as infinity where the force becomes zero.

**Calculation of Absolute P.E.**

- Let a body of mass  $m$  is taken from the surface of the earth to a point at distance  $r$  from the center of the earth.

- At infinity, the force becomes zero.

$$(F_{grav} = \frac{GMm}{r^2} \Rightarrow F_{grav} \propto \frac{1}{r^2})$$

As the body moves away from the surface of the earth, the force decreases.

- The work done in moving the body from the surface of the earth to a point at distance  $r$  from the center of the earth is given by:

Work done during 1<sup>st</sup> step (1 to 2)

Suppose

$m = \text{mass of body}$

$M = \text{mass of earth}$

$r = \text{distance of point 1 from the center of the earth}$

$r_1 = \text{distance of point 1 from the center of the earth}$

**Calculation of  $r_1$** 

Let  $r_1$  be the distance of point 1 from the center of the earth.

$$r_1 = \frac{r - r_2}{2} \quad (1)$$

Now, displacement of body from point 1 to 2 is

$$\Delta r = r - r_1 \quad (2)$$

$$r_1 = r - \Delta r \quad (3)$$

$$r_1 = r - \Delta r$$

$$r_1 = r - \Delta r$$

$$r_1 = r - \Delta r$$

$$r_1 = r - \Delta r$$

$$r_1 = r - \Delta r$$

$$r_1 = r - \Delta r$$

$$r_1 = r - \Delta r$$

$$r_1 = r - \Delta r$$

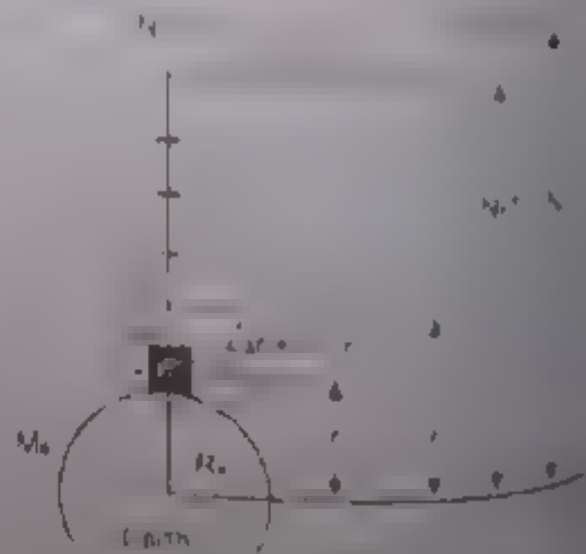


Fig. 4.10

$$\Rightarrow r_{av} = \frac{2r}{2} + \frac{\Delta r}{2}$$

$$\Rightarrow r_{av} = r + \frac{\Delta r}{2} \quad (4)$$

Squaring both sides, we have

$$\Rightarrow r_{av}^2 = \left( r + \frac{\Delta r}{2} \right)^2$$

$$\Rightarrow r_{av}^2 = r^2 + \frac{\Delta r^2}{4} + 2r \cdot \frac{\Delta r}{2}$$

$$\Rightarrow r_{av}^2 = r^2 + \frac{\Delta r^2}{4} + r_1 \Delta r$$

Since  $(\Delta r)^2 \ll r_1^2$ , so this term can be neglected as compared to  $r_1^2$

$$\Rightarrow r_{av}^2 = r_1^2 + r_1 \Delta r$$

$$\Rightarrow r_{av}^2 = r^2 + r(r_2 - r)$$

$$\Rightarrow r_{av}^2 = r^2 + r r_2 - r^2$$

$$\boxed{r_{av} = \frac{r_1 + r_2}{2}} \quad (5)$$

Now, the gravitational force  $F$  at the center of this step is

$$F_{av} = \frac{GMm}{r_{av}^2} \quad (6)$$

Putting value from equation (5) in (6), we have

$$F_{av} = \frac{GMm}{\left( \frac{r_1 + r_2}{2} \right)^2} \quad (7)$$

Thus the work done during 1<sup>st</sup> step is

$$W_{1 \rightarrow 2} = \vec{F} \cdot \vec{\Delta r}$$

$$= F_{av} \Delta r \cos 180^\circ \quad (\text{Since } \vec{F} \text{ is opposite to } \vec{\Delta r})$$

$$= -F_{av} \Delta r = -F \Delta r$$

$$= -G \frac{Mm}{r_1 r_2} (r_2 - r_1) \quad (\text{using equations (2) \& (7)})$$

$$W_{1 \rightarrow 2} = -GMm \left[ \frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2} \right]$$

$$W_{1 \rightarrow 2} = -GMm \left[ \frac{1}{r_1} - \frac{1}{r_2} \right]$$

Work done during 2<sup>nd</sup> step (2 to 3)

$$W_{2 \rightarrow 3} = -GMm \left[ \frac{1}{r_2} - \frac{1}{r_3} \right]$$

Work done during last step (N-1 to N)

For last step work done is,

$$W_{N-1 \rightarrow N} = -GMm \left[ \frac{1}{r_{N-1}} - \frac{1}{r_N} \right]$$



| Question                                                                                                             | Answer                                     |
|----------------------------------------------------------------------------------------------------------------------|--------------------------------------------|
| 1. The work done in moving a body of mass m from a distance r <sub>1</sub> to r <sub>2</sub> from a fixed mass M is  | GMm [1/r <sub>1</sub> - 1/r <sub>2</sub> ] |
| 2. The work done in moving a body of mass m from a distance r <sub>1</sub> to r <sub>2</sub> from a fixed mass M is  | GMm [1/r <sub>1</sub> - 1/r <sub>2</sub> ] |
| 3. The work done in moving a body of mass m from a distance r <sub>1</sub> to r <sub>2</sub> from a fixed mass M is  | GMm [1/r <sub>1</sub> - 1/r <sub>2</sub> ] |
| 4. The work done in moving a body of mass m from a distance r <sub>1</sub> to r <sub>2</sub> from a fixed mass M is  | GMm [1/r <sub>1</sub> - 1/r <sub>2</sub> ] |
| 5. The work done in moving a body of mass m from a distance r <sub>1</sub> to r <sub>2</sub> from a fixed mass M is  | GMm [1/r <sub>1</sub> - 1/r <sub>2</sub> ] |
| 6. The work done in moving a body of mass m from a distance r <sub>1</sub> to r <sub>2</sub> from a fixed mass M is  | GMm [1/r <sub>1</sub> - 1/r <sub>2</sub> ] |
| 7. The work done in moving a body of mass m from a distance r <sub>1</sub> to r <sub>2</sub> from a fixed mass M is  | GMm [1/r <sub>1</sub> - 1/r <sub>2</sub> ] |
| 8. The work done in moving a body of mass m from a distance r <sub>1</sub> to r <sub>2</sub> from a fixed mass M is  | GMm [1/r <sub>1</sub> - 1/r <sub>2</sub> ] |
| 9. The work done in moving a body of mass m from a distance r <sub>1</sub> to r <sub>2</sub> from a fixed mass M is  | GMm [1/r <sub>1</sub> - 1/r <sub>2</sub> ] |
| 10. The work done in moving a body of mass m from a distance r <sub>1</sub> to r <sub>2</sub> from a fixed mass M is | GMm [1/r <sub>1</sub> - 1/r <sub>2</sub> ] |

Total work done from point 1 to N

$$W_{\text{total}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + \dots + W_{N-1 \rightarrow N}$$

Putting values

$$\begin{aligned} \Delta W_{\text{total}} &= -GmM_e \left( \frac{1}{r_1} - \frac{1}{r_2} \right) - GmM_e \left( \frac{1}{r_2} - \frac{1}{r_3} \right) - \dots - GmM_e \left( \frac{1}{r_{N-1}} - \frac{1}{r_N} \right) \\ &= -GmM_e \left[ \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + \dots + \frac{1}{r_{N-1}} - \frac{1}{r_N} \right] \\ \Delta W_{\text{total}} &= -GmM_e \left( \frac{1}{r_1} - \frac{1}{r_N} \right) \end{aligned}$$

As point N is at infinity

$$\Delta W_{\text{total}} = -GmM_e \left[ \frac{1}{r_1} - \frac{1}{r_N} \right]$$

$$\frac{1}{r_N} = 0$$

$$\Delta W_{\text{total}} = -GmM_e \left[ \frac{1}{r_1} - 0 \right]$$

$$\Delta W_{\text{total}} = \frac{-GmM_e}{r_1}$$

$\Delta W_{\text{total}}$  is called absolute P.E. therefore

$$\text{Absolute P.E.} = \frac{-GmM_e}{R_e}$$

Where  $R_e$  is radius of earth

$$W_{\text{total}} = -GmM_e \left[ \frac{1}{r} - \frac{1}{r_N} \right]$$

### Absolute P.E.

If N lies at infinity, then

$$r_N = \infty \text{ and } \frac{1}{r_N} = \frac{1}{\infty} = 0$$

$$\text{So } W_{\text{total}} = -GmM_e \left[ \frac{1}{r} - \frac{1}{\infty} \right] = -GmM_e \left[ \frac{1}{r} - 0 \right]$$

$$\text{OR } W_{\text{total}} = \frac{-GmM_e}{r} \quad (8)$$

In general, the absolute gravitational potential of body at distance  $r$  from the center of the earth is

$$U = \frac{-GmM_e}{r} \quad (9)$$

### Absolute P.E. on the surface of Earth

When the body lies at the surface of the earth then,  $r = R$ . So, equation (9) becomes

$$U = \frac{-G_e Mm}{R} \quad (10)$$

Where  $R$  is the radius of the earth and the negative sign represents that gravitational field of earth's mass  $m$  is attractive

Note

- When the body moves away from earth's surface,  $r$  increases,  $U$  increases (i.e. it becomes less negative)
- When the body falls towards the earth's surface,  $r$  decreases,  $U$  decreases (i.e. it becomes more negative)

### NOTE - Tip-Note

More work has been done since 1945 than was done in the whole of history before that

**Note:** To calculate P.E. choice of zero reference point

We can take the *surface* of the Earth or the point at *infinity* as zero P.E. reference

As we go away from earth, gravitational force decreases and it becomes zero at a very far point from earth surface (where earth cannot attract anybody towards itself) at this point P.E. becomes zero. so in case of absolute P.E. zero reference point lies at infinity

**Q 15** Define gravitational potential. Write its formula

**Ans:** Gravitational Potential:

It is gravitational potential energy per unit mass of the body in the gravitational field

OR

The potential energy per unit mass at that point which is at distance  $r$  from the centre of earth and it is shown as

$$V(r) = \frac{\text{P.E. at distance } r}{\text{mass}}$$

$$GmM_2$$

$$V(r) = - \frac{GM}{m}$$

$$V(r) = - \frac{GM_1}{r}$$

At distance  $r = R_e$  (on the surface of earth)

$$V(r) = - \frac{GM_1}{R_e}$$

**Q 16** Define escape velocity and derive the mathematical expression for escape velocity

**Ans:** Escape Velocity

The initial velocity of a body with which it goes out of the earth's gravitational field is called escape velocity

OR

The initial velocity, which a projectile must have at earth surface in order to go out of earth's gravitational field.

**Explanation**

When a body is thrown upward it returns back after reaching a certain height. This is due to gravitational force acting downward. If we ~~increase~~ the initial velocity of the body then it gains more height. If we go on increasing the initial velocity then at certain velocity the body it will not return back to the ground. This particular velocity is called *escape velocity*.

**Expression for escape velocity**

We know that the absolute P.E. of a body of mass  $m$  on the surface of earth is

$$P.E. = U_e = - \frac{GMm}{R} \quad \dots \dots \dots (1)$$

And P.E. of the body at infinity is

$$P.E. = 0$$

As the body goes out of gravitational field, its P.E. becomes zero

$$\text{So, Change in P.E.} = P.E. - P.E. = 0 - \left( - \frac{GMm}{R_e} \right) = \frac{GMm}{R_e}$$



Fig 8.11 Gravitational field around the earth

From law of conservation of energy, The increase in P.E. is equal to initial K.E.

i.e. Initial K.E. = Change in P.E.

$$\text{Initial K.E.} = \frac{GmM_e}{R_e}$$

$$\text{OR} \quad \frac{1}{2}mv_{\text{esc}}^2 = \frac{GmM_e}{R_e}$$

Where  $m$  = the mass of the body,  $M_e$  = mass of the earth and,  $R_e$  = radius of earth ;

$$\text{OR} \quad v_{\text{esc}} = \sqrt{\frac{2GM_e}{R_e}}$$

$$\text{OR} \quad v_{\text{esc}} = \sqrt{\frac{2GM_e}{R_e}} \quad \text{--- (2)}$$

This is expression for the escape velocity of a body

#### Another expression for escape velocity

As the gravitational force for a mass  $m$  placed on the surface of the earth of mass is

$$F = \frac{GMm}{R_e^2}$$

$$\text{But } F = mg$$

$$\text{So } mg = \frac{GMm}{R_e^2}$$

$$\text{OR } g = \frac{GM}{R_e^2}$$

$$\text{OR } GM = gR_e^2$$

Thus equation (2) becomes,

$$v_{\text{esc}} = \sqrt{\frac{2gR_e^2}{R_e}}$$

$$\text{OR } v_{\text{esc}} = \sqrt{2gR_e} \quad \text{--- (3)}$$

#### Factors.

It depends upon

a. Mass of planet      b. Radius of planet      c. gravitational acceleration  $g$  at that planet

**Note:** escape velocity does not depend upon mass of body and angle of projection

#### Value of escape velocity on Earth

$$\text{As } g = 9.8 \text{ m/s}^2 \quad \text{and} \quad R_e = 6.4 \times 10^6 \text{ m}$$

$$\text{So } v_{\text{esc}} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$= 11.2 \times 10^3 \text{ m/s}$$

$$\text{OR } = 11.2 \times 10^3 \text{ m/s}$$

$$\text{OR } v_{\text{esc}} = 11.2 \text{ km/s}$$

#### For Your Information

##### Some Escape speeds (km/s)

|         |      |
|---------|------|
| Moon    | 2.4  |
| Mercury | 4.3  |
| Mars    | 5    |
| Venus   | 10.4 |
| Earth   | 11.2 |
| Neptune | 23.5 |
| Uranus  | 25   |
| Saturn  | 36   |
| Jupiter | 60   |

#### Assignment 4.2:

How fast would the moon need to travel in order to escape the gravitational pull of Earth. Earth has a mass of  $5.98 \times 10^{24} \text{ kg}$  and the distance from Earth to the moon is  $3.84 \times 10^8 \text{ m}$ .

**Solution:**

$$\text{Mass of earth} = M = 5.98 \times 10^{24} \text{ kg}$$

$$\text{Distance from earth to moon} = r = R + h = 3.84 \times 10^8 \text{ m}$$

$$\text{Escape velocity of moon} = v_{\text{esc}} = ?$$

Formula

$$V_{\text{rel}} = \frac{1}{2}mv^2$$

By putting values

$$V_{\text{rel}} = \frac{1}{2} \times 500 \times 2^2$$

$$V_{\text{rel}} = 1000 \text{ J}$$

Q 17 Discuss Inter-conversion of potential energy and kinetic energy

**Ans** Inter Conversion of Potential Energy and Kinetic EnergyLet us consider a piece of wood of mass  $m$  lying at B at height  $h$  as shown in Fig.

At Point B

It possesses potential energy. Its P.E. at point B w.r.t. point A is

$$P.E. = mgh$$

Its kinetic energy at point B is zero as it is at rest.

$$K.E. = \frac{1}{2}mv^2 = 0$$

Now, let us let it fall down in the form of potential energy.

At Point A

➤ It possesses kinetic energy. It can be used to drive the nail into the wall.

➤ It possesses zero potential energy at point B. It will do work on the nail as it falls down to point A.

Thus,

When a body falls from a height  $h$ , its P.E. is converted into kinetic energy and it starts moving with a velocity  $v$ .

- Under such conditions, we can drive the hammer as shown in picture. Here, the potential energy of a falling hammer is converted into kinetic energy. This is the inter-conversion of the potential energy of the hammer as it falls down to point A. This is if there is no friction.

We can say that  $P.E. = K.E.$  if there is no friction.

This means that when potential energy of a body decreases there is an equal increase in its kinetic energy.

$$P.E. = K.E. \quad \text{if there is no friction}$$

- **Work-Energy Theorem in Resistive Medium**

When Friction is present

In many situations, friction cannot be ignored. Because frictional forces reduce the mechanical energy of a system, these forces are called dissipative forces. Because some part of mechanical energy is used against friction and dissipates as heat and sound energy.

$$P.E. = K.E. + \text{Work done against friction}$$

$$\text{OR } [ \text{Loss in P.E.} = \text{K.E. gained} + (\text{Work done against friction}) ] \quad \text{--- (2)}$$

Since  $W = mgh$  is potential energy,  $W = mgh = \text{work done against friction}$ 

This is a very important work-energy equation.

- Similarly, when the body moves up, then

$$\text{Loss in K.E. at A} = \text{Gain in P.E. at B} + \text{work done against friction}$$

This is law of conservation of energy in terms of K.E. and P.E.

FIGURE 4.14

**Do you know?**

When a body falls from a height  $h$ , its potential energy is converted into kinetic energy and it starts moving with a velocity  $v$ .

**Q 18** State law of conservation of energy. Why new sources of energy has to be developed if energy is conserved?

### Conservation of Energy

#### Statement

Energy cannot be destroyed. It can be transformed from one form into another, but the total amount of energy remains constant.

#### Conservation of mechanical energy

The K.E. and P.E. are the different forms of mechanical energy

The total mechanical energy of the body is equal to the sum of K.E. and P.E.

P.E. may change into K.E. Similarly the K.E. may also be

change into P.E. but total energy remains constant

Mathematically,

$$\text{Total energy} = \text{P.E.} + \text{K.E.} = \text{constant}$$

This is the special case of conservation of energy

#### Need of New Energy Sources

In daily life we observe many energy changes from one form to another

At last all energy transfers heat the environment which is useless. So, useful energy is decreasing though total energy is conserved. That is why we need to develop new sources of energies



Fig. 4.14 which shows the energy flow in a house

#### Four Major Sources of Energy

| Source of energy | Original Source |
|------------------|-----------------|
| Sun              | Sun             |
| Biomass          | Sun             |
| Fossil fuels     | Sun             |
| Wind             | Sun             |
| Wave             | Sun             |
| Hydroelectricity | Sun             |
| Tides            | Moon            |
| Geothermal       | Earth           |

#### Assignment 4.3:

Consider a person on a sled sliding down a 100 m long hill on a  $30^\circ$  incline. The mass is 20 kg and the person has a velocity of  $2 \text{ ms}^{-1}$  down the hill when they're at the top. (a) How fast is the person travelling at the bottom of the hill? (b) If, the velocity at the bottom of the hill is  $10 \text{ ms}^{-1}$  because of friction. How much work is done by friction?

**Solution:**

(a) Velocity at the bottom  $v_b = ?$

Length of hill  $L = 100 \text{ m}$ , then height of the hill can be calculated as follows

$$h = L \sin \theta = 100 \sin 30^\circ = 100 \times 0.5 = 50 \text{ m}$$

Now calculating velocity at the bottom,

$$\begin{aligned} K.E. &= P.E. + K.E. \\ \frac{1}{2} m v_b^2 &= m g h + \frac{1}{2} m v_i^2 \\ \frac{1}{2} m v_b^2 &= m (g h + \frac{1}{2} v_i^2) \\ v_b^2 &= 2(g h + \frac{1}{2} v_i^2) \\ v_b^2 &= 2 g h + v_i^2 = 2 \times 9.8 \times 50 + 2^2 = 9800 + 40 = 9840 \\ v_b &= \sqrt{9840} = 31.368 \text{ m/s} = 31.4 \text{ m/s} \end{aligned}$$

(b) Work done against friction = ?

If velocity at the bottom  $v_b = 10 \text{ m/s}$

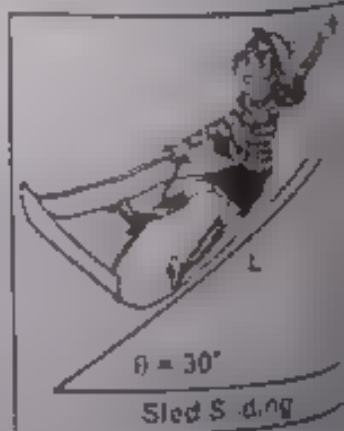
Total energy at top = K.E. at bottom + Work against friction

$$P.E. + K.E. = K.E. + W$$

$$\Rightarrow W = P.E. + K.E. - K.E. = m g h + \frac{1}{2} m v_i^2 - \frac{1}{2} m v_b^2$$

$$\Rightarrow W = 20 \times 9.8 \times 50 + \frac{1}{2} \times 20 \times 2^2 - \frac{1}{2} \times 20 \times 10^2$$

$$\Rightarrow W = 9800 + 40 - 1000 = 8840 \text{ J}$$



**Q.19** What are nonrenewable energy sources? Shortly brief different nonrenewable energy sources

**Ans** Sources of Energy

### Non-Renewable Energy Sources

These are the natural energy sources from earth that exist in limited amount and cannot be replaced if it is used up. These are such sources which cannot be refilled by natural means at the same rate at which it is consumed.

| Energy Sources                              |               |
|---------------------------------------------|---------------|
| Renewable                                   | Non-renewable |
| Hydroelectric                               | Fossil        |
| Wind                                        | Nuclear Gas   |
| Tides                                       |               |
| Geothermal*                                 | Solar**       |
| Biomass                                     | Oil           |
| Bio-fuel                                    | Coal          |
| Energy Storage                              |               |
| * Under development                         |               |
| ** Non-renewable unless solar panel is used |               |
| CO <sub>2</sub> Molecules                   |               |

- Energy can be generally classified as non-renewable and renewable. Over 85% of the energy used in the world is from non-renewable supplies.
- Most developed nations are dependent on non-renewable energy sources such as fossil fuels (coal, oil and gas) and nuclear power. These sources are called non-renewable because they cannot be renewed.
- Fossil fuels are the most commonly used types of non-renewable energy. They were formed when incompletely decomposed plant and animal matter was buried in the earth's crust. This process occurred over millions of years. The three main types of fossil fuels are coal, oil, and natural gas.

#### a. Coal

- Coal is the most abundant fossil fuel in the world with an estimated reserve of one trillion metric tons.
- Coal formed slowly formed over millions of years from the buried remains of ancient swamp plants.
- Different types of coal resulted from differences in the pressure and temperature that prevailed during formation.
- The softest coal, about 80% carbon, which also has the lowest energy output, is called lignite.
- Currently, the world is consuming coal at a rate of about 5 billion metric tons per year.
- The main use of coal is for
  - power generation
  - heating
  - cooking
- If consumption continues at the same rate, the current reserves will last for more than 200 years.
- The burning of coal results in significant atmospheric pollution.

#### b. Oil

- Oil is available in abundance in most of the middle east countries such as Saudi Arabia, Kuwait, Iraq and Iran etc.
- Like coal, it was also made out of dead plants and animals that had lived millions of years ago. When plants and animals died they were covered with thick layer of mud and sand which created huge pressure and temperature.
- Most known oil reserves are already being exploited, and oil is being used at a rate that exceeds the rate of discovery of new sources.
- If oil consumption rate remains same, oil supplies may be exhausted in another 50 years or so.

#### c. Natural Gas

- Natural Gas is the gaseous form of fossil fuels.
- It is a mixture of several gases including methane, ethane, propane and butane.
- It burns completely and leaves no ashes.
- It causes almost no pollution and is one of the cleanest form of fossil fuel.
- The natural gas is made into liquefied petroleum gas (LPG).



Energy From Fossil Fuels

**Uses**

- In developed countries, natural gas is used primarily for heating, cooking and powering vehicles.
- It is also used in excess for making ammonia fertilizer.
- The current estimate of natural gas reserves is about 100 million metric tons.
- At current usage, even this supply will last an estimated 40 years.

**1. Nuclear Energy**

- When atoms are split apart, this energy can be used to make electricity. This process is called **fusion**.
- Fusion takes place in the sun and in nuclear reactors.
- Nuclear power is produced in nuclear reactors that use fuel to generate electricity. The process is called **fusion**.

**Q 20** What are renewable energy sources? Shortly brief different renewable energy sources  
**OR** Describe briefly various non-conventional sources of energy

**ANS: Renewable Energy Sources:**

The resources that are naturally replenished by natural processes are called **Renewable Resources**.

- Sun, wind, water, geothermal energy, etc. are the non-conventional sources of energy.
- They are inexpensive in nature.
- These are such energy sources that are not exhausted by use. They are called **renewable energy sources**.
- Some of these are listed below:

**1. Energy from Biomass**

- Biomass energy is energy generated from plants and animals, and it is a renewable source of energy. Biomass is the organic material is produced by plants and animals. It is made up of wood, crops, and other organic materials. Biomass is a renewable source of energy because it can be replenished by natural processes.
- The trucks bring the waste from factories and transport it to a biomass power plant. This is used to generate a furnace where it is burned. The heat is used to heat water in the boiler and the energy in the steam is used to turn turbines and generators.

**FIGURE 4.16**

- Technologies today are able to recover the energy contained in plastics. Plastics have a high energy content that can be converted to electricity synthetically. These and recycled feedstocks are new raw materials.
- About 8 million-ton annual biomass production has a huge potential to produce enough energy employing different technologies, especially with pyrolysis in pyrolysis processes.
- Similarly, as with the dump from 72 million tonnes of waste and by-products and from 78 million tonnes of waste can produce considerable biogas production per year.

### 3 Energy from waves

- Ocean waves are caused by the wind as it blows across the sea.
- The speed of waves is as the speed of the wind.
- These waves are powerful sources of renewable energy.
- There are many devices which are designed to convert wave energy into electricity.
- Ocean Water Column (OWC) is one of them as shown in Figure 4.17.

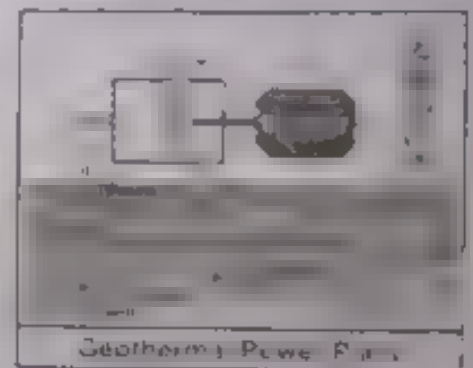
FIGURE 4.17



- An Ocean Water Column (OWC) is a partially submerged structure that uses the kinetic energy of waves.
- When these waves enter the structure, they create a pressure difference.
- This difference causes the structure to move up and down.
- This movement is used to generate electricity.

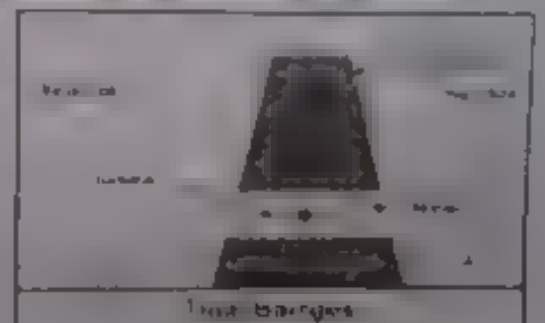
### 4 Geothermal energy

- The earth's interior is a source of energy. It is called geothermal energy.
- Geothermal power plants work by tapping into the heat of the earth to generate electricity, as shown in Figure 4.18.
- At a geothermal power plant, we are drilled 1 or 2 miles deep into the earth to pump out steam. It is then used to generate electricity.
- The water is pumped to the deep underground and then it comes out at high pressure.
- When the water reaches the surface, the pressure is dropped, which causes the water to turn into steam.
- The steam turns a turbine, which is connected to a generator that produces electricity.
- The steam cools off in a cooling tower and condenses back to water.
- The cooled water is pumped back into the earth to begin the process again.
- There are many types of geothermal power plants in an area that has a lot of volcanic activity, because these are places where the earth is particularly hot.



### 5 Tidal Energy

- Using the power of the tides, energy is produced from the gravitational pull from both the moon and the sun, which pulls water upwards, while the Earth's rotation and gravitational power pulls water down, thus creating a tidal wave.
- This movement of water from high to low and back up again is called tidal energy, a renewable energy.
- Tidal barrages are the most efficient tidal energy sources as shown in Figure 4.19.



- A tidal barrage is a dam that utilizes the potential energy generated by the change in height between high and low tides.
- This energy turns a turbine or compresses air which generates electricity.
- The Oxford University engineers calculated that underwater turbines strung across the entire width of the narrow inlet of the sea could generate a maximum  $1.9 \text{ EJ}$  (giga watt) of power averaged across the fortnightly tidal cycle. That is equivalent to  $16.5 \text{ TWh}$  (terawatt hr) of electricity a year, almost half Scotland's entire annual electricity consumption in 2011.

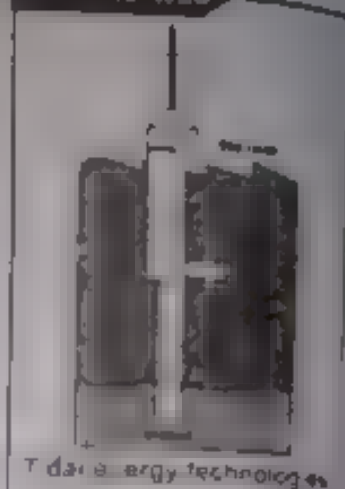
## 7. Solar Energy

- Solar energy is the radiant light and heat from the Sun that has been harnessed by humans since ancient times using a range of ever-evolving technologies.
- According to the International Energy Agency global capacity of solar PV had reached 462.8 GW at the end of 2017.
- The research and data indicate that solar energy is the best renewable energy option for Pakistan due to many factors such as price, availability, low maintenance cost, etc.
- Pakistan is blessed with  $5.4 \text{ kWh/m}^2$  solar exposure with an annual mean sunshine duration of 8-9 hours throughout the country.

## 8. Wind Energy

- Wind energy describes the process which wind is used to generate electricity.
- Wind turbines convert the kinetic energy in the wind into mechanical power.
- An equivalent of 1100 billion watts per year of power in the form of wind energy is available on the earth.
- In the windy regions, wind mills are used to produce mechanical energy.
- This mechanical energy may be used in tube wells or flour mills.
- Wind speed  $5-7 \text{ ms}^{-1}$  persists in the coastal regions of Sindh and Baluchistan provinces with more than 20,000 MW of economically feasible wind power potential.

FIGURE 4.20



Tidal energy technology

FIGURE 4.21



Wind energy

## MCQ's From Past Board Papers

- Biomass is the potential source of \_\_\_\_\_.  
 (A) Renewable energy (B) Non-renewable energy (C) Both A and B (D) Tidal energy
- A solar cell converts the light energy into \_\_\_\_\_.  
 (A) Heat energy (B) Thermal energy (C) Electrical energy (D) Atomic energy
- Work done by gravitational field in displacing object up to certain height is \_\_\_\_\_.  
 (A) Negative (B) Zero (C) Minimum (D) Virtual
- Absolute P.E. of an object at an infinite height w.r. to earth is taken as \_\_\_\_\_.  
 (A) Negative (B) Zero (C) Minimum (D) Virtual
- Escape velocity on the surface of earth is  $11.2 \text{ km/s}$ . The escape velocity on the surface of another planet of same mass as that of earth but of  $\frac{1}{4}$  times of the radius of earth is \_\_\_\_\_.  
 (A)  $5.6 \text{ km/s}$  (B)  $11.2 \text{ km/s}$  (C)  $22.4 \text{ km/s}$  (D)  $44.8 \text{ km/s}$
- Escape velocity of a body of mass  $1000 \text{ kg}$  is  $11 \text{ km/s}$ . If the mass of the body is doubled then its escape velocity will be \_\_\_\_\_.  
 (A)  $5.5 \text{ km/s}$  (B)  $11 \text{ km/s}$  (C)  $22.4 \text{ km/s}$  (D)  $44 \text{ km/s}$
- What is the value of escape velocity on earth?  
 (A)  $11 \text{ km/s}$  (B)  $13 \text{ km/s}$  (C)  $15 \text{ km/s}$  (D)  $17 \text{ km/s}$

8. What is the required power to lift a mass of 5000g to height of 1 m in 2 second?  
(A) 240 W (B) 24 N watt (C) 245 watt (D) 246 N watt
9. Which one is non-renewable source of energy?  
(A) Wind (B) Petroleum (C) Coal (D) Sunlight

Answers Key:

|     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 A | 2 C | 3 A | 4 B | 5 C | 6 B | 7 B | 8 B | 9 C |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|

### FORMULAE

|    |                                |                                       |                                          |
|----|--------------------------------|---------------------------------------|------------------------------------------|
| 1  | Work done by constant force    | $W = \vec{F} \cdot \vec{d}$           |                                          |
| 2  | Work done by variable force    | $W = \int \vec{F} \cdot d\vec{r}$     |                                          |
| 3  | Power                          | $P = \frac{W}{t}$                     | $P = \frac{W}{t}$                        |
| 4  | Average Power                  | $P_{avg} = \frac{\Delta W}{\Delta t}$ |                                          |
| 5  | Instantaneous Power            | $P = \frac{dW}{dt}$                   |                                          |
| 6  | Kinetic Energy                 | $K = \frac{1}{2}mv^2$                 | $K = \frac{1}{2}mv^2$                    |
| 7  | Gravitational Potential Energy | $U = mgh$                             |                                          |
| 8  | Work Energy Principle          | $W = \Delta K$                        |                                          |
| 9  | Absolute P.E                   | $U = mgh$                             | $U = \frac{GMm}{R}$ at surface of earth, |
| 10 | Escape Velocity                | $v = \sqrt{2gR}$                      | $v = \sqrt{\frac{2GM}{R}}$               |
| 11 | Conservation of Energy         | $mgh = \frac{1}{2}mv^2$               | $mg = \frac{mv^2}{r}$                    |

### Key Points

- The work done on a body by a constant force is defined as the product of the displacement and the components of the force in the direction of the displacement.
- $W = \vec{F} \cdot \vec{d} = Fd \cos \theta$  where  $\theta$  is angle between  $\vec{F}$  and  $\vec{d}$
- When an object is moved in the gravitational field of the Earth, the work is done by the gravitational force. The work done in the Earth's gravitational field is independent of the path followed and the work done along a closed path is zero. Such a force field is called conservative field.
- Power is defined as the rate of doing work and is expressed as  $P = \frac{W}{t}$

- ❖ Energy of a body is its capacity to do work. The kinetic energy is  $K.E. = \frac{1}{2}mv^2$
- ❖ The potential energy is possessed by a body because of its position in a force field
- ❖ The absolute P.E. of a body on the surface of Earth is  
 Absolute Potential Energy =  $\frac{GM_2m}{R_e}$
- ❖ The min. velocity of a body with which it should be projected upwards so that it does not come back is called escape velocity  
 $v_{esc} = \sqrt{2gR} = 11 \text{ km s}^{-1}$
- ❖ Some of the non-conventional energy sources are
 

|                               |                        |
|-------------------------------|------------------------|
| a) Energy from the tides      | (b) Energy from waves  |
| c) Solar energy               | d) Energy from biomass |
| e) Energy from waste products | f) Geothermal energy   |



Calculate the power required of a 1400 kg car under the following circumstances (a) the car climbs a 10° hill (fairly steep hill) at a steady and (b) the car accelerates along a level road from 90 kmh<sup>-1</sup> to 100 kmh<sup>-1</sup> in 5.0 s to pass another car. Assume the average retarding force on the car is throughout.

Given data

Required

Solution

For (a) the car is moving up the hill at a steady speed. The work done by the engine is equal to the increase in potential energy of the car.

$$W = mgh \sin \theta$$

$$h = 100 \text{ m} \times \sin 10^\circ = 17.36 \text{ m}$$

Work done by the engine =  $1400 \text{ kg} \times 9.8 \text{ m s}^{-2} \times 17.36 \text{ m} = 237000 \text{ J}$

$$W = 237000 \text{ J}$$

For (b) the car is moving on a level road. The work done by the engine is equal to the increase in kinetic energy of the car.

Work done by the engine =  $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

$$W = \frac{1}{2} \times 1400 \times (100^2 - 90^2)$$

$$W = 1400 \times 450 = 630000 \text{ J}$$

$$W = 630000 \text{ J}$$

$$W = 630000 \text{ J} = 630 \text{ kJ} = 0.63 \text{ MJ}$$

The required power increases with speed and the motor must be able to provide a constant torque.

$$P = \frac{W}{t} = \frac{630000 \text{ J}}{10 \text{ s}} = 63000 \text{ W} = 63 \text{ kW} = 84 \text{ hp}$$



Find the work require to lift a mass of 5 tonnes to a height of 30m. If this is done in 2 minutes what power is being used?

Given Data

Mass  $m = 500 \text{ kg}$  height  $h = 30 \text{ m}$  time  $t = 2 \text{ min} = 120 \text{ s}$

Required,

power  $P = ?$  work  $W = ?$

Solution

$W = mgh = 500 \text{ kg} \times 9.8 \text{ m/s}^2 \times 30 \text{ m} = 1.47 \times 10^5 \text{ J}$

$P = \frac{W}{t} = \frac{1.47 \times 10^5 \text{ J}}{120 \text{ s}}$

$P = 1225 \text{ W}$

A machine needed 1000J of energy to raise a 10 kg block at distance of 6.0m. What is the machine efficiency?

Given Data

Input work = 1000J, mass  $m = 10 \text{ kg}$ , distance  $d = 6.0 \text{ m}$

Required

Machine efficiency  $\eta = ?$

Solution

First find the work done to raise the block:  $W = mgh$

$= 10 \text{ kg} \times 9.8 \text{ m/s}^2 \times 6.0 \text{ m} = 588 \text{ J}$

$\frac{W}{\text{Input work}}$

$\frac{588 \text{ J}}{1000 \text{ J}} = 58.8\%$

Indeterminate are complicated form of pulley system.

A machine is used to lift a mass of 5 kg to a height of 20m. Calculate the work done by the machine.

Given Data

Mass  $m = 5 \text{ kg}$  height  $h = 20 \text{ m}$

Required

Work done  $W = ?$

Solution

$W = mgh = 5 \text{ kg} \times 9.8 \text{ m/s}^2 \times 20 \text{ m} = 980 \text{ J}$

$W = 980 \text{ J}$

Efficiency  $\eta = \frac{\text{Output work}}{\text{Input work}} \times 100\%$

$\eta = \frac{980 \text{ J}}{1000 \text{ J}} \times 100\% = 98\%$

The moon is at a distance of  $3.84 \times 10^8 \text{ m}$  and the acceleration due to gravity  $g = 1.6 \text{ m/s}^2$  on the surface. Find the escape velocity from the moon's surface.

Given Data

Radius of moon  $R_m = 1.74 \times 10^6 \text{ m}$

Acceleration due to gravity  $g = 1.6 \text{ m/s}^2$

Required

Escape velocity  $v_e = ?$

**Solution:**

$$v_{\text{esc}} = \sqrt{\frac{2GM_m}{R_m}}$$

$$v_{\text{esc}} = \sqrt{2g_m R_m} \quad \text{putting the values}$$

$$v_{\text{esc}} = \sqrt{2 \times 1.6 \times 1.74 \times 10^6}$$

$$v_{\text{esc}} = 2.360 \times 10^3 \text{ ms}^{-1}$$

**Compare the escape speed of a rocket launched from the moon with Earth. The mass of the moon is  $7.35 \times 10^{22}$  kg and the radius is  $1.74 \times 10^6$  m.**

**Given Data**

$$\text{Mass of moon} = M_m = 7.35 \times 10^{22} \text{ kg}$$

$$\text{Radius } R = 1.74 \times 10^6 \text{ m}$$

**Required:**

$$\text{Speed } v = ?$$

**Solution:**

$$v = \sqrt{\frac{2GM_m}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{1.74 \times 10^6}} = 2370 \text{ ms}^{-1}$$

Notice that you can escape from the moon by travelling much more slowly than you must leave to escape the gravitational pull of Earth. This is why launching a Lunar Module from the moon's surface was so much easier than launching an Apollo spacecraft from Earth.

$$v = 2.370 \times 10^3 \text{ ms}^{-1}$$

**A ball of mass 100 g is thrown vertically upward at a speed of  $25 \text{ m s}^{-1}$ . If no energy is lost, determine the height it would reach. If the ball only rises to 25 m, calculate the work done against air resistance. Also calculate the force of friction?**

**Given Data:**

$$(i) \quad \text{Height} = h = ?$$

$$\text{Mass} = m = 100 \text{ g} = 0.1 \text{ kg}$$

$$(ii) \quad \text{Friction} = f = ?$$

$$\text{Speed} = v = 25 \text{ m s}^{-1}$$

**Required:**

$$(i) \quad \text{Height} = h = 25 \text{ m}$$

$$\text{Work done against air resistance} = W = ?$$

**Solution:**

$$(i) \quad \text{As } f = 0 \text{ so}$$

$$\text{Loss in K.E.} = \text{Gain in P.E.}$$

$$\frac{1}{2}mv^2 = mgh$$

$$\Rightarrow h = \frac{v^2}{2g}$$

$$\text{Putting the values then}$$

$$h = \frac{(25)^2}{(2 \times 9.8)} = 31.9 \text{ m}$$

$$(ii) \quad \text{Loss in K.E.} = \text{Gain in P.E.} + gh$$

$$\frac{1}{2}mv^2 = mgh + fh \quad \Rightarrow fh = \frac{1}{2}mv^2 - mgh$$

$$fh = 0.5 \times 0.1 \times 25 = 25 - 0.1 \times 9.8 \times 25 = 6.75 \text{ J}$$

$$\text{Putting the value of } h \text{ we get } f = \frac{6.75}{25} = 0.27 \text{ N}$$



## Text Book Exercises

Q.1 Select the correct answer of the following questions

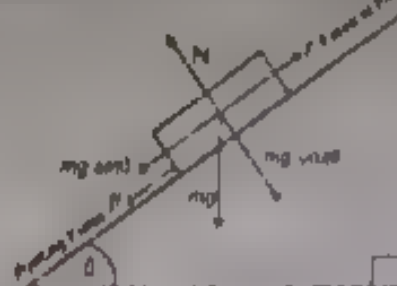
Choose the best possible answer

- You push a heavy crate down a ramp at a constant velocity. (Only four forces act on the crate). Which force does the greatest magnitude of work on the crate?  
A. The force of friction B. The force of gravity C. The normal force D. The force you push it
- The force constant of a wire is  $k$  and that of another wire is  $3k$  when both the wires are stretched through same distance, if work done are  $W_1$  and  $W_2$ , then:  
A.  $W_1 = W_2$  B.  $W_2 = 9W_1$  C.  $W_1 = 3W_2$  D.  $W_2 = 3W_1$
- Escape velocity on the surface of the earth is  $11.2 \text{ km/s}$ . If the mass of the earth increases to twice its value and the radius of the earth becomes half, the escape velocity is:  
A.  $5.6 \text{ km/s}$  B.  $11.2 \text{ km/s}$  C.  $22.4 \text{ km/s}$  D.  $23.6 \text{ km/s}$
- An example of non-conservative force is:  
A. Gravitational force B. Gravitational force C. Frictional force D. Magnetic force
- When the speed of your car is doubled, by what factor does its kinetic energy increase?  
A.  $\sqrt{2}$  B. 2 C. 4 D. 8
- One horse power is given by:  
A.  $746 \text{ W}$  B.  $746 \text{ KW}$  C.  $746 \text{ MW}$  D.  $746 \text{ GW}$
- Work is said to be negative when  $\vec{F}$  and  $\vec{s}$  are:  
A. Parallel B. Anti-parallel C. Perpendicular D. at  $45^\circ$
- Two bodies of masses  $m_1$  and  $m_2$  have equal momentum their kinetic energies  $E_1$  and  $E_2$  are in the ratio:  
A.  $\sqrt{m_1} : \sqrt{m_2}$  B.  $m_1 : m_2$  C.  $m_2 : m_1$  D.  $\sqrt{m_2} : \sqrt{m_1}$
- The atmosphere is held to the earth by:  
A. Gravity B. Gravity C. Gravity D. The rotation of earth
- If momentum is increased by  $50\%$  then K.E. increases by:  
A.  $44\%$  B.  $55\%$  C.  $66\%$  D.  $77\%$
- If the K.E. of a body becomes nine times of the initial value, then new momentum will:  
A. Become three times its initial value B. Become three times its initial value C. Remain constant D. Remain constant
- Two bodies with kinetic energies in the ratio of  $4:1$  are moving with equal linear momentum. The ratio of their masses is:  
A. 1:2 B. 1:1 C. 4:1 D. 1:4
- A body of mass  $4 \text{ kg}$  is moving with a momentum of  $10 \text{ kg ms}^{-1}$ . A force of  $0.2 \text{ N}$  acts on it in the direction of motion of the body for  $10 \text{ s}$ . The increase in its kinetic energy is:  
A.  $2 \text{ J}$  B.  $3.2 \text{ J}$  C.  $3.8 \text{ J}$  D.  $4.1 \text{ J}$
- If force and displacement of particle in the direction of force are doubled, Work would be:  
A. Double B. 4 times C. Half D.  $\frac{1}{4}$  times

## SOLUTIONS

## SOLUTIONS

|    |    |                   |                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|----|----|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. | A) | Force of friction | <ul style="list-style-type: none"> <li>Since normal force and my weight forces are perpendicular to motion of crate, so they don't do any work. (Other forces do work on crate).</li> <li>Crate is moving with constant velocity so net force is zero. Along the ramp, two forces my weight and Pushing force <math>F</math> is balanced by frictional force <math>f</math> (i.e. <math>f = mg \sin \theta + F</math>)</li> </ul> |
|----|----|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|



So, friction is greatest force,

- Therefore, it performs greater work than other forces acting on block

|    |     |                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|----|-----|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2. | (D) | $W_2 = 3W_1$     | <p>Work done in stretching the first spring <math>W_1 = \frac{1}{2} k x^2</math></p> <p>For second spring, we can write as <math>W_2 = \frac{1}{2} (3k) x^2 = 3 \left( \frac{1}{2} (k) x^2 \right)</math></p> <p><math>\Rightarrow W_2 = 3W_1</math></p>                                                                                                                                                                                                                           |
| 3. | (C) | 22.4 km/s        | <p><math display="block">v_{esc} = \sqrt{\frac{2GM}{R}}</math></p> <p>Since <math>M' = 2M</math> and <math>R' = \frac{R}{2}</math></p> <p>So,</p> <p><math display="block">v'_{esc} = \sqrt{\frac{2(2GM)}{\frac{R}{2}}}</math></p> <p><math>\Rightarrow v'_{esc} = \sqrt{\frac{4(2GM)}{R}}</math></p> <p><math>\Rightarrow v'_{esc} = 2 \sqrt{\frac{2GM}{R}} = 2 v_{esc}</math></p> <p><math>\Rightarrow v'_{esc} = 2 (11.2 \text{ km s}^{-1}) = 22.4 \text{ km s}^{-1}</math></p> |
| 4. | (C) | Frictional Force | <p>Frictional force is non conservative.</p> <p>Magnetic force under certain conditions can be considered as conservative</p> <p>So, best option is (c).</p>                                                                                                                                                                                                                                                                                                                       |
| 5. | (C) | 4                | <p><math>K E = \frac{1}{2} m v^2</math></p> <p><math>\Rightarrow K E' = \frac{1}{2} m (2v)^2 = 4 \left( \frac{1}{2} m v^2 \right) = 4 K.E.</math></p>                                                                                                                                                                                                                                                                                                                              |
| 6. | (A) | 746 watts        | <p>1 hp = 550 foot pound per second (550 ft lb/s).</p> <p>Since 1 pound (1 lb) = 4.44822 N</p> <p>1 foot (1 ft) = 0.3048 m</p> <p>So, 1 hp = 550 x 0.3048 m x 4.44822 N/s</p> <p><math>\Rightarrow 1 \text{ hp} = 745.69 \text{ Nm} = 746 \text{ J/s}</math></p> <p>1 hp = 746 W</p>                                                                                                                                                                                               |
| 7. | (B) | Anti-parallel    | $W = Fd (\cos 180) = Fd (-1) = -Fd$                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| 8. | (C) | $m_1 = m_2$      | <p><math display="block">v^2 = \frac{p^2}{2m}</math></p> <p><math>\Rightarrow p^2 = 2mE</math></p> <p><math>\therefore p_1 = p_2</math></p> <p><math>\sqrt{2m_1 E_1} = \sqrt{2m_2 E_2}</math></p>                                                                                                                                                                                                                                                                                  |

|     |     |                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-----|-----|------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|     |     |                                    | $m_1 E_1 = m_2 E_2$<br>$\frac{E_1}{E_2} = \frac{m_2}{m_1}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 9.  | (B) | Gravity                            | Atmosphere consists of gases attracted by earth.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| 10. | (A) | 44%                                | <p>Initial K.E. = <math>\frac{p^2}{2m}</math></p> <p>New momentum = <math>p + \frac{p}{5} = \frac{6p}{5}</math></p> <p>Final K.E. = <math>\frac{\left(\frac{6p}{5}\right)^2}{2m} = \frac{36p^2}{25 \times 2m}</math></p> <p>Increase in K.E. = <math>K.E._f - K.E._i = \frac{36p^2}{25 \times 2m} - \frac{p^2}{2m}</math></p> <p style="text-align: center;">= <math>\frac{p^2}{2m} \left( \frac{36}{25} - 1 \right)</math></p> <p style="text-align: center;">Increase in K.E. = <math>\frac{11}{25} \left( \frac{p^2}{2m} \right)</math></p> <p>% increase in K.E. = <math>\frac{\text{Increase in K.E.}}{\text{Initial K.E.}} \times 100</math></p> <p style="text-align: center;">= <math>\frac{\frac{11}{25} \left( \frac{p^2}{2m} \right)}{\frac{p^2}{2m}} \times 100</math></p> <p style="text-align: center;">= <math>\frac{11}{25} \times 100</math></p> <p style="text-align: center;">= 44%</p> |
| 11. | (A) | Becomes twice of its initial value | <p><math>p = \sqrt{2m K.E.} \rightarrow (1)</math></p> <p>So, <math>p' = \sqrt{2m (4K.E.)}</math></p> <p><math>\Rightarrow p' = 2\sqrt{2m (K.E.)}</math></p> <p>Putting value from Eq. (1)</p> <p><math>p' = 2p</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 12. | (D) | 1 : 4                              | <p style="text-align: right;"><math>K.E. = \frac{p^2}{2m}</math></p> <p><math>K.E. \propto \frac{1}{m}</math> (Since <math>p</math> is same)</p> <p style="text-align: center;"><math>\frac{m_1}{m_2} = \frac{K.E._2}{K.E._1}</math></p> <p style="text-align: center;"><math>\frac{m_1}{m_2} = \frac{1}{4}</math></p> <p><math>m_1 : m_2 = 1 : 4</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| 13. | (D) | 4.4 J                              | <p style="text-align: center;"><math>K.E._1 = \frac{p^2}{2m} = \frac{10^2}{2 \times 5} = 10 \text{ J}</math></p> <p style="text-align: center;"><math>\Delta p = F \times \Delta t = 0.2 \times 10 = 2 \text{ Ns}</math></p> <p style="text-align: center;"><math>p_1 = p + \Delta p = 10 + 2 = 12</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |

**Alternate Soln :**

Initial momentum =  $p$

Increase in momentum

$$= 20\% = \frac{20}{100} p = \frac{1}{5} p$$

|     |     |         |                                                                                                                                            |
|-----|-----|---------|--------------------------------------------------------------------------------------------------------------------------------------------|
|     |     |         | $K E_f = \frac{p_f^2}{2m} = \frac{4^2}{2 \times 5} = \frac{16}{10} = 1.6 \text{ J}$ $\Delta K E = K E_f - K E_i = 1.6 - 0 = 1.6 \text{ J}$ |
| 14. | (b) | 4 times | $W = F \cdot d$<br>When force and displacement are doubled<br>$W = (2F)(2d) = 4 \times W$                                                  |



## Q 2 Write short answers of the following questions

1. A bucket is taken to the bottom of a well, does the bucket possess any P.E. Explain?

**Ans** Yes bucket possess P.E. When a bucket is taken to the bottom of a well.

- ▶ Bucket possess P.E. because a bucket is taken to the bottom of a well and some work is done against the gravity that is why bucket possess work done is stored in the form of energy in the bucket.
- ▶ We can also say that the bucket possesses some negative potential energy as it is at the ground level.
- ▶ And we can say that the bucket is at a lower potential energy level than it was at the top.



2. When an arrow is shot from a bow, it has P.E. Explain why?

**Ans** When bow is stretched backward with arrow it is then elastic potential energy is stored in the bow and arrow.

- ▶ Elastic P.E. stored in bow and arrow is equal to the work done to stretch the bow.
- ▶ When arrow is shot then its stored elastic P.E. converts into kinetic energy and the arrow moves in the forward direction.



3. Does a hydrogen filled balloon possess any P.E. Explain.

**Ans** Yes a hydrogen filled balloon possesses potential energy. The reason is that it has work to do against gravity. Hydrogen is the lightest gas and as it is lighter than air, there is the up thrust force acting on hydrogen filled balloon, so upward motion of weight of the balloon and the net force acting on the balloon is:

$$F_{\text{net}} = \text{upthrust force} - \text{weight}$$

$$F_{\text{net}} = F_u - W$$

$$F_{\text{net}} = F_u - mg$$

If a net force acting on hydrogen filled balloon moves it upward.

### Alternate Answer:

When hydrogen filled balloon moves up against the gravity its position changes with ground and P.E. increases.

$$\text{Since } P.E. = mgh$$

- ▶ P.E. is high, from ground.

4. Is K.E. a vector quantity?

**Ans** No, K.E. is a scalar quantity.

**Reason** K.E. is scalar quantity so it can be expressed in term of scalar product.



$$KE = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2}mv^2$$

$$\text{Since } (v \cdot v) = v^2$$

As mass of body is scalar and  $(v \cdot v) = v^2$  also provide scalar  $KE$  of body is scalar quantity

**6 What happens to  $KE$  of a bullet when it penetrates into a target?**

**Ans** Possible Energy changes.

Moving bullet possess  $KE$  when it hits the target and penetrate, some part of its  $KE$  is used against the friction provided by target material and it also dissipates energy in the form of heat and sound.

**Mathematically**

According to work energy principle

$$W = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2$$

When bullet comes to rest,  $v_f = 0$

$$W = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2$$

$$W = -\frac{1}{2}mv_i^2$$

$$\text{Work done} = -\text{Initial } KE$$

The  $KE$  is used to work done to penetrate the bullet in to target



**6 Does the tension in the string of a swinging pendulum do any work? Explain**

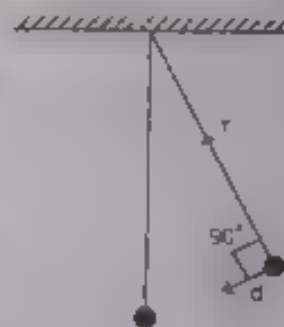
**Ans** No, it does not do any work.

**Reason**

The tension in the string does not do work because it is always perpendicular to the curved path (i.e. instantaneous displacement).

Since  $W = \int \mathbf{F} \cdot d\mathbf{s} = 0$

$$W = 0$$



**7 A meteor when enters into the earth's atmosphere burns. What happens to its energy?**

**Ans** When meteor enters the earth's atmosphere then air friction acts on it. The huge amount of energy is used against the resistive force of the atmosphere in bringing the meteor to rest. The loss of  $KE$  appears in the form of heat energy which makes the meteor to burn.

**8 What type of energy is stored in the spring of watch?**

**Ans** A compressed spring of watch stores elastic  $PE$  in it, which is given by formula

$$E_{PE} = \frac{1}{2}kx^2$$

This elastic  $PE$  stored in the spring is equal to work done to compress the spring

This elastic  $PE$  stored in the spring again convert into mechanical energy to move the arms of the watch

**9 A man drops a cup from a certain height, which breaks into pieces. What energy changes are involved?**

**Ans** Energy changes

A cup thrown from certain height losses its gravitational potential energy and gain its  $KE$ . When it strikes the ground then a part of its kinetic energy is used to break the cup and rest of the energy converts into

- (i) Sound energy
- (ii)  $KE$  of scattered moving pieces
- (iii) Heat energy dissipated against friction

10. A man rowing boat upstream is at rest with respect to shore, is he doing work?

**Ans:** No work is done as boat is at rest with respect to shore

**Reason:**

As boat is at rest with respect to shore so it covers no displacement with respect to shore

Therefore  $d = 0$

Since  $W = F d \cos \theta$

$$\rightarrow W = F (0) \cos \theta = 0$$

11. Why energy savers are instead of normal bulbs?

**Ans:** Energy savers are used instead of normal bulb due to the following reasons.

- The consumption of energy in energy savers is far less than normal light bulbs.
- Energy savers produces light of high power and intensity as compared to normal light bulbs
- Energy savers produce very little amount of heat, while 98% of input is converted into beam of light.



### Comprehensive Questions

Q3 Give a short response to the following questions.

1. Define work and show that it is the dot product of force and displacement. At what condition work done will be maximum or minimum?

**Ans:** See Q # 1 from book

2. Define power and show that power is the dot product of force and velocity. What are different units of power used in our daily life?

**Ans:** See Q # 8-9 from book

3. Prove that Absolute P.E. =  $\frac{GmM_1}{R_0}$

**Ans:** See Q # 15 from book

4. Calculate the values of the escape velocity of a body and show that it is equal  $11.2 \text{ km/s}$

**Ans:** See Q # 17 from book

5. Describe briefly various non-conventional sources of energy

**Ans:** See Q # 21 from book

# Numerical Problems

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1. A 70 kg man runs up a long flight of stairs in 4 s. the vertical height of the stair is 4.5m. Calculate his power.

Given Data:

Mass of the man =  $m = 70 \text{ kg}$   
 Time taken =  $t = 4 \text{ s}$   
 Height of the stairs =  $h = 4.5 \text{ m}$

To Find:

Power output =  $P = ?$

Calculation:

As

$$\text{Power} = P = \frac{W}{t}$$

or

$$P = \frac{mgh}{t}$$

Putting values, we get

$$P = \frac{70 \times 9.8 \times 4.5}{4}$$

$$P = \frac{3087}{4}$$

$$P = 771.5 \text{ watt}$$

Or

$$P = 7.7 \times 10^2 \text{ watt}$$



2. A body of mass 2.0 kg is dropped from a rest position 5m above the ground. What is its velocity at height of 3.0 m above the ground?

Given Data:

Mass of brick =  $m = 2 \text{ kg}$   
 Initial velocity of brick =  $v = 0$   
 Initial height =  $h = 5 \text{ m}$   
 Final height =  $h = 3 \text{ m}$

To Find:

Velocity at height 3m above the ground =  $v_2 = ?$

Calculation:

Loss of P.E = Gain in K.E

$$mg(h_1 - h_2) = \frac{1}{2} m(v_2^2 - v_1^2)$$

Putting values, we get

$$2 \times 9.8(5 - 3) = \frac{1}{2} \times 2(v_2^2 - 0^2)$$

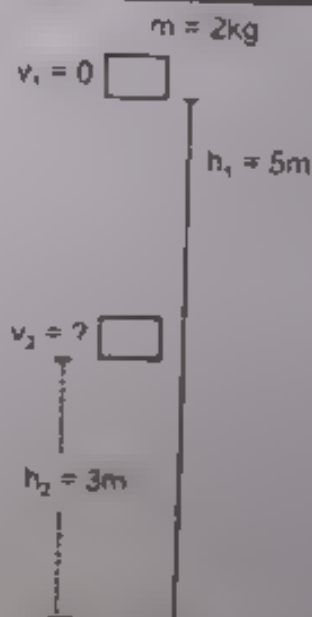
$$2 \times 9.8 \times 2 = v_2^2$$

Or

$$v_2 = 19.2 \text{ (As } v_2 = v)$$

So

$$v = 6.3 \text{ m s}^{-1}$$



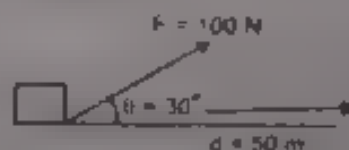
3. A man pulls a trolley through a distance of 50m by applying a force of 100N which makes an angle of  $30^\circ$  with horizontal. Calculate the work done by the man.

Given Data:

Displacement =  $d = 50 \text{ m}$   
 Applied force =  $F = 100 \text{ N}$   
 Angle =  $\theta = 30^\circ$

To Find:

Work done =  $W = ?$



Calculation:

$$W = P d \cos \theta$$

$$\Rightarrow W = 100 \times 50 \times \cos 30^\circ$$

$$\Rightarrow W = 100 \times 50 \times 0.866$$

$$\Rightarrow W = 4330 \text{ J}$$

4. The roller-coaster car starts its journey from a vertical height of 40m on the first hill and reaches a vertical height of only 25 on the second hill, where it slows to a momentary stop. It travels a total distance of 400m. Determine the thermal energy produced and estimate the average friction force on the car whose mass is 1000 kg.

Given Data:

$$\begin{aligned} 1^{\text{st}} \text{ Hill} = h_1 &= 40\text{m} \\ 2^{\text{nd}} \text{ Hill} = h_2 &= 25\text{m} \\ \text{At } 2^{\text{nd}} \text{ Hill velocity is equal to zero} \\ v &= 0 \\ g &= 9.8 \text{ m/s}^2 \\ m &= 1000 \text{ kg} \\ s &= 400 \text{ m} \end{aligned}$$

To Find:

- (a)  $f = ?$   
(b) Thermal Energy = ?

Solution:

According to Question

Law of Conservation of Energy

$$PE_{\text{initial}} = (KE_{\text{final}} + \text{Work Done against friction})$$

$$mg(h_1 - h_2) = \frac{1}{2}mv^2 + w_f$$

$$\Rightarrow 1000 \times 9.8 \times (40 - 25) = \frac{1}{2} \times 1000 \times (0) + w_f$$

$$\Rightarrow 147000 \text{ J} = w_f$$

Now

$$w_f = fh = 147000$$

$$fs = 147000$$

$$f = \frac{147000}{s} = \frac{147000}{480}$$

$$f = 306.25 \text{ N}$$

This work done against friction produces same amount of thermal energy (Heat Energy)  
Heat energy =  $w_f = 147000 \text{ J}$

5. A man whose mass is 70kg walks up to the third floor of a building which is 12m above ground in 20s. Find his power in watts and hp.

Given Data:

$$\begin{aligned} \text{Mass of man} = m &= 70\text{kg} \\ \text{Height of building} = h &= 12\text{m} \end{aligned}$$

To Find:

- Time taken =  $t = 20 \text{ sec}$ ,  
(a) power in watt = ?  
(b) power in horse power = ?

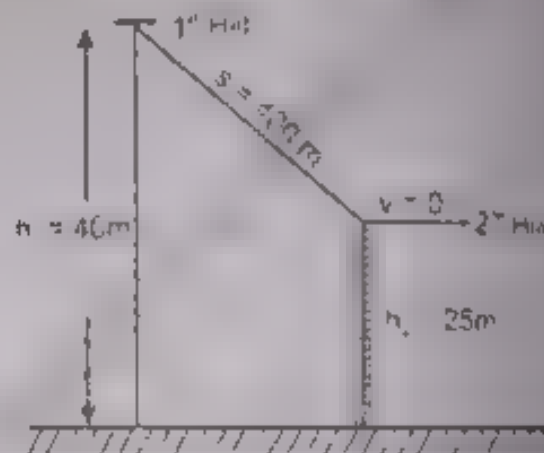
Solution:

- (a) We know that,

Calculation:

$$P = \frac{W}{t}$$

$$[\because W = PE = mgh]$$



$$P = \frac{P \cdot t}{t}$$

$$P = \frac{mgh}{t}$$

$$P = \frac{(10 \times 9.8 \times 12)}{20}$$

$$P = 41.6 \text{ watt}$$

$$[1 \text{ hp} = 746 \text{ watts}]$$

$$P = \frac{41.6}{746} \text{ hp}$$

$$P = 0.05 \text{ hp}$$

6. To what height can a 400W engine lift a 100 kg mass in 3s?

Given Data:

$$\text{Power} = P = 400 \text{ W}$$

$$\text{mass} = m = 100 \text{ kg}$$

$$\text{time} = t = 3 \text{ s}$$

$$g = 9.8 \text{ ms}^{-2}$$

To Find:

$$h = ?$$

Solution:

$$P = \frac{mgh}{t}$$

$$P \times t = mgh$$

$$h = \frac{P \times t}{mg} = \frac{400 \times 3}{100 \times 9.8}$$

$$h = 1.22 \text{ m}$$

7. A ball of mass 100 g is thrown vertically upward at a speed of  $25 \text{ ms}^{-1}$ . If no energy is lost, determine the height it would reach. If the ball only rises to 25m, calculate the work done against air resistance. Also calculate the force of friction.

Given Data: Mass of ball  $m = 100 \text{ g} = 0.1 \text{ kg}$

$$\text{Speed of ball } v = 25 \text{ ms}^{-1}$$

To Find:

$$(a) \text{ Height to which the ball would reach } h = ?$$

$$(b) \text{ If height } h = 25 \text{ m then}$$

$$(i) \text{ work done } W = ? \quad (ii) \text{ Force of friction } F = ?$$

Calculation: (a) If we ignore the air resistance then

$$\text{Loss in K.E.} = \text{Gain in P.E.}$$

$$\Rightarrow \frac{1}{2}mv^2 = mgh \quad \Rightarrow v^2 = 2gh$$

$$h = \frac{v^2}{2g}$$

$$\Rightarrow h = \frac{25^2}{2 \times 9.8} = 31.9 \text{ m}$$

$$(b) (i) h = 25 \text{ m. In case of air resistance we have,}$$

$$\text{Loss in K.E.} = \text{gain in P.E.} + \text{work done against friction}$$

$$\Rightarrow \text{work done} = \text{loss in K.E.} - \text{gain in P.E.}$$

$$\Rightarrow W = \frac{1}{2}mv^2 - mgh$$

$$\Rightarrow W = \frac{1}{2} \times 0.1 \times (25)^2 - 0.1 \times 9.8 \times 25 = (31.25 - 24.5) \text{ J}$$

$$\Rightarrow \boxed{W = 6.7 \text{ J}}$$

(ii) we know that, work =  $Fd$

$$\Rightarrow W = Fh' \quad [d = h]$$

$$\Rightarrow F = \frac{W}{h'} = \frac{6.7}{25}$$

$$\Rightarrow F = 0.26 \text{ N} \quad \Rightarrow \boxed{F = 0.3 \text{ N}}$$

8. An object of mass 1000 g falls from a height of 30m on the sand below. If it penetrates 4cm into the sand, what opposing force is exerted on it by the sand? Neglect air friction

Data:  $m = 1000 \text{ g} = 1 \text{ kg}$

Height =  $h = 30 \text{ m}$

Distance through which object penetrates in sand =  $S = 4 \text{ cm}$

$$S = 0.04 \text{ m}$$

Opposing force =  $f = ?$

Solution:

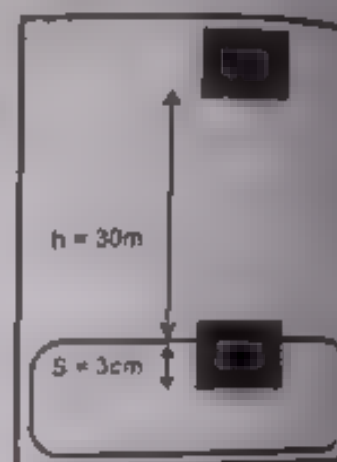
Loss of P.E. = work done

$$mgh = fS$$

$$f = \frac{mgh}{S}$$

$$= \frac{1 \times 9.8 \times 30}{0.04}$$

$$= 7350 \text{ N}$$



9. A body of mass 'm' drops from Bridge into water of the river. The bridge is 10m high from the water surface.

(a) Find the speed of the body 5m above the water surface.

(b) Find the speed of the body before it strikes the water.

Data: Mass = m

Height of bridge from water =  $h_1 = 10 \text{ m}$

Initial velocity =  $v_i = 0$

(a)  $h_2 = 5\text{m}$

$v_f = ?$

(b) final velocity at surface of water =  $v_f = ?$

Solution:

Given of K.E. = Loss of P.E.

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = mgh_1 - mgh_2$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}m(0) = mg(h_1 - h_2)$$

$$\frac{1}{2}mv_f^2 = mg(h_1 - h_2)$$

$$v_f^2 = 2g(h_1 - h_2)$$

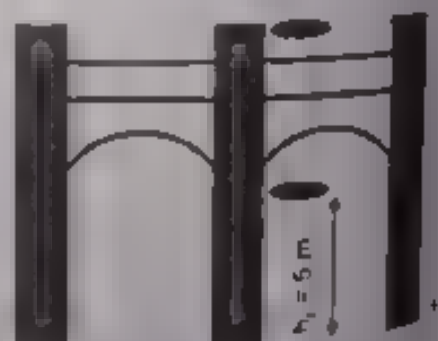
Taking square root of both sides

$$v_f = \sqrt{2g(h_1 - h_2)}$$

$$= \sqrt{2 \times 9.8(10 - 5)}$$

$$v_f = \sqrt{2 \times 9.8 \times 5}$$

$$\boxed{v_f = 9.9 \text{ m/s}}$$



(b)  $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = mgh$

$\frac{1}{2}mv_f^2 - \frac{1}{2}m(0)^2 = mgh$

$\frac{1}{2}mv_f^2 = mgh$

$v_f^2 = 2gh$

Taking square root of both sides

$v_f = \sqrt{2gh}$

$= \sqrt{2 \times 9.8 \times 10}$

$v_f = 14 \text{ m/s}$

10. The engine of a JF - Thunder fighter develops a thrust of 3000N. What horse power does it at a velocity of 600 m/s.

Solution.

Given Data Thrust of engine =  $F = 3000\text{N}$

Velocity =  $v = 600 \text{ m/s}$

To Find Power in horse power =  $P = ?$

Calculation.

We know that,  $P = \vec{F} \cdot \vec{v}$

$\Rightarrow P = Fv \cos \theta$

$\Rightarrow P = 3000 \times 600 \times \cos 0^\circ$

$\{\theta = 0^\circ \text{ and } \cos 0 = 1\}$

$\Rightarrow P = 1800000 \times 1 = 1800000 \text{ watt}$

$\Rightarrow P = 1800000 \text{ watt}$

$\{1 \text{ hp} = 746 \text{ watt}\}$

$\Rightarrow P = 1800000 \div 746 \text{ h.p.}$

$\Rightarrow P = 2412.9 \text{ h.p.}$

$\Rightarrow \boxed{P = 2413 \text{ h.p.}}$

11. The mass of the moon  $1/80$  of the mass of the earth and corresponding radius is  $1/4$  of the earth. Calculate the escape velocity on the surface of moon.

Solution:

Given Data:

Mass of moon  $M_m = \frac{1}{80} \times M_e$

Radius of moon  $R_m = \frac{1}{4} \times R_e$

To Find:

$V_{\text{esc}} = ?$  (at moon)

Solution:

$V_{\text{esc}} = \sqrt{\frac{2GM_m}{R_m}} \Rightarrow \sqrt{\frac{2G \times M_e}{80 \times \frac{1}{4} \times R_e}}$

$V_{\text{esc}} = \sqrt{\frac{2 \times 4G M_m}{80 \times R_e}} \Rightarrow \sqrt{\frac{8 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{80 \times 0.4 \times 10^6}}$

$V_{\text{esc}} = 2500 \text{ ms}^{-1}$

$\boxed{V_{\text{esc}} = 2.5 \text{ km/s}}$



## Additional Conceptual Short Questions With Answers

- 1 A car is accelerated from rest to a speed of  $10 \text{ ms}^{-1}$ . Let the energy spent be  $E$ . Then if we accelerate the car from  $10 \text{ ms}^{-1}$  to  $20 \text{ ms}^{-1}$ . How much energy will be spent?

**Ans:** For 1st case

From work - Energy theorem

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$E = \frac{1}{2} m [v_f^2 - v_i^2]$$

$$E = \frac{1}{2} m [0 - 0]$$

$$= \frac{1}{2} m (10^2 - 0)$$

$$E = \frac{1}{2} m v_i^2$$

Now for 2nd case

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$E = \frac{1}{2} m [(20)^2 - (10)^2]$$

$$E = \frac{1}{2} m (400 - 100)$$

$$E = \frac{1}{2} m (300)$$

$$E = \frac{3}{2} m (100) = 3 \left( \frac{1}{2} m (100) \right)$$

$$[E = 3E] \text{ Energy will increase by } 3 \text{ times}$$

- 2 Show that  $K.E. = \frac{P^2}{2m}$  where  $P$  is momentum and  $m$  is mass of body

**Ans:**  $K.E. = \frac{1}{2} mv^2$

Multiplying and dividing R.H.S by  $m$

$$K.E. = \frac{1}{2} mv^2 \times \frac{m}{m}$$

$$K.E. = \frac{m^2 v^2}{2m} = \frac{mv^2}{2m}$$

$$K.E. = \frac{P^2}{2m}$$

Putting  $P = mv$

- 3 A light body and a heavy body have equal momenta. Which of the two has larger K.E? Also find ratios of their energies?

**Ans:** We know that

$$E = \frac{P^2}{2m}$$

$$\Rightarrow P = \sqrt{2mE}$$

- ⇒ Energy and mass are inversely proportional to each other if momentum is kept constant for lighter body  
 • has greater energy

Let for lighter body

$$E = \frac{p^2}{2m}$$

Let for heavier body

$$E_2 = \frac{p^2}{2m_2}$$

$$\rightarrow \frac{E_1}{E_2} = \frac{\frac{p^2}{2m_1}}{\frac{p^2}{2m_2}}$$

$$\rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1}$$

It also shows that energy and mass of the body are inversely proportional if  $p$  is constant

What is the velocity of the particle if momentum and K.E. are numerically equal?

Ans. From the given condition

$$p = K.E.$$

$$mv = \frac{1}{2}mv^2$$

$$1 = \frac{1}{2}v$$

$$\Rightarrow v = 2 \text{ m/s}$$

A heavy body and a light body have equal K.E. which of the two has a greater momentum?

Ans. Let two bodies of masses  $m_1$  and  $m_2$  ( $m_1 > m_2$ ) and  $p_1$  and  $p_2$  respectively

$$K.E. = \frac{p^2}{2m}$$

$$K.E. = \frac{p^2}{2m}$$

As their K.E. is equal

$$K.E. = K.E.$$

$$\frac{p^2}{2m_1} = \frac{p_2^2}{2m_2}$$

$$\frac{p^2}{m_1} = \frac{p_2^2}{m_2}$$

$$\frac{p^2}{m_1} = \frac{p_2^2}{m_2}$$

$$\frac{p^2}{m_1} = \frac{p_2^2}{m_2}$$

$$\frac{p^2}{m_1} = \frac{p_2^2}{m_2}$$

$$\frac{p^2}{m_1} = \frac{p_2^2}{m_2}$$

$$\frac{p^2}{m_1} = \frac{p_2^2}{m_2}$$

$$\frac{p^2}{m_1} = \frac{p_2^2}{m_2}$$

$$\frac{p^2}{m_1} = \frac{p_2^2}{m_2}$$

If  $m_1 > m_2$ , then  $p_1 > p_2$ , i.e. the heavier body has more momentum



# MCQ's From Past FBISE Papers (FEDERAL BOARD)

1. SI unit of energy is:
  - A erg
  - B calorie
  - C joule
  - D none of these
2. 1 Kwh = \_\_\_\_\_:
  - A 3.6 MJ
  - B  $3.6 \times 10^3$  J
  - C  $3.6 \times 10^6$  J
  - D  $3.6 \times 10^9$  J
3. If body of mass of 2 kg is raised vertically through 2m, then the work done will be:
  - A 4 J
  - B 38 J
  - C 19.2 J
  - D 4 J
4. Two masses 1g and 4g are moving with K.E the ratio of their momenta are:
  - A 6
  - B 2
  - C 4
  - D None
5. 1 kg block is dropped from a height of 8m, its velocity just before hitting the ground is
  - A  $\sqrt{9.8}$
  - B 4
  - C 9.8
  - D  $2\sqrt{2}$
6. Brick of mass 2kg dropped rest from height of 4m then its velocity at height 2m above ground is
  - A 1.6 m/s
  - B 1.6 m/s
  - C 1.6 m/s
  - D 2.8 m/s
7. KWH is unit of:
  - A Energy
  - B Power
  - C Time
  - D None
8. Energy released by burning of 1 liter of petrol:
  - A 100 J
  - B  $7 \times 10^4$  J
  - C  $5 \times 10^4$  J
  - D  $4 \times 10^4$  J
9. On a clear day intensity of solar energy reaching earth surface is about
  - A 4 kw/m<sup>2</sup>
  - B 1 kw/m<sup>2</sup>
  - C 2 kw/m<sup>2</sup>
  - D 1.4 kw/m<sup>2</sup>
10. If K.E of moving body is doubled then momentum becomes
  - A  $\sqrt{2}$  times
  - B 2 times
  - C 3 times
  - D 4 times
11. Which of the following is the example of conservative force?
  - A Tension in the string
  - B Propulsion force of rocket
  - C Gravitational field
  - D Magnetic force in compressed spring
12. Anybody requires \_\_\_\_\_ escape velocity to escape from the gravitational pull of the earth
  - A 2.4 km/s
  - B 4.1 km/s
  - C 5 km/s
  - D 11.2 km/s
13. A brick of mass 2 kg is dropped from a rest position 8 m above the ground. What is its velocity at a height of 3 m above the ground?
  - A 2.4 m/s
  - B 6.3 m/s
  - C 7 m/s
  - D 1.2 m/s
14. Anybody requires \_\_\_\_\_ escape velocity, to escape from the gravitational pull of Venus.
  - A 5.5 km/s
  - B 2.4 km/s
  - C 4.1 km/s
  - D 10.4 km/s
15. When the angle between force and displacement is greater than  $90^\circ$ , the work done is
  - A Negative
  - B Positive
  - C Maximum
  - D Zero
16. Which of the following is non-conservative force?
  - A Electric force
  - B Elastic spring force
  - C Gravitational force
  - D None of these
17. 1kWh is equal to:
  - A  $3.6 \times 10^3$  J
  - B  $3.6 \times 10^6$  J
  - C  $3.60 \times 10^3$  J
  - D  $3.6 \times 10^9$  J
18. If angle ' $\theta$ ' is greater than  $90^\circ$ , the work done is
  - A Maximum
  - B Positive
  - C Zero
  - D Negative

19. One horse power is equal to:  
 A. 746 joules B. 746 KW C. 746 N D. 746 W s (PBIQR 2018)
20. An example of non-conservative force is:  
 A. Electric force B. Magnetic force C. Gravitational force D. Frictional force (PBIQR 2018)
21. The expression for escape velocity is given by:  
 A.  $2gR$  B.  $\sqrt{2gR}$  C.  $\frac{gR}{2}$  D.  $2gR$  (PBIQR 2018)

|            |   |   |   |   |   |
|------------|---|---|---|---|---|
| Answer Key | C | D | C | B | D |
|            | C | A | C | B | A |
|            | C | C | B | D | A |
|            | D | A | D | D | D |
|            | B |   |   |   |   |



# SELF ASSESSMENT PAPER

Question No. 1 Choose the correct answer from the given options. Total Marks 40

## SECTION - A

1. The area under the curve of force-displacement graph represents  
 A Force B Work C Power D Displacement
2. Which one is a conservative force?  
 A Frictional Force B Spring Force C Magnetic Force D Electrostatic Force
3. Escape velocity on the surface of the earth is 11.2 km/s. If the mass of the earth becomes twice its value and the radius of the earth becomes half, the escape velocity is  
 A 4.6 km/s B 11.2 km/s C 22.4 km/s D 5.6 km/s
4. One horse power is given by  
 A 746 W B 746 kW C 746 J/s D 746 hp
5. If momentum is increased by 20%, then K.E. increases by  
 A 44% B 40% C 42% D 46%
6. Which one is the biggest unit of energy?  
 A eV B Joule C kcal D kWh
7. A body of mass 1 kg drops from the top of tower of height 50 m. What is the velocity at the bottom?  
 A 31.3 m/s B 31.3 m/s C 31.3 m/s D 31.3 m/s

Question No. 2 Choose the correct answer from the following options. Total Marks 40

## SECTION - B

- i. A body of mass 2 kg is moving with a velocity of 10 m/s. Find its kinetic energy.
- ii. A body of mass 5 kg is moving with a velocity of 10 m/s. Find its momentum.
- iii. A body of mass 10 kg is moving with a velocity of 10 m/s. Find its kinetic energy.
- iv. A body of mass 10 kg is moving with a velocity of 10 m/s. Find its momentum.
- v. A body of mass 10 kg is moving with a velocity of 10 m/s. Find its kinetic energy.
- vi. A body of mass 10 kg is moving with a velocity of 10 m/s. Find its momentum.
- vii. A body of mass 10 kg is moving with a velocity of 10 m/s. Find its kinetic energy.
- viii. A body of mass 10 kg is moving with a velocity of 10 m/s. Find its momentum.
- ix. A body of mass 10 kg is moving with a velocity of 10 m/s. Find its kinetic energy.
- x. A body of mass 10 kg is moving with a velocity of 10 m/s. Find its momentum.

## SECTION - C

- a. A body of mass 10 kg is moving with a velocity of 10 m/s. Find its kinetic energy.
- b. A body of mass 10 kg is moving with a velocity of 10 m/s. Find its momentum.

## CHAPTER

## 5

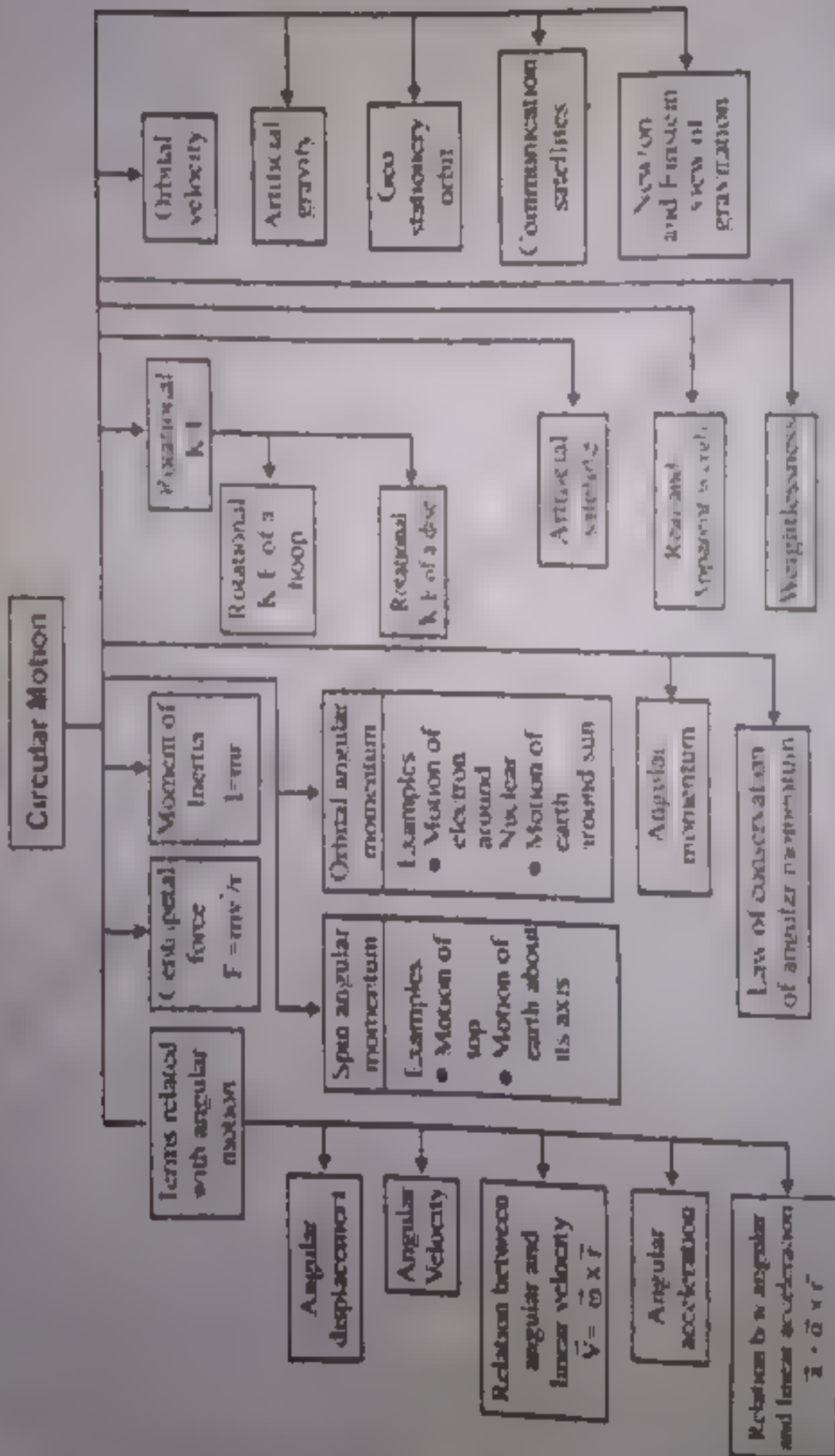
## ROTATIONAL &amp; CIRCULAR MOTION

Learning Objectives

- ◆ Define angular displacement, angular velocity and angular acceleration and express angular displacement in radians
- ◆ Solve problems by using  $S = r\theta$  and  $v = r\omega$
- ◆ State and use of equations of angular motion to solve problems involving rotational motions
- ◆ Describe qualitatively motion in a curved path due to a perpendicular force
- ◆ Derive and use centripetal acceleration  $a = r\omega^2$   $a = \frac{v^2}{r}$
- ◆ Solve problems using centripetal force  $F = m\omega^2 r$   $F = \frac{mv^2}{r}$
- ◆ Describe situations in which the centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, or a normal force
- ◆ Explain when a vehicle travels round a banked curve at the specified speed for the banking angle, the horizontal component of the normal force on the vehicle causes the centripetal acceleration
- ◆ Describe the equation  $\tan \theta = \frac{v^2}{rg}$  relating banking angle  $\theta$  to the speed  $v$  of the vehicle and the radius of curvature  $r$
- ◆ Explain that satellites can be put into orbits round the earth because of the gravitational force between the earth and the satellite
- ◆ Explain that the objects in orbiting satellites appear to be weightless
- ◆ Define the term orbital velocity and derive relationship between orbital velocity, the gravitational constant, mass and the radius of the orbit
- ◆ Analyze that satellites can be used to send information between places on the earth which are far apart, to monitor conditions on earth including the weather, and to observe the universe without the atmosphere getting in the way
- ◆ Describe that communication satellites are usually put into orbit high above the equator and that they orbit the earth once a day so that they appear stationary when viewed from earth
- ◆ Define moment of inertia of a body and angular momentum
- ◆ Derive a relation between torque, moment of inertia and angular acceleration
- ◆ Explain conservation of angular momentum as a universal law and describe examples of conservation of angular momentum
- ◆ Use the formulae of moment of inertia of various bodies for solving problems

## Chapter No. 5

## CONCEPT MAP



Q 1 Define and explain angular displacement.

Ans: **Angular Displacement**

The angle subtended at the center of a circle by an arc along which a body moves on the circumference in a given time is called angular displacement.

OR

When a rigid body rotates about a fixed axis, the angular displacement is the angle swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly.

It is denoted by  $\Delta\theta$

For small value of  $\Delta\theta$ , the angular displacement is a vector quantity.

This angle  $\theta = \angle AOB$  is the angular displacement of wheel after given a small push.

**Sign Convention:**

For anticlockwise rotation of a body between two points on circumference, the angular displacement  $\Delta\theta$  is positive.

For the clock-wise rotation, the angular displacement  $\Delta\theta$  is negative.

**Direction of Angular Displacement:**

In order to determine the direction of angular displacement, we use the 'right hand rule'.

**Right hand Rule:**

"Grasp the axis of rotation in right hand with fingers curling in the direction of rotation then the erect thumb indicates the direction of angular displacement."

**Units:**

Angular displacement is measured in degrees, or revolutions or radians.

**Radian:** The SI unit of angular displacement is radian.

"It is the angle subtended by an arc at the center of the circle whose length is equal to the radius of circle."

Its other its formal units are degrees and revolution.

**Degree:**

"It is the angle subtended at centre of circle by  $\frac{1}{360}$  th part of its circumference."

**Revolution:**

The angle subtended by a complete round trip of the body along the circumference of the circle is called one revolution.

**Dimension:** Angular displacement has no dimensions.

Q 2 Derive the relation between linear displacement and angular displacement OR prove that  $s = r\theta$

Ans: **Relation between linear displacement and angular displacement**

Consider a particle that is moving in a circle of radius 'r' with center at O. Let particle moves from point "A" to point "B" in a circle such that

$\angle AOB = 1 \text{ rad.}$

arc AB = r = radius of circle

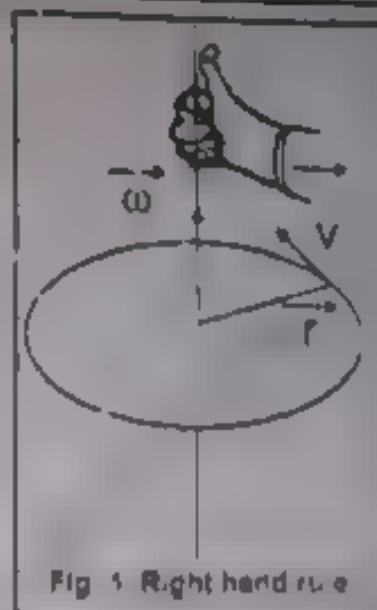
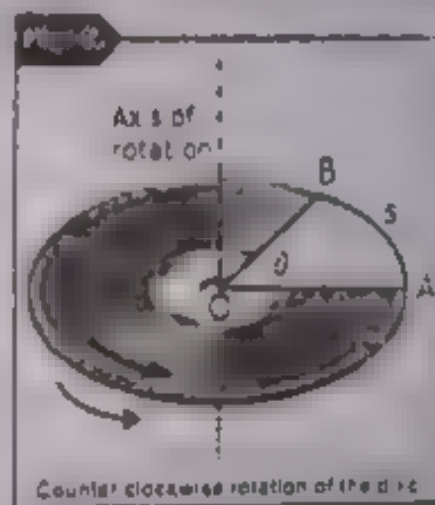


Fig. 1 Right hand rule



Counter clockwise rotation of the disk

We take point D very near to B so that arc DB = S approximately. Angle subtended by arc DB is  $\angle DOB = \theta$

As has geometry we know that

Are length or angle subtended

So their ratios will be equal

$$\frac{\text{Arc DB}}{\text{Arc AB}} = \frac{\angle DOB}{\angle AOB}$$

In other words

$$\frac{S}{r} = \frac{\theta}{1 \text{ rad}} \quad (\text{using diagram})$$

If  $S = r$  then  $\theta = 1 \text{ radian}$

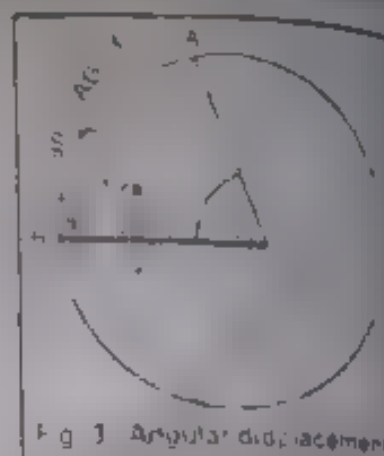


Fig 3 Angular displacement

**Q 3** Define different units of angular displacement, radian, degree and revolution. Also derive different relations between them

**Ans:** Relation between radian, degree and revolution

**Q 1** If the radius of a circle is  $r$  and the arc length is  $S$ , then the angle subtended by the arc at the center is  $\theta$  radian.

are  $AB = AO = BO = r = \text{radius of circle}$

Thus angle  $\angle AOB = \theta$

$$\text{By definition } \theta = \frac{S}{r} = \frac{\text{arc length}}{\text{radius}} = 1 \text{ radian}$$

**Q 2** Number of radians in one revolution  $\theta = \frac{\text{Circumference}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi \text{ rad}$

**Relation between radian and degree**

For one revolution  $\theta = 2\pi \text{ rad}$

So,  $2\pi \text{ rad} = 360^\circ$  1 revolution

Or  $1 \text{ rad} = \frac{360^\circ}{2\pi}$

Or  $1 \text{ rad} = \frac{360^\circ}{2 \times 3.14}$

Or  $1 \text{ rad} = 57.3^\circ$

Since  $1 \text{ rad} = \frac{360^\circ}{2\pi}$

So,  $1^\circ = \frac{2\pi}{360} \text{ rad}$

Or  $1^\circ = \frac{2 \times 3.14}{360} \text{ rad}$

Or  $1^\circ = 0.0174 \text{ rad}$

**Q 4** Define and explain the term angular velocity

**Ans:** Angular Velocity

Time rate of change of angular displacement is called as angular velocity

$$\vec{\omega} = \frac{d\theta}{dt}$$



### Average Angular velocity

The change in angular displacement in time  $t$  taken is called average angular velocity. Suppose  $\theta$  is the angular displacement during the time  $t$ . So the average angular velocity can be expressed as

$$\omega_{av} = \frac{\theta}{t}$$

### Instantaneous Angular Velocity

The limiting value of  $\omega_{av}$  as time approaches zero

OR

The instantaneous angular velocity can be defined as the limiting value of  $\omega_{av}$  as the time  $t$  approaches to zero.

$$\omega = \lim_{t \rightarrow 0} \frac{\theta}{t}$$

### Direction

Angular velocity is a vector quantity. Its direction is along the axis of rotation and can be determined by right hand rule.

### Unit

The SI unit of angular velocity is rad/s. It is also measured in revolution per second (rev/s) or rpm (rev/min).

### Note

If a body rotates with uniform angular motion, its average angular velocity will be equal to its instantaneous angular velocity.

### Assignment 5.1:

A rotating pulley completes 12 rev in 4s. Determine the average angular velocity in rev/s, rpm, and rad/s?

### Given Data

$$\begin{aligned} \theta &= 12 \text{ rev} = 12 \times 2\pi \text{ rad} = 24\pi \text{ rad} \\ t &= 4 \text{ s} \\ \omega_{av} &= \frac{\theta}{t} = \frac{24\pi \text{ rad}}{4 \text{ s}} = 6\pi \text{ rad/s} \\ &= 6 \times 180^\circ/\pi \text{ rad} = 1080^\circ/\text{s} \\ &= 18 \text{ rev/s} = 18 \times 60 \text{ rpm} = 1080 \text{ rpm} \end{aligned}$$

### Q5 Define and explain the angular acceleration?

### Ans Angular Acceleration

The rate of change of angular velocity is called angular acceleration.

### Average Angular Acceleration

When we switch on electric fan, the angular velocity goes on increasing. If  $\omega_1$  be the initial angular velocity and  $\omega_2$  is the final angular velocity at time  $t_1$  and  $t_2$  respectively. Then the average angular acceleration during time  $t_2 - t_1$  is can be defined as "the ratio of total change in angular velocity to the total time interval".

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

### FOR YOUR INFORMATION

When angular velocity is increasing, the angular acceleration is positive and if angular velocity is decreasing then it is negative. The direction of angular velocity.

## Instantaneous angular acceleration

The instantaneous angular acceleration can be defined as the limiting value of  $\frac{\Delta\omega}{\Delta t}$  as the time interval  $\Delta t$  approaches to zero. It is called instantaneous angular acceleration.

No.

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t}$$

Direction

It is a vector quantity. The direction of angular acceleration is along the axis of rotation.

Unit

The S.I. unit of angular acceleration is  $\text{rad s}^{-2}$ . Its dimension are  $[L^0 T^{-2}]$ .

## Axis of Rotation

A particle of a rotating body moves in a circle the line joining the centre of the circle is called axis of rotation.

## MCQ's From Past Board Papers

- Which of the following is equal to one radian?
  - $90^\circ$
  - $57.3^\circ$
  - $180^\circ$
  - $360^\circ$
- $30^\circ$  is equal to \_\_\_\_\_ radian.
  - $\frac{1}{6}$
  - $\frac{2}{3}$
  - $\frac{5}{6}$
  - $\frac{1}{3}$
- One radian equal to
  - $2\pi$  rev
  - $\frac{1}{2\pi}$  rev
  - $\frac{1}{2}$  rev
  - $\frac{1}{2\pi}$  rev
- The angle subtended at the center by the circumference of a circle is
  - $\pi$  radian
  - $2\pi$  radian
  - $2\pi$  radian
  - $2\pi$  radian
- $100$  radians equals to
  - $573^\circ$
  - $5730^\circ$
  - $573^\circ$
  - $5730^\circ$
- Which of the following are the dimensions of angular acceleration?
  - $[L^0 T^{-2}]$
  - $[L T^{-2}]$
  - $[T^{-2}]$
  - $[T^{-1}]$
- The angle through which a body moves is:
  - Angular displacement
  - Angular acceleration
  - Angular displacement
  - Angular momentum
- The ratio of circumference of a circle to its diameter is equal to
  - $\pi$
  - $2\pi$
  - $\frac{\pi}{2}$
  - One revolution
- How many degrees are in 1 revolution?
  - $90^\circ$
  - $180^\circ$
  - $57.3^\circ$
  - $360^\circ$
- Which of the following quantity is dimensionless?
  - Angular velocity
  - Angular acceleration
  - Centripetal force
  - Angular displacement
- A wheel of radius  $2\text{ m}$  turns through an angle of  $57.3^\circ$ . It lays out a tangential distance
  - $2\text{ m}$
  - $4\text{ m}$
  - $5.73\text{ m}$
  - $11.46\text{ m}$
- $2$  radian =
  - $114.6^\circ$
  - $57.3^\circ$
  - $75.3^\circ$
  - $37.6^\circ$
- Direction of angular acceleration is always along
  - axis
  - axis
  - axis
  - axis

## Answers Key

|       |      |      |      |      |      |      |      |      |       |       |       |
|-------|------|------|------|------|------|------|------|------|-------|-------|-------|
| 1. C  | 2. B | 3. D | 4. C | 5. C | 6. C | 7. C | 8. B | 9. D | 10. D | 11. D | 12. A |
| 13. A | 14.  |      |      |      |      |      |      |      |       |       |       |

Q 6 Define and explain angular displacement. Show that  $1 \text{ radian} = 57.3^\circ$

## Relation between Angular and Linear motions

### Relation between Angular and Linear velocities

Consider a particle that is moving in a circle of radius  $r$  with center at  $O$ . A particle moves from point  $A$  to point  $B$  in a circle such that it

If we take  $\theta$  in radians then  $s = r\theta$   $v = r\omega$   $a = r\alpha$

Similarly, in linear motion, when a body moves with uniform velocity  $\vec{v}$ , in time 't', its linear displacement will be

$$S = vt \quad \text{-----(2)}$$

Comparing the above equations, we can derive

$$v = r\omega$$

$$v = r \left( \frac{\theta}{t} \right)$$

Putting

$$\frac{\theta}{t} = \omega$$

$$v = r\omega$$

In vector form, we can write

$$\vec{v} = \omega \times \vec{r}$$

In magnitude form,

$$v = r\omega \sin \theta$$

For circular motion,  $\theta = 90^\circ$  i.e.  $\vec{r}$  is perpendicular to  $\vec{v}$

Above vector form shows that  $\vec{\omega}$  is perpendicular to the plane formed by  $\vec{r}$  and  $\vec{v}$  (it is always along axis of rotation)

$$\Rightarrow v = r\omega \sin 90 = r\omega (1) = r\omega$$

**Relation between Angular and Linear accelerations:**

By definition of acceleration of the body is

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t}$$

Where  $\vec{v}_1$  is initial linear velocity,  $\vec{v}_2$  is final linear velocity, this change in velocity occurs in time 't'

Putting

$$\vec{v}_1 = \omega_1 \times \vec{r} \text{ and } \vec{v}_2 = \omega_2 \times \vec{r}$$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t} = \frac{\omega_2 \times \vec{r} - \omega_1 \times \vec{r}}{t}$$

$$\vec{a} = \left( \frac{\omega_2 - \omega_1}{t} \right) \times \vec{r}$$

$$\frac{\omega_2 - \omega_1}{t} = \alpha$$

Putting

$$\Rightarrow \vec{a} = \alpha \times \vec{r}$$

Or

$$a = \alpha r \sin \theta$$

Which relates linear acceleration and angular acceleration

In special case when angle between  $\vec{r}$  and  $\alpha$  is  $90^\circ$  in case of circular motion, then

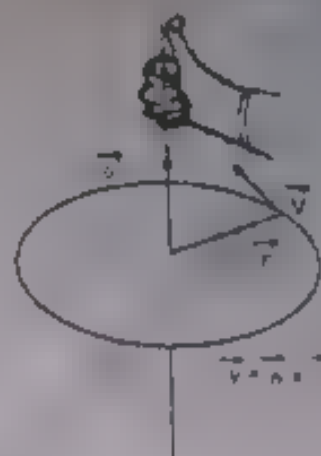
$$a = r\alpha, \quad \sin 90^\circ = 1$$

The direction of  $\vec{a}$  will be along thumb using right hand rule when  $\vec{r}$  and  $\alpha$  are multiplied  
This is the tangential acceleration  $a_t$

**Q 7** How can we write the equations of motion in case of angular motion?

**Ans** Equation of Angular Motion

Equation of angular motion are similar to those in linear motion except that  $S$ ,  $v$  and  $a$  have been replaced with  $\theta$ ,  $\omega$  and  $\alpha$  respectively. Thus



## Equations for Linear Motion • Equations for Angular Motion

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

When an object is moving in a circle of radius  $r$  and its angular velocity is  $\omega$ , then its linear velocity  $v$  is given by  $v = r\omega$ . Such an object can be treated as a particle.

## MCQ's From Past Board Papers

- Which of the following is correct?  
(A)  $\omega = vr$  (B)  $v = r\omega$  (C)  $v = r\omega$  (D)  $\omega = vr$
- A wheel of radius 50 cm having the angular speed of 5 rad/s will have linear speed in m/s  
(A) 1.5 (B) 2.5 (C) 3.5 (D) 4.5
- 1 rev/min is equal to  
(A)  $\frac{\pi}{2}$  rad/s (B)  $\frac{\pi}{5}$  rad/s (C)  $\frac{\pi}{20}$  rad/s (D)  $\frac{\pi}{30}$  rad/s
- The dimensions of angular velocity are  
(A) [T] (B) [T<sup>-1</sup>] (C) [T<sup>-1</sup>] (D) [L<sup>-1</sup>T]
- The rate of change of angular velocity is called  
(A) angular velocity (B) angular acceleration (C) angular displacement (D) angular momentum

## Answers Key

1 C

2 B

3 D

4 A

5 B

Q 8 Define centripetal force and centripetal acceleration. Derive their equations.

## XIII Centripetal Force and Centripetal Acceleration

## Centripetal Force

A force which compels a body to move in a circular path is called centripetal force.

(1)

A force which compels a body to move in a circular path is called centripetal force.

## Explanation

If an object is moving in a circle or along the arc of a circle, it follows that there must be a force acting on it to change its direction. Moving in a circle means that the direction of motion is constantly changing.

From fig. we see that at each instant and at each point the direction of velocity  $v$  changes.

## Formula

$$F = \frac{mv^2}{r}$$

## Centripetal Acceleration

The change in velocity of body produces acceleration directing towards the centre of circle. Such acceleration is known as centripetal acceleration. It is always directed towards the centre of circle.

$$\text{Mathematically centripetal acceleration is } a_c = \frac{v^2}{r} \quad \text{--- (1)}$$



At each instant and at each point the direction of velocity  $v$  changes.

force is the same as that of the centripetal force. Hence

$$\text{Centrifugal force} = \frac{mv^2}{r}$$

### Assignment 5.2:

An aeroplane dives along a curved path of radius  $R$  and velocity  $\vec{v}$ . The centripetal acceleration is  $10\text{ms}^{-2}$ . If both the velocity and the radius are doubled, what will be the new acceleration?

Given Data :  $a_c = 10\text{ms}^{-2}$

As velocity and radius of curved path is doubled i.e.  $v = 2v$  and  $R = 2R$ , then

$$a_c = \frac{(2v)^2}{2R} = \frac{4v^2}{2R} = 2 \times \frac{v^2}{R}$$

$$\therefore a_c = 2a_c = 2 \times 10 = 20\text{ms}^{-2}$$

**Q 10** What is meant by banking of road? Why banked roads are necessary at turns? Derive the formula for necessary centripetal force and angle of banking?

**Ans:** Banking of road

If outer edge of the road is higher than inner edge of the road at turns then it is called banking of road.

**Reason for Banking of Road:**

If a car is moving round a circular path then to it at uniform speed on horizontal road the resultant force acting on it must be directed to the centre of its circular path. This is the centripetal force. This force arises from the interaction of the car with air and the ground.

(i) The direction of the force exerted by the air on the car will be more or less opposite to the just necessary direction. If the car breaks due to side wind on turn.

(ii) The vertical and more important horizontal force is the frictional force inward by ground on the tyres of car. If  $\mu$  is the coefficient of friction, these two forces give the centripetal force.

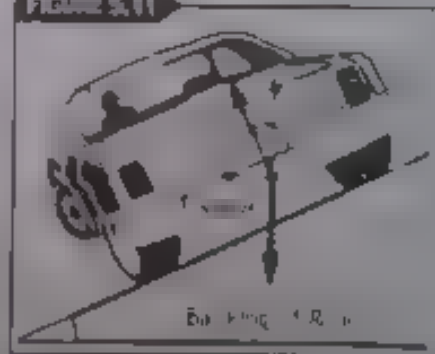
If the inner edge and road is not sufficient to provide necessary centripetal force then vehicle will skid on road. So, road is banked to provide necessary centripetal force without depending on friction.



FIGURE 5.10



FIGURE 5.11



## Formula for Necessary Centripetal Force and Angle of Banking

Let us consider a car moving on a banked curve with radius  $r$  and angle  $\theta$ . The forces acting on the car are weight  $mg$  acting vertically downwards and the normal reaction  $F$  acting perpendicular to the surface of the road. The centripetal force  $F_c$  is directed towards the center of the circle.

$$F \sin \theta = F_c$$

$$F \sin \theta = \frac{mv^2}{r} \quad \text{-----(1)}$$

From Eq (b)

vertical

$$F \cos \theta = mg$$

by division equation (1) by (2)

$$\frac{F \sin \theta}{F \cos \theta}$$

$$\frac{F \sin \theta}{F \cos \theta}$$

$$\tan \theta = \frac{v^2}{rg}$$

From Eq (3) we can write

$$r = \frac{v^2}{g \tan \theta}$$

## Example 6.3:

At what speed (in km/h) is a bank angle of  $45^\circ$  required for an aeroplane to turn on a radius of 600 m?

Given Data

Q 11 Define Moment of Inertia and give its significance

**ANS** Moment of Inertia (Rotational Inertia)

It is a scalar quantity which measures the resistance of a body to rotational motion. It is denoted by  $I$ .

It is the sum of the products of the mass of each particle and the square of its perpendicular distance from the axis of rotation.

Mathematically,

Physical Significance

At  $m = 1$  if inertia property same for all objects in a system then  $m = 1$  is the unit of mass.

Example

Consider the bike wheel as shown in figure. When we apply the force  $F$  at a distance  $r$  from the axis of rotation  $O$ , it rotates.

More massive wheel, smaller the angular acceleration  $\alpha$  in a given time.

If we push it to rotate clockwise the axle the  $\alpha$  is negative. If we push it to rotate counter-clockwise the  $\alpha$  is positive.

Dependence of moment of inertia

It is shown that moment of inertia depends on

- mass  $m$  of body
- square of perpendicular distance from axis of rotation  $r$
- shape and size of the body and the position of the axis of rotation.

Q 12 Derive the formula of torque in term of moment of inertia

**Life** Torque and Moment of Inertia

Consider the bike wheel as shown in figure, which we apply the force to rotate it. Let the  $I$  is dependent on the wheel at a point  $O$  is at distance  $r$  from its axis.

From Newton's 2<sup>nd</sup> law  $F = ma$

We know that  $a = r\alpha$  where  $\alpha$  is angular acceleration.

$$F = m r \alpha$$

$$F r = m r^2 \alpha$$

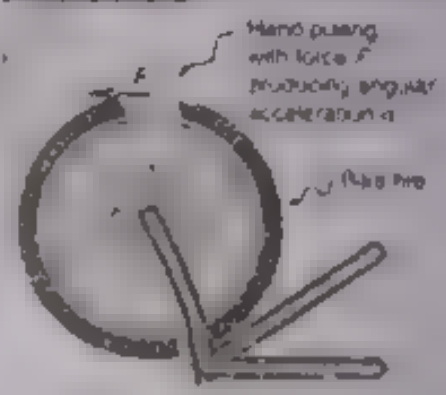
Multiplying both sides

$$r I = m r^2 \alpha$$

$$\tau = I \alpha$$

$$\tau = I \frac{d\omega}{dt}$$

FIGURE 12



In above equation  $m r^2$  is a constant for a given body.  $m r^2 = I$  is called

moment of inertia or rotational inertia of a body

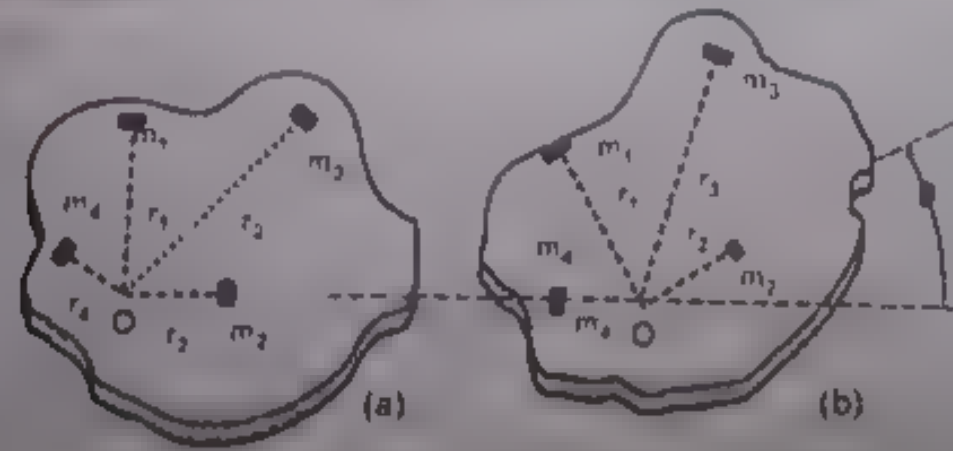
$$\text{Thus } \tau = I \alpha \quad (2)$$

The product of the moment of inertia  $I$  and the angular acceleration  $\alpha$  is called the torque  $\tau$  in  $\text{N m}$ .

Q 13 Derive the formula for Moment of inertia of Rigid Body

**Life** Moment of inertia of a rigid body

Mostly the bodies have no uniform mass distribution. Consider a rigid body made up of small pieces of masses  $m_1, m_2, \dots$  at distance  $r_1, r_2, \dots$  from axis of rotation O.



In this case the sum of products of masses of the particles in a body and the squares of their respective perpendicular distance from the axis of rotation is called moment of inertia.

$$I = \sum m_i r_i^2$$

is the moment of inertia of rigid body.

### Moment of Inertia of Different Objects or Shapes:

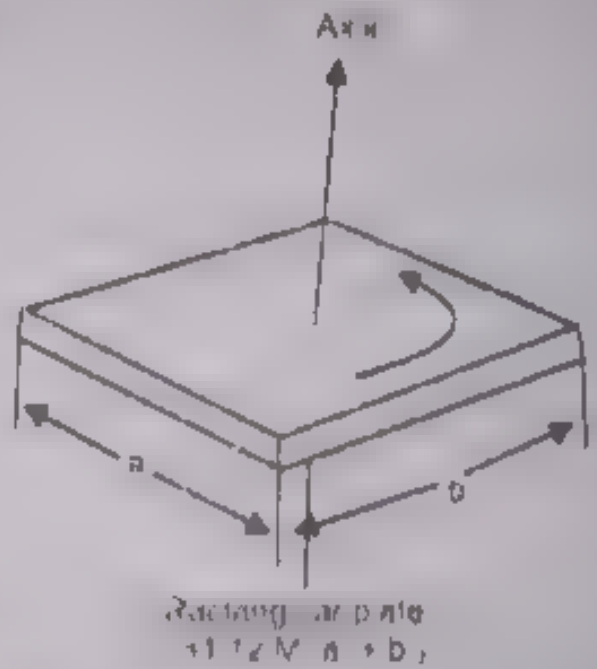
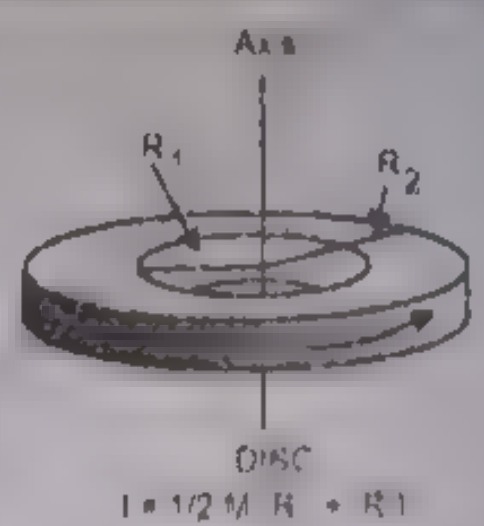
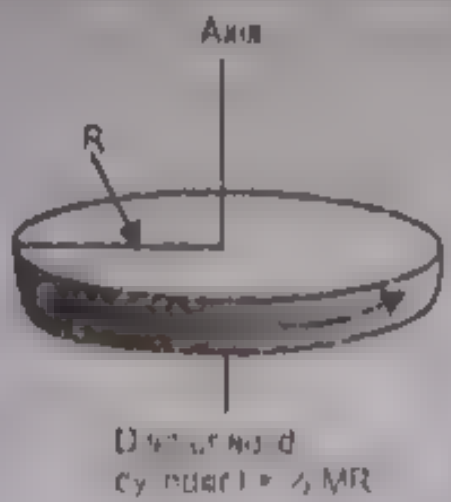
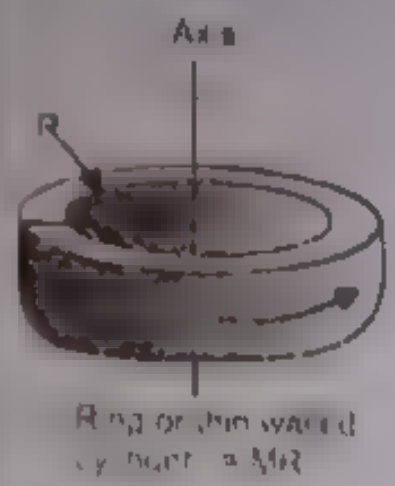


Fig 8.14 Moment of Inertia of various bodies about vertical axis

In angular motion the centripetal force is

(A)  $m^2$

(B)  $m^2 \omega$

(C)  $m \omega^2$

(D)  $m \omega^2 r$

The angular acceleration is produced by

(A) Momentum

(B) Torque

(C) Pressure

(D) Power

Choose the quantity which plays the same role in angular motion as that of mass in linear motion

(A) angular acceleration

(B) Torque

(C) Moment of inertia

(D) Angular velocity

The ratio of moment of inertia of disc and hoop is

(A)  $\frac{1}{2}$

(B)  $\frac{1}{4}$

(C)  $\frac{2}{3}$

(D)  $\frac{1}{3}$

### Answers Key

|      |      |      |      |      |
|------|------|------|------|------|
| 1. A | 2. C | 3. B | 4. C | 5. A |
|------|------|------|------|------|

### Assignment 5.0

A belt is wrapped around the edge of a pulley that is 40 cm in diameter. The pulley rotates with a constant angular acceleration of  $3.50 \text{ rad/s}^2$ . At  $t = 0$ , the rotational speed is  $2 \text{ rev/s}$ . What is the angular displacement and angular velocity of the pulley 2 s later?

Given Data:

$$d = 40 \text{ cm}$$

$$\alpha = 3.50 \text{ rad/s}^2$$

$$\omega_i = 2 \text{ rad/s}$$

$$t = 2 \text{ s}$$

(a)  $\omega_f = ?$

(b)  $\theta = ?$

Solution: (a)

$$\omega_f = \omega_i + \alpha t$$

$$\Rightarrow \omega_f = 2 + (3.50) 2$$

$$\Rightarrow \omega_f = 2 + 7 = 9 \text{ rad/s}$$

(b)  $\theta = \omega_i t + \frac{1}{2} \alpha t^2$

$$\Rightarrow \theta = (2)(2) + \frac{1}{2}(3.50)(2)^2 = 11 \text{ rad}$$

$$\Rightarrow \theta = 11 \text{ rad}$$

Q 14 (a) What is angular momentum? What are its formulae? Write its units

(b) Derive angular momentum of a spinning rigid body?

(c) Show that rate of change of angular momentum of a body is equal to applied torque on it?

Ans (a) Definition:

Quantity of angular motion in a body is called angular momentum

OR

If a body changes its angular position w.r.t its reference axis then it is said that it has angular momentum.

OR

Angular momentum ( $\vec{L}$ ) is the cross product of position vector ( $\vec{r}$ ) and linear momentum  $\vec{p}$

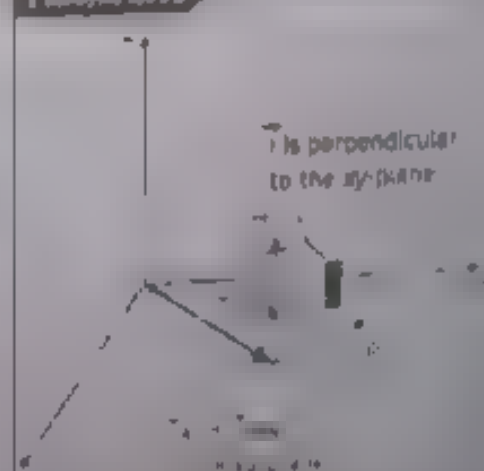
$$\vec{L} = \vec{r} \times \vec{p} \quad \text{--- (1)}$$

Magnitude of angular momentum is

$$L = r p \sin \theta \quad \text{--- (2)}$$

Consider the special case of motion in a circle where  $\vec{r}$  is always perpendicular to  $\vec{p}$  i.e.  $\theta = 90^\circ$

### Diagram 5.0



In three-dimensional space the position vector  $\vec{r}$  lies in the xy-plane and the linear momentum  $\vec{p}$  is perpendicular to  $\vec{r}$ . The angular momentum  $\vec{L}$  with respect to the origin is  $\vec{L} = \vec{r} \times \vec{p}$  where  $\vec{L}$  is in the z-direction. The direction of  $\vec{L}$  is given by the right-hand rule.

Then the magnitude of angular momentum is

$$L = r P \sin \theta$$

$$L = r P \sin(90^\circ)$$

$$L = r P$$

$$P = mv$$

$$P = mv$$

$$L = mvr \quad (3)$$

$$P = mv$$

$$L = I \omega$$

$$\Rightarrow L = m (r \omega) r = mr^2 \omega$$

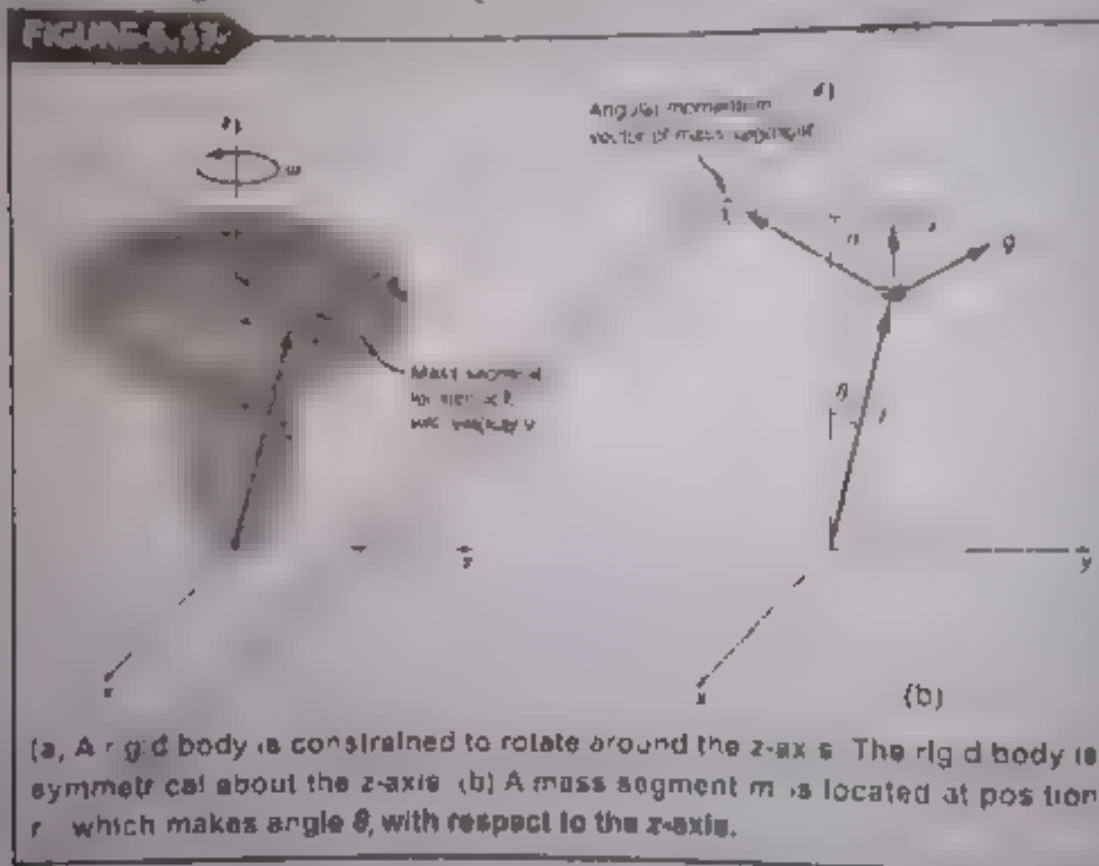
$$\Rightarrow L = I \omega$$

▶ Angular momentum is vector quantity and its direction is along axis of rotation ( $\hat{L}$  is perpendicular to plane containing vectors  $\vec{r}$  and  $\vec{p}$ )

▶ SI unit of angular momentum is  $\text{kg m}^2 \text{s}^{-1}$  or Js.

▶ Dimensions of angular momentum =  $[\text{ML}^2 \text{T}^{-1}]$

FIGURE 5.37



(a) A rigid body is constrained to rotate around the z-axis. The rigid body is symmetrical about the z-axis. (b) A mass segment  $m$  is located at position  $r$  which makes angle  $\theta$ , with respect to the z-axis.

### (a) Angular Momentum of a Rigid Body:

The rigid body is made of many particles and the sum of angular momenta of all the particles is the total angular momentum of the rigid body.

$$L = L_1 + L_2 + \dots + L_n$$

Then in terms of the masses and velocities of individual particles, we can write the total angular momentum as

$$L = m_1 R_1 v_1 + m_2 R_2 v_2 + \dots + m_n R_n v_n$$

$$L = \sum m_i v_i R_i \quad \text{Since } (v_i = R_i \omega)$$

$$\Rightarrow L = \sum m_i v_i R_i = \sum m_i (R_i \omega) R_i$$

$$\Rightarrow L = \sum (m_i R_i^2) \omega$$

$$\Rightarrow L = I \omega \quad \text{Where } I = \sum (m_i R_i^2) = \text{M.O.I.}$$

### (c) Angular Momentum is given by:

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\Delta \vec{L} = \vec{r} \times \Delta \vec{p}$$

Dividing both sides by  $\Delta t$

$$\frac{\Delta L}{\Delta t} = r \times \frac{\Delta p}{\Delta t}$$

$$\frac{\Delta L}{\Delta t} = r \times F$$

$$\frac{\Delta L}{\Delta t} = \tau$$

$$\tau = \frac{\Delta L}{\Delta t}$$

$$L = I\omega$$

$$\tau = \frac{\Delta L}{\Delta t} = I \frac{\Delta \omega}{\Delta t}$$

Rate of change in angular momentum of the body is equal to applied torque on the body.

Q.16 State and explain law of conservation of angular momentum of the body Give some examples to explain this law

### Law of Conservation of Angular Momentum

Law of conservation of angular momentum which states that in the absence of any external torque the angular momentum of a system remains constant.

If  $\tau = 0$  then

$$\text{Then } \frac{\Delta L}{\Delta t} = 0 \Rightarrow L = \text{Constant}$$

In a closed system in which no force and no torque acts on system, hence the angular momentum of an isolated system is conserved.

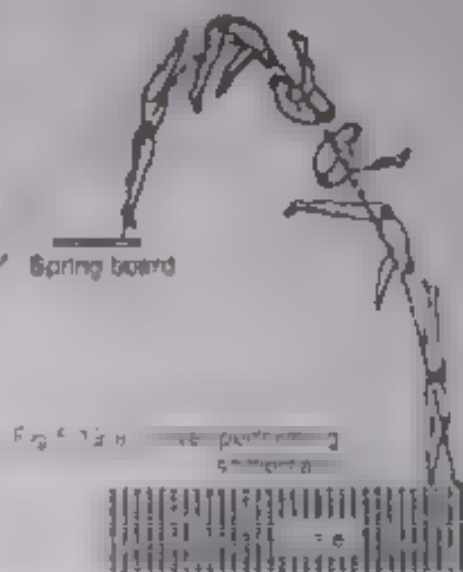
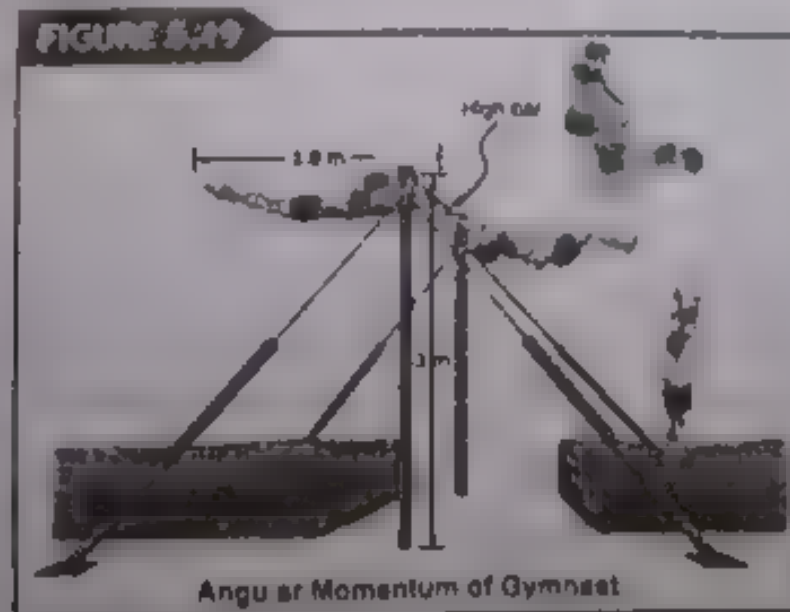


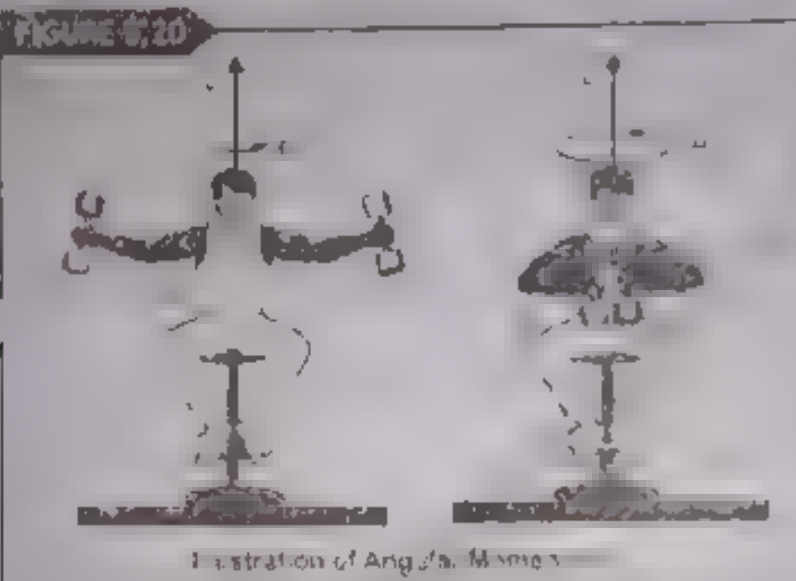
Fig. 5.48 A person performing somersault

### Applications

- This law is often used by circus acrobats, divers, ballet dancers, ice skaters and gymnasts to perform break in angles.
- In Fig. 5.49 a diver leaves the spring board with his arms and legs extended and a small angular speed about a horizontal axis through his centre of gravity. When he pulls his arms and legs in, his moment of inertia becomes smaller. In order to keep his angular momentum,  $L = I\omega$  constant, his angular velocity increases. He can thus make one or two extra somersaults.



- A person starts the diameter of his feet, extension and then by pulling it in a very short time, he moves his hands towards the centre of the axis of rotation, thereby decreasing the moment of inertia. As a result, his angular velocity increases in order to keep his angular momentum  $L = I\omega$  constant. This is called angular velocity and we can extend it to complete the rotation.
- The figure skater is standing on a turntable and holds a heavy weight in each hand. When he starts to stretch his arms, he is first set rotating slowly. Then, as he pulls his arms in, his angular velocity is considerably increased. He starts with a low rotating speed by stretching his hands, but as he pulls his arms in, his angular momentum is conserved.



### Assignment

A DVD disc has a radius of 0.0600 m and a mass of 0.0200 kg. The moment of inertia of a solid disc is  $I = \frac{1}{2}MR^2$  where  $M$  is the mass of the disc, and  $R$  is the radius. When a DVD in a certain machine starts playing, it has an angular velocity of  $1600 \text{ rad s}^{-1}$ . What is the angular momentum of the disc?

**Given Data**  $r = 0.0600 \text{ m}$   
 $m = 0.0200 \text{ kg}$   
 $\omega = 1600 \text{ rad s}^{-1}$

**Solution**  $L = I\omega$   
 $I = \frac{1}{2}MR^2$  or  $\text{For Disc } I = \frac{1}{2}mr^2$   
 $\therefore L = \frac{1}{2}(0.0200)^2(1600) = 0.00576 \text{ kg m}^2/\text{s}$

Q 16 Define rotational K.E. Derive its formula.

### ANSWER Rotational Kinetic Energy

The energy possessed by the body due to its rotation about an axis is called rotational kinetic energy.

**Proof**

Let us derive the energy in a body due to its linear motion, called 'K.E.'

$$K.E. = \frac{1}{2}mv^2 \quad (1)$$

Where  $m$  is mass of body,  $I$  is inertia and  $v$  is linear velocity of body.



Similarly, the energy in a body due to its angular motion is called rotational kinetic energy and is given by equation

$$K.E._{\text{rot}} = \frac{1}{2} I \omega^2$$

Where 'I' is the moment of inertia of body and ' $\omega$ ' stands for angular velocity of body.

### Rotational K.E. of A Irregular Rigid Body

Let us apply a force ' $F$ ' on a rigid body as shown. We have divided the whole body into a number of small pieces of masses  $m_1, m_2, m_3, \dots, m_n$  having distances from C.G. point as  $r_1, r_2, r_3, \dots, r_n$  respectively.

The total K.E. of body will be  $K.E. = K.E._1 + K.E._2 + \dots + K.E._n$

$$\text{Or } K.E._r = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots + \frac{1}{2} m_n v_n^2$$

Using equation  $v = r\omega$ , then

$$K.E._{\text{rot}} = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \dots + \frac{1}{2} m_n r_n^2 \omega^2$$

Here, each piece of body moves with same angular velocity  $\omega$

$$\therefore \text{as } K.E._r = \frac{1}{2} \sum m r^2$$

$$\text{Or } K.E._{\text{rot}} = \frac{1}{2} I \omega^2$$

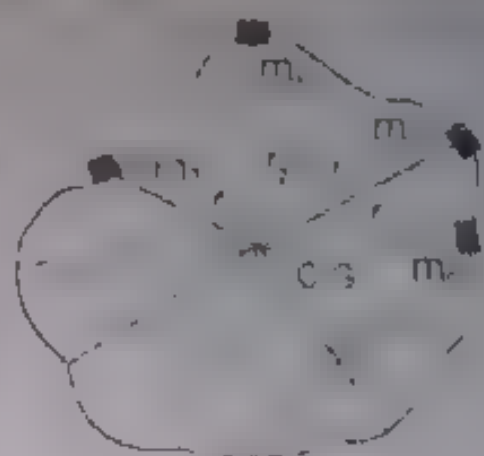


Fig 5.21

Q 17. Compare the equations of motions for linear motion and angular motion

Ans. Comparing angular and linear motions

| Linear Equations for linear motion           | Angular Equations for angular motion                                                     |
|----------------------------------------------|------------------------------------------------------------------------------------------|
| (i) $S = vt$                                 | (i) $\theta = \omega t$                                                                  |
| (ii) $v = u + at$                            | (ii) $\omega_f = \omega_i + \alpha t$                                                    |
| (iii) $v^2 - u^2 = 2aS$                      | (iii) $\omega_f^2 - \omega_i^2 = 2\alpha\theta$                                          |
| (iv) $S = ut + \frac{1}{2}at^2$              | (iv) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$                                       |
| (v) Inertia = $m$                            | (v) Moment of inertia $I = mr^2$                                                         |
| (vi) Force = $ma$                            | (vi) Torque = $\tau = I\alpha$                                                           |
| (vii) Linear momentum = $\vec{p} = m\vec{v}$ | (vii) Angular momentum = $\vec{L} = \vec{r} \times \vec{p}$ or $\vec{L} = I\vec{\omega}$ |
| (viii) $K.E._{\text{lin}} = \frac{1}{2}mv^2$ | (viii) $K.E._{\text{rot}} = \frac{1}{2}I\omega^2$                                        |

**Q 18** Discuss the K.E. of the rolling body from an inclined plane

**Ans:** K.E. of Rolling Body

When a body is rolling down an inclined plane, then P.E. of the rolling body converts into two types of kinetic energies i.e. it has linear K.E. due to its linear motion i.e.  $K.E. = \frac{1}{2}mv^2$  and it has rotational K.E. due to its rotation i.e.  $K.E. = \frac{1}{2}I\omega^2$ . So, total K.E. of the body is the sum of its linear and rotational kinetic energies

$$K.E._{total} = K.E._{linear} + K.E._{Rotational}$$

$$Or \quad P.E._p = K.E._{linear} + K.E._{Rotational}$$

**Q 19** Find the rotational K.E. of hoop, disc and sphere?

**Ans:** Rotational K.E. of a Hoop:

As we know

$$(K.E.)_{rot} = \frac{1}{2}I\omega^2$$

As moment of inertia of a hoop is

$$I = mr^2$$

$$So, \quad (K.E.)_{rot} = \frac{1}{2}(mr^2)\omega^2$$

$$(K.E.)_{rot} = \frac{1}{2}mr^2\omega^2$$

$$K.E._{rot} = \frac{1}{2}m(r\omega)^2$$

$$(K.E.)_{rot} = \frac{1}{2}mv^2$$

$$v = r\omega$$

Rotational K.E. of a Disk

As we know

$$(K.E.)_{rot} = \frac{1}{2}I\omega^2$$

As moment of inertia of a disc is  $I = \frac{1}{2}mr^2$

$$So \quad (K.E.)_{rot} = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2$$

$$K.E._{rot} = \frac{1}{4}mr^2\omega^2$$

As we know  $v = r\omega$

$$(K.E.)_{rot} = \frac{1}{4}mv^2$$

Rotational K.E. of a Sphere

As we know

$$(K.E.)_{rot} = \frac{1}{2}I\omega^2$$

As moment of inertia of a sphere is  $I = \frac{8}{5}mr^2$

$$So \quad K.E._{rot} = \frac{1}{2}\left(\frac{8}{5}mr^2\right)\omega^2$$

$$\Rightarrow K.E._{rot} = \frac{4}{5}m(r^2\omega^2)$$

Putting  $r\omega = v$

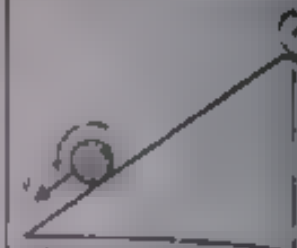
$$\Rightarrow K.E._{rot} = \frac{4}{5}mv^2$$

**Q 20** Derive the relations for the velocities of disc and hoop moving down an inclined plane.

**Ans:** Hoop and disc are placed at height  $h$  on an inclined plane as shown. When these are released both roll down the hill, they move forward with translational or linear K.E. and it has rotational K.E. as it also rotates. So from law of conservation of energy

$$P.E. = K.E._{trans} + K.E._{rot}$$

For Your Information



As the sphere rolls down the incline, its potential energy is converted into translational kinetic energy and rotational kinetic energy.

**EXPLANATION**  
 $P.E. = K.E._{trans} + K.E._{rot}$

**EXPLANATION**  
 $P.E. = K.E._{trans} + K.E._{rot}$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \dots\dots (1)$$

For Hoop

$$I = mr^2$$

Put in eq. (1)

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\omega^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}m(r\omega)^2 \quad (\text{Since } v = r\omega)$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\Rightarrow mgh = mv^2$$

$$\Rightarrow \frac{gh}{1} = \frac{v^2}{1}$$

$$\Rightarrow \sqrt{gh} = \sqrt{v^2}$$

$$\Rightarrow v = \sqrt{gh}$$

$$\Rightarrow \boxed{v = \sqrt{gh}}$$

For Disc

$$I = \frac{1}{2}mr^2$$

Put in eq. (1)

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}mr^2)\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 \quad (\text{Since } v = r\omega)$$

$$mgh = \frac{3}{4}mv^2$$

$$mgh = \frac{3}{4}mv^2$$

$$\Rightarrow gh = \frac{3}{4}v^2$$

$$\Rightarrow \frac{4}{3}gh = v^2$$

$$\Rightarrow \sqrt{\frac{4}{3}gh} = \sqrt{v^2}$$

$$\Rightarrow \boxed{v = \sqrt{\frac{4}{3}gh}}$$

$$\Rightarrow \boxed{v_{disc} = \sqrt{\frac{4}{3}gh}}$$

$$\Rightarrow \boxed{v_{disc} = \sqrt{\frac{4}{3}gh}}$$

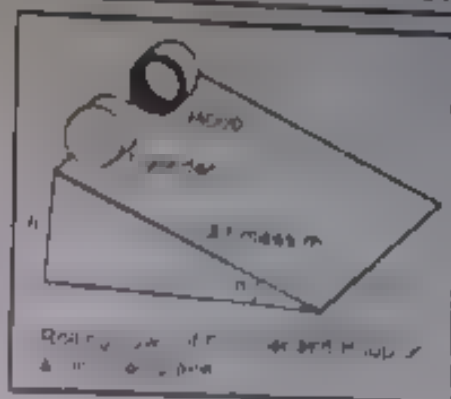
Now since  $v_{disc} > v_{hoop}$

$$v_{disc} > v_{hoop}$$

As we can see

$$v_{disc} = \sqrt{\frac{4}{3}gh} > v_{hoop} = \sqrt{gh}$$

$$v_{disc} > v_{hoop}$$



## SOME IMPORTANT POINTS RELATED TO ROTATIONAL K.E. OF DIFFERENT BODIES

| Body            | Moment of Inertia  | Rotational K.E.        | Translational K.E. | Total K.E.         | Rotational BW K.E. and H.E. | Ratio | Ratio | Ratio | Ratio         | Ratio |
|-----------------|--------------------|------------------------|--------------------|--------------------|-----------------------------|-------|-------|-------|---------------|-------|
| Thin rod        | $\frac{1}{12}ml^2$ | $\frac{1}{2}I\omega^2$ | $\frac{1}{2}mv^2$  | $\frac{1}{2}mv^2$  | $K.E. = \frac{1}{2}K.E.$    | 2     | 1     |       |               | 1     |
| Thin disc       | $\frac{1}{2}mr^2$  | $\frac{1}{2}I\omega^2$ | $\frac{1}{2}mv^2$  | $\frac{3}{2}mv^2$  | $K.E. = \frac{1}{2}K.E.$    | 2     |       | 1     | $\frac{1}{2}$ |       |
| Thin sphere     | $\frac{2}{5}mr^2$  | $\frac{1}{2}I\omega^2$ | $\frac{1}{2}mv^2$  | $\frac{7}{10}mv^2$ | $K.E. = \frac{2}{5}K.E.$    | 5     |       | 2     | $\frac{2}{5}$ |       |
| Spherical shell | $\frac{2}{3}mr^2$  | $\frac{1}{2}I\omega^2$ | $\frac{1}{2}mv^2$  | $\frac{5}{6}mv^2$  | $K.E. = \frac{1}{3}K.E.$    | 3     |       | 2     | $\frac{2}{3}$ |       |

**Q 21** What are real and apparent weight? Find the apparent weight in different cases for an object suspended by a string and spring balance in a lift?

### Real and Apparent Weight

**Real Weight (W)**

It is the real pull of the earth on the object.

$$W = mg$$

**Apparent Weight (T)**

It is the weight as felt by the person in the lift.

It is given by

$$T = W$$

It is the weight as felt by the person in the lift. It is the weight as felt by the person in the lift.

It is the weight as felt by the person in the lift. It is the weight as felt by the person in the lift.

**Apparent weight of an object in lift**

Consider a person weighs a fish with a spring balance attached to a ceiling of the lift.

Let the person is

1. At rest or moving with uniform velocity

2. Accelerating upwards

3. Accelerating downwards

When the lift is at rest or moving with uniform velocity, the apparent weight is equal to the real weight.

When the lift is accelerating upwards, the apparent weight is greater than the real weight.

When the lift is accelerating downwards, the apparent weight is less than the real weight.

### CASE 1

When the lift is at rest or moving with uniform velocity i.e.  $a = 0$

When the lift is at rest, Newton's second law tells us that the acceleration of the object is zero. So the net force becomes zero.

If  $W$  is the gravitational force acting on the object (real weight) and  $T$  is the tension in the string (apparent weight), then the net force on the person is

Then  $T = W$

As  $a = 0$

$0 = W - T$

$T = W$

Apparent Weight = Real Weight

Result

Hence the apparent weight of person is equal to the real weight, for observer inside the lift.

FIGURE 8.23



When the lift is at rest or moving with uniform velocity, the spring scale reads the weight of the fish.

### CASE I

When the lift is moving upward with acceleration 'a'

When the lift is moving upwards with an acceleration 'a'

So the upward force i.e. Tension in string is greater than downward force i.e.  $W$

Then the net force acting on the body is

$$T - W = F_{\text{net}}$$

$$W - mg = T$$

OR  $T = W + ma$

Apparent weight = Real weight +  $ma$

#### Result

When the lift is moving upwards with an acceleration 'a', the apparent weight of object is increased by a amount of ' $ma$ ' than its actual weight.

Above equation can be written as

$$T = mg + ma$$

### CASE III

When the lift is moving downward with acceleration 'a'

When the lift is moving downwards with an acceleration 'a', the downward force i.e.  $T$  is less than downward force i.e.  $W$

So  $T < W$

Then the net force acting on the body is

$$W - T = F_{\text{net}}$$

$$W - T = ma$$

$$T = W - ma$$

OR  $T = W - ma$

Apparent weight = Real weight -  $ma$

#### Result

When the lift is moving downwards with an acceleration 'a', the apparent weight of object is less than the actual weight by a amount of ' $ma$ '.

Above equation can be written as

$$T = mg - ma$$

$$T = (m)g - (a)$$

### CASE IV

When Lift is falling freely under gravity i.e.  $a = g$

Now we consider that the lift is falling freely under gravity. Then

$$a = g$$

As  $T = W - ma$

$$T = mg - ma$$

$$T = mg - mg$$

$$T = 0$$

Apparent weight = 0

#### Result

So the apparent weight of object shown by the balance is zero. The object seems to be weightless. Thus, it is in state of weightlessness.

FIGURE 5.20



When the lift accelerates upward the spring scale reads a value greater than the weight of the fish.

FIGURE 5.21



When the lift accelerates downward the spring scale reads a value less than the weight of the fish.

#### Do you know?

Your apparent weight differs from your true weight when the velocity of the lift changes at the start and end of a ride, not during the rest of the ride when that velocity is constant.



- The orbiting spaceship is accelerating towards the center of the earth at all the times exactly the same way as cannon balls orbiting round the earth as shown in figure. Its radial acceleration is simply  $g$  the free fall acceleration.
- So the orbiting satellite or space station is a falling free object in space. Everything within it will appear to be weightless.

### Example

A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of  $5 \text{ m/s}^2$  what will be the reading of the spring balance?

**Given Data**  
 $W = 49 \text{ N}$   
 $m = \frac{W}{g} = \frac{49}{9.8} = 5 \text{ kg}$   
 and  $a = 5 \text{ m/s}^2$

**Solution** When lift moves downward  
 $T = m(g - a)$   
 $\Rightarrow T = 5(9.8 - 5) = 5(4.8)$   
 $\Rightarrow T = 24 \text{ N}$

Q.24 What are artificial satellites? Find the expression for minimum velocity to put a satellite into the orbit?

### Artificial Satellite

An object in orbit in the outer mode is the artificial satellite.

Satellite can be launched from earth's surface to circle the earth by means of rocket. They are kept into orbit by gravitational attraction of earth. The satellites which are near the earth have the acceleration  $9.8 \text{ m/s}^2$  otherwise they would fly off in the straight line tangent to earth.

#### Critical orbital velocity

The minimum velocity required to put a satellite into an orbit above the earth is called critical velocity.

#### Expression

is determined by mass  $m$  moving with velocity  $v$  close to the earth in a circle of radius  $R$ . The centripetal force acting on the satellite is provided by gravitational force acting on satellite by earth. So the centripetal acceleration of the orbiting satellite is,

$$a_c = \frac{v^2}{R} \quad (1)$$

This force being provided by its weight. So, its acceleration must be equal to gravitational acceleration.

$$a_c = g = \frac{v^2}{R} \quad (2)$$

$$v^2 = R \cdot g \quad \text{or} \quad v = \sqrt{R \cdot g}$$

Where  $v$  is the critical velocity and  $R$  is the radius of earth.  
 Thus from equation (2)

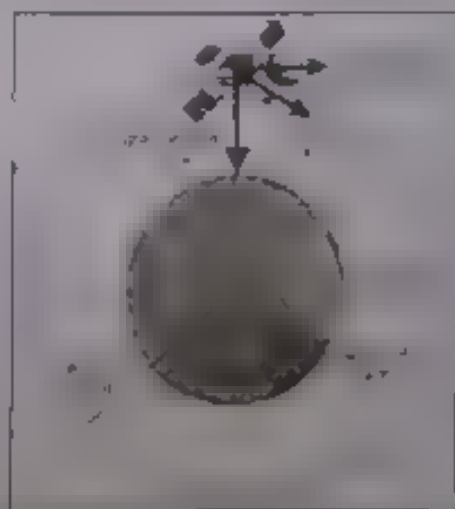
$$\Rightarrow v^2 = gR$$

$$\Rightarrow v = \sqrt{gR}$$

$$v = \sqrt{9.8 \times 6.4 \times 10^6} \text{ m/s}$$

Putting values, we get

$$v = 8.98 \times 10^3 \text{ m/s}$$



critical velocity



Quiz

If astronaut is cut there hairs in space station  
(a) will it fall to the floor?  
(b) if not, what are the reasons?

Ans

As it is in free space station  
the hairs are stationary falling object, so it will  
not move across the floor any thing in it will appear  
weightless. This is a free fall condition.

Do you know?

When a person is in a space station, they are in a state of weightlessness. This is because the station is in a free fall orbit around the Earth. The person and the station are both falling towards the Earth at the same rate, so the person does not feel any weight.

Q 25 What is Artificial Gravity? Derive expression for frequency of spaceship required to provide the artificial gravity?

Ans Artificial Gravity

When we have a spaceship in space, we have to provide a force of the spaceship. Such provided gravity is known as artificial gravity.

OR

Artificial gravity is the gravity like effect produced in an orbit by spinning it about its own axis.

Explanation:

When a spaceship is in space, it is in a state of weightlessness. To provide artificial gravity, the spaceship must rotate around its own axis. The rotation creates a centrifugal force that acts outwards from the axis of rotation. This force is what provides the artificial gravity. The faster the spaceship rotates, the stronger the artificial gravity will be.

Expression for frequency

The expression for the frequency of rotation required to provide artificial gravity is given by:

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{r}}$$

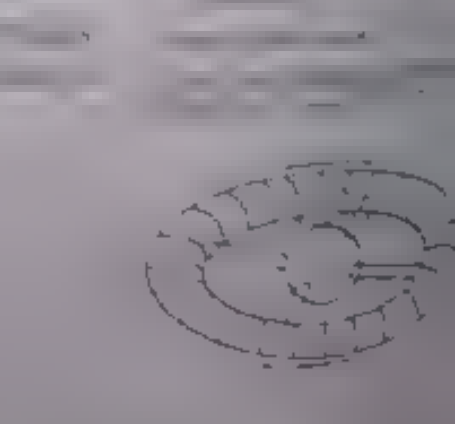
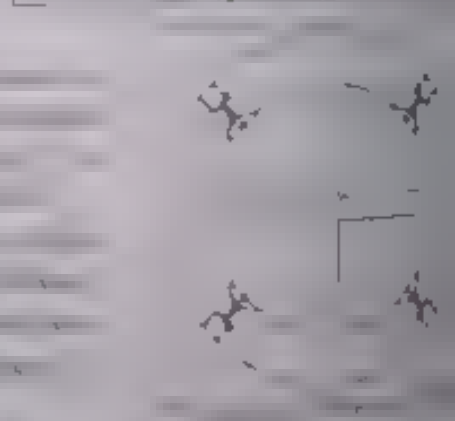
where  $f$  is the frequency of rotation,  $g$  is the acceleration due to gravity, and  $r$  is the radius of the spaceship.

This expression shows that the frequency of rotation required to provide artificial gravity is inversely proportional to the square root of the radius of the spaceship.

Therefore, if the radius of the spaceship is increased, the frequency of rotation required to provide artificial gravity will decrease.

Conversely, if the radius of the spaceship is decreased, the frequency of rotation required to provide artificial gravity will increase.

This expression is useful for determining the required rotation rate for a given artificial gravity level and spaceship radius.



$$g = \frac{GM}{r^2}$$

$$K = \frac{4\pi^2}{T^2} r^3 \quad \left[ \text{As } \frac{1}{T^2} = f^2 \right]$$

$$g = \frac{4\pi^2}{T^2} r$$

$$\therefore T = \frac{2\pi}{\sqrt{g}} \sqrt{r}$$

Taking square root of both sides

$$T = \frac{2\pi}{\sqrt{g}} \sqrt{r}$$

FIGURE 6.27



When the spaceship or satellite revolves with this frequency, the astronauts can help the astronauts perform the work easily.

### POINT-TO-POINTER

Astronauts who live with micro-gravity like on the ISS, are weightless. They can sleep or eat anything.

However, when they are in space, they have to attach themselves so they don't float around and bump into something. ISS astronauts usually sleep in sleeping bags located in small crew cabins.

Each crew cabin is just big enough for one astronaut.

Source: NASA, National Aeronautics and Space Administration



**Q 26** What is orbital velocity of a satellite? Derive its formula.

### 6.7 Orbital Velocity

1. A satellite is a body that revolves in a circular orbit around the earth.

2. The distance travelled by a satellite in one revolution around the earth is an orbital velocity. The earth and some other planets revolve round the sun in nearly circular orbits. The speed of the planet is called orbital velocity. All the satellites also revolve around the sun.

#### Expression for Orbital Velocity

Consider a satellite of mass  $m$  moving with orbital velocity  $v$  around the earth of mass  $M$ . If  $r$  is the radius of the orbit, then centripetal force  $F_c$  can be expressed as

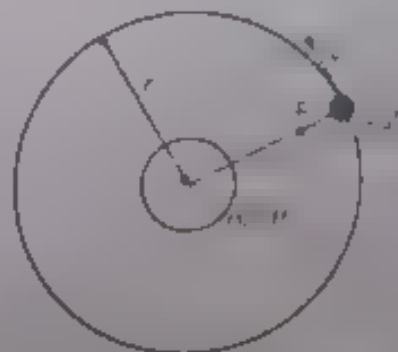
$$F_c = \frac{mv^2}{r} \quad (1)$$

This force is provided by gravitational force of attraction between earth and satellite and is given by

$$F_g = G \frac{Mm}{r^2} \quad (2)$$

Comparing equation (1) and (2) we get

$$\frac{mv^2}{r} = G \frac{Mm}{r^2}$$





**Q 27** What are geo-stationary satellites and geo-stationary orbits? Find the orbital radius of geo-stationary satellites?

**Geo-stationary Orbit**

The orbit in which the time period of revolution of satellite is equal to the time period of rotation of earth about its axis is called geo-stationary orbit.

**Geo-stationary Satellites**

The satellite which completes its one revolution around earth in 24 hours is called geo-stationary satellite.

The circular motion of satellite is such that the position of the earth about its axis is unchanged. Geo-stationary satellite.

This type of satellite is the one whose orbital period is equal to the period of rotation of earth. So the satellite remains always over the same point on the equator as the earth spins on its axis. These are satellites which remain stationary with earth.

**Applications**

Such satellites are used in communication systems, weather forecasting, military uses.

**Expression of orbital radius of geo-stationary satellite**

As we know, speed necessary for circular orbit is given by,

$$v = \frac{2\pi r}{T}$$

Here,  $v$  is speed that be equal to the orbital speed of satellite.

$$v = \frac{2\pi r}{T}$$

Or  $v = \frac{2\pi r}{T} \quad (2)$

Where  $T$  is period of revolution of satellite. As satellite is in the same orbit as earth, its period of revolution is exactly one day.

Equating eq. (1) and (2) we get

$$\frac{2\pi r}{T} = \frac{GM}{r^2}$$

Squaring both sides we get

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r^2}$$

$$r^4 = \frac{GMT^2}{4\pi^2}$$

Taking cube root on both sides

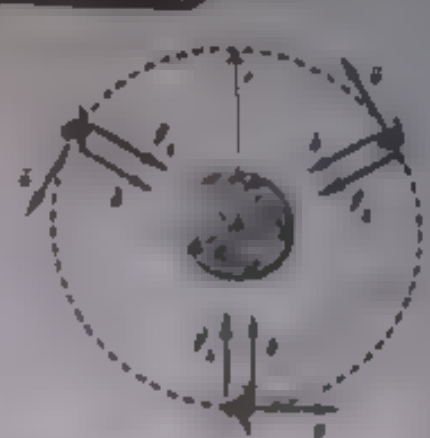
$$r = \left( \frac{GMT^2}{4\pi^2} \right)^{1/4}$$

This equation gives the orbital radius of the geo-stationary satellite.

Substituting the values

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2, M = 6 \times 10^{24} \text{ kg}$$

**FIGURE 5.29**

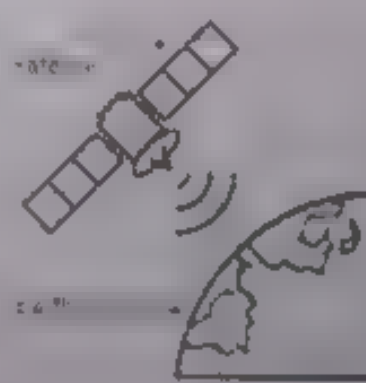


The satellite is in a circular orbit. It is always over the same point on the equator. So, it appears stationary.



**For Your Information**

The satellite is in a circular orbit. It is always over the same point on the equator. So, it appears stationary.



$$T = 1 \text{ day} = 24 \text{ hours} = 24 \times 60 \times 60 \text{ s} = 86400 \text{ s}$$

$$r = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 86400^2}{4 \times \pi^2}$$

$$= 4.23 \times 10^7 \text{ m}$$

$$= 4.23 \times 10^4 \text{ km}$$

$$r = 21 \times 10^3 \text{ km}$$

A satellite orbit has as measured from center of the earth 1 if geo-stationary satellite

- A satellite at a height will always stay directly above a particular point on the surface of earth.
- This type of orbit is used for many communication, ocean and weather satellites.

A communication satellite has an altitude of  $3.6 \times 10^4 \text{ km}$  or  $36,000 \text{ km}$ .

A satellite is an object in space moving at a speed of  $6.98 \text{ km per second}$ .

## Q 28 Describe different types of artificial satellites

### ARTIFICIAL SATELLITES

A satellite is any object that orbits around another object. Satellites are not natural objects that exist in space whereas planets and moons are natural objects that exist in space.



How the earth can be covered by just three geostationary satellites

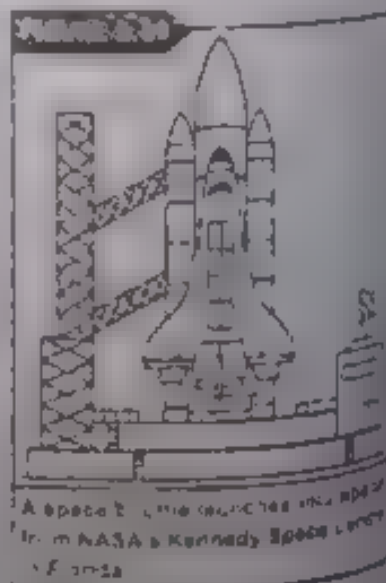
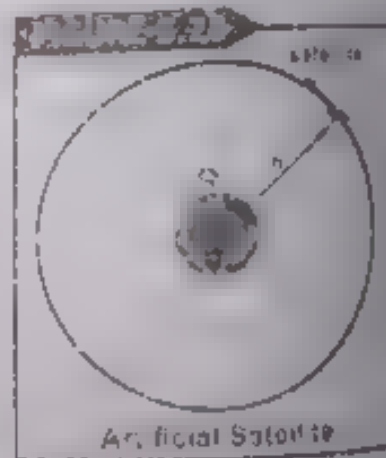
### Types of Satellites

#### Navigation satellites

Navigation satellites are used to determine the location of a receiver on the earth's surface. They are used in GPS systems and other navigation systems.

#### Communication satellites

Communication satellites are used to relay signals between different parts of the world. An example is the Optus B1 satellite, which is a geostationary satellite above the equator with a coverage footprint to provide signals to Australia and New Zealand.



#### Weather satellites

These satellites are used to image clouds and measure temperature and rainfall.

1. The path of a satellite in a circular orbit is a circle. The satellite is moving with a constant speed. The satellite is moving in a circular orbit. The satellite is moving in a circular orbit. The satellite is moving in a circular orbit.

### Ques from Past Board Papers

Weight of a block at the centre of the earth is

- (A)  $W$  (B)  $2W$  (C)  $4W$  (D)  $0$

Weight of 50 kg man in moving lift downwards, with constant acceleration of  $g$  where  $g = 10 \text{ m/s}^2$

- (A)  $500 \text{ N}$  (B)  $450 \text{ N}$  (C)  $600 \text{ N}$  (D)  $250 \text{ N}$

The apparent weight of a man in a lift moving downwards with an acceleration of  $9.8 \text{ m/s}^2$  is

- (A)  $0$  (B)  $W$  (C)  $2W$  (D)  $3W$

If a body of mass  $m$  is lifted upwards with an acceleration of  $9.8 \text{ m/s}^2$  its apparent weight be observed as

- (A)  $0$  (B)  $2mg$  (C)  $mg$  (D)  $3mg$

The weight of a man in a lift ascending with an acceleration of  $4.9 \text{ m/s}^2$  will become

- (A)  $W$  (B)  $\frac{W}{2}$  (C)  $0$  (D)  $2W$

A man of mass  $m$  is standing on the floor of a lift which is ascending with an acceleration of  $a$ . The apparent weight of the man is

- (A)  $W$  (B)  $W - ma$  (C)  $W$  (D)  $W + ma$

Rotational inertia of two equal masses of any shape but one has larger diameter will be

- (A)  $0$  (B)  $\text{Larger}$  (C)  $\text{Same}$  (D)  $\text{Smaller}$

Which is important in describing the satellite's orbit?

- (A)  $\text{Distance of satellite from earth's center}$  (B)  $\text{Angle of inclination of orbit}$

- (C)  $\text{Speed of satellite}$  (D)  $\text{Direction of motion}$

The ratio of the velocity of a satellite in a circular orbit to its orbital velocity is

- (A)  $1$  (B)  $2$  (C)  $3$  (D)  $4$

The velocity of a satellite in a circular orbit is

- (A)  $v = \sqrt{\frac{GM}{r}}$  (B)  $v = \sqrt{\frac{GM}{R}}$  (C)  $v = \sqrt{\frac{GM}{R^2}}$  (D)  $v = \sqrt{\frac{GM}{R^3}}$

A body of mass  $m$  is suspended from the ceiling of an elevator moving up with an acceleration  $g$ . Its apparent weight in the elevator will be

- (A)  $W$  (B)  $2W$  (C)  $3W$  (D)  $4W$

### Answers

|      |      |      |      |      |      |      |      |      |       |
|------|------|------|------|------|------|------|------|------|-------|
| 1. C | 2. D | 3. A | 4. D | 5. A | 6. D | 7. C | 8. D | 9. C | 10. D |
|------|------|------|------|------|------|------|------|------|-------|



### FORMULAE

|   |                                         |                               |                               |
|---|-----------------------------------------|-------------------------------|-------------------------------|
| 1 | Relation between $S$ , $r$ and $\theta$ | $S = r\theta$                 | $\omega = \frac{S}{t}$        |
| 2 | Average angular velocity                | $\omega = \frac{\theta}{t}$   |                               |
| 3 | Instantaneous angular velocity          | $\omega = \frac{d\theta}{dt}$ |                               |
| 4 | Average angular acceleration            | $\alpha = \frac{\omega}{t}$   | $\alpha = \frac{d\omega}{dt}$ |

|    |                                                               |                                                        |                                               |                                          |
|----|---------------------------------------------------------------|--------------------------------------------------------|-----------------------------------------------|------------------------------------------|
| 6  | Instantaneous angular acceleration                            | $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ |                                               |                                          |
| 6  | Relation between $v$ , $r$ and $\omega$                       | $v = r\omega$                                          |                                               | $\omega = \frac{v}{r}$                   |
| 7  | Relation between $a$ , $r$ and $\alpha$                       | $a_t = r\alpha$                                        |                                               | $a_r = r\omega^2$                        |
| 8  | Equations of angular motion                                   | $\omega = \omega_0 + \alpha t$                         | $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ | $2\alpha\theta = \omega^2 - \omega_0^2$  |
| 9  | Centripetal acceleration                                      | $a_c = \frac{v^2}{r}$                                  |                                               |                                          |
| 10 | Centripetal force                                             | $F_c = \frac{mv^2}{r}$                                 |                                               |                                          |
| 11 | Centripetal force (in angular measures)                       | $F_c = m r \omega^2$                                   |                                               |                                          |
| 12 | Moment of inertia of a particle                               | $I = mr^2$                                             |                                               |                                          |
| 13 | Torque on a particle rotating about a fixed point             | $\tau = r F \sin\theta$                                |                                               |                                          |
| 14 | Moment of inertia of a thin rod of length $L$                 | $I = \frac{1}{12} m L^2$                               |                                               |                                          |
| 15 | Moment of inertia of a hoop                                   | $I = m r^2$                                            |                                               |                                          |
| 16 | Moment of inertia of a disc                                   | $I = \frac{1}{2} m r^2$                                |                                               |                                          |
| 17 | Moment of inertia of a sphere                                 | $I = \frac{2}{5} m r^2$                                |                                               |                                          |
| 18 | 2 <sup>nd</sup> law of motion for rotational motion           | $\tau = I \alpha$                                      |                                               |                                          |
| 19 | Angular momentum                                              | $\vec{L} = \vec{r} \times \vec{p}$                     | $L = r p \sin\theta$                          | $L = r m v \sin\theta$                   |
| 20 | Angular momentum (in angular measures)                        | $L = m r^2 \omega$                                     |                                               | $L = I \omega$                           |
| 21 | Rotational K.E.                                               | $K E_{rot} = \frac{1}{2} I \omega^2$                   |                                               | $K E_{rot} = \frac{1}{2} m r^2 \omega^2$ |
| 22 | Rotational K.E. of disc                                       | $K E_{rot} = \frac{1}{4} m r^2 \omega^2$               |                                               | $K E_{rot} = \frac{1}{4} m v^2$          |
| 23 | Rotational K.E. of hoop                                       | $K E_{rot} = \frac{1}{2} m r^2 \omega^2$               |                                               | $K E_{rot} = \frac{1}{2} m v^2$          |
| 24 | Velocity of hoop falling from an inclined plane of height $h$ | $v = \sqrt{2gh}$                                       |                                               |                                          |
| 25 | Velocity of disc falling from an inclined plane of height $h$ | $v = \sqrt{\frac{4}{3}gh}$                             |                                               |                                          |

|    |                                                                         |                                        |                           |
|----|-------------------------------------------------------------------------|----------------------------------------|---------------------------|
| 26 | Velocity of sphere falling from an inclined plane of height $h$         | $v = \sqrt{2gh \sin^2 \theta}$         |                           |
| 27 | Critical orbital velocity                                               | $v_c = \sqrt{\frac{GM}{R}}$            | $v = \sqrt{\frac{GM}{R}}$ |
| 28 | Time period of close orbiting satellite                                 | $T = 2\pi \sqrt{\frac{R^3}{GM}}$       |                           |
| 29 | Apparent weight of an object at rest or moving up with uniform velocity | $W_a = W$                              |                           |
| 30 | Apparent weight of an object moving up with uniform acceleration $a$    | $W_a = W(1 + \frac{a}{g})$             |                           |
| 31 | Apparent weight of an object moving down with uniform acceleration $a$  | $W_a = W(1 - \frac{a}{g})$             |                           |
| 32 | Apparent weight of an object falling freely                             | $W_a = 0$                              |                           |
| 33 | Orbital velocity of a satellite                                         | $v_o = \sqrt{\frac{GM}{R}}$            |                           |
| 34 | Spinning frequency of a satellite about its own axis                    | $f_s = \frac{1}{T_s}$                  |                           |
| 35 | Orbital radius of a geostationary satellite                             | $R_o = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$ |                           |



## Key Points

Angular displacement, angular velocity and angular acceleration are related with a body moves in circle as follows, angular displacement by equation

$$\theta = r \phi \quad \text{or} \quad \phi = \frac{\theta}{r}$$

Angular velocity of a body is the change in angular velocity of a body in particular time. It is related as follows

$$\omega = \frac{d\theta}{dt} \quad \text{or} \quad \theta = \int \omega dt$$

A force which attracts a body towards the centre of circle when a body moves in circle is called centripetal force. Its formula is  $F_c = \frac{mv^2}{r}$

$$F_c = m\omega^2 r \quad \text{or} \quad \omega = \sqrt{\frac{F_c}{mr}}$$

The angular momentum of a body is  $L = mvr$

Relation between torque and angular momentum is  $\tau = \frac{dL}{dt}$

The orbital speed of a satellite in its orbit is given by  $v_o = \sqrt{\frac{GM}{R}}$

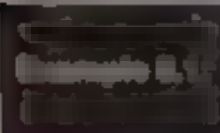
Velocity along a satellite's orbit whose angular velocity is synchronous with angular velocity of earth

The gravity provided to the spacecraft of a spaceship is called artificial gravity

- ❖ Apparent weight of a body is the force needed to prevent the body falling in the gravitational field of earth
- ❖ The whole surface of earth can be covered by  $\approx 3600$  geostationary satellites
- ❖ The state of a body in which it becomes weightless is called weightlessness
- ❖ The torque produced in body of mass  $m$  when a force  $F$  is applied on it at position vector  $r$  is given by  $\tau = r \times F \sin \theta$



## Solved Examples



An electric motor turns at 400 rpm. What is the angular velocity? What is the angular displacement after 4 s?

**Given Data**

$$\omega = 400 \text{ rpm} = 400 \times \frac{2\pi}{60} \text{ rad/s}$$

**Required**

$$\omega = ? \text{ rad/s}$$

$$\theta = ? \text{ rad}$$

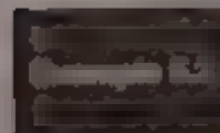
**Solution**

$$\omega = 400 \times \frac{2\pi}{60} \text{ rad/s}$$

$$\omega = 16.76 \text{ rad/s}$$

$$\theta = \omega t = 16.76 \times 4$$

$$\theta = 67.04 \text{ rad}$$



In a carnival ride, the passenger travels in a circle of radius 5.0 m, making one complete circle in 4.0 s. What is its acceleration?

**Given Data**

$$r = 5.0 \text{ m}, \quad T = 4.0 \text{ s}$$

**Required**

$$a = ? \text{ m/s}^2$$

**Solution**

$$\text{period } T = 4.0 \text{ s} \Rightarrow \text{frequency } f = \frac{1}{T} = \frac{1}{4.0} \text{ per s}$$

$$a = \omega^2 r = (2\pi f)^2 r$$

$$a = (2\pi \times \frac{1}{4.0})^2 \times 5.0$$

$$a = \frac{4\pi^2 \times 5.0}{16.0}$$

$$a = 12.36 \text{ m/s}^2$$



The curved roadway is designed in such a way that a car will not have to rely on friction to round the curve even when the road is covered with ice. Suppose the designated speed for the road is to be 12 m/s and the radius of the curve is 36.0 m. What angle should the curve be banked?

**Given Data**

$$v = 12 \text{ m/s}, \quad r = 36.0 \text{ m}$$

**Required**

$$\theta = ? \text{ banked}$$

**Solution**

$$\tan \theta = \frac{v^2}{rg}$$

A 2 kg mass swings in a circle of radius 50 cm at the end of a light rod. What resultant torque is required to give an angular acceleration of  $2.5 \text{ rad/s}^2$ ?

Given Data

Radius  $r = 50 \text{ cm} = 0.5 \text{ m}$   
 Mass  $m = 2 \text{ kg}$   
 Angular acceleration  $\alpha = 2.5 \text{ rad/s}^2$

Required

torque

Solution

$$I = mr^2 = 2 \text{ kg} \times (0.5 \text{ m})^2 = 0.5 \text{ kg m}^2$$

$$\tau = I\alpha = (0.5 \text{ kg m}^2)(2.5 \text{ rad/s}^2)$$

$$= 1.25 \text{ Nm}$$

$\therefore 1.25 \text{ Nm}$

Find Earth's angular momentum using Earth-Sun distance and mass of Earth.  
 Earth-Sun distance  $149.6 \times 10^9 \text{ m}$   
 Mass of the Earth  $5.9742 \times 10^{24} \text{ kg}$

Given Data

Distance from Earth to Sun  $= 149.6 \times 10^9 \text{ km}$   
 Mass of Earth  $= 5.9742 \times 10^{24} \text{ kg}$

Required

Angular momentum

Solution

Angular momentum  $L = I\omega$

or

$L = Mr^2 \omega$  (if  $M$  and  $r$  are constant and  $\omega$  is constant, then  $L$  is constant)

Angular momentum  $L = Mr^2 \omega$

or

$$L = 5.9742 \times 10^{24} \text{ kg} \times (149.6 \times 10^9 \text{ m})^2 \times 2\pi \times 10^{-8} \text{ s}^{-1}$$

$\therefore L = 2.63 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}$

Angular momentum  $L = 2.63 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}$

$$L = 5.9742 \times 10^{24} \text{ kg} \times 2.98 \times 10^8 \text{ m s}^{-1} \times 149.6 \times 10^9 \text{ m}$$

$\therefore L = 2.63 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}$

Earth angular momentum is  $2.63 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}$

A body of moment of inertia  $0.80 \text{ kg m}^2$  about a fixed axis rotates with constant angular velocity of  $100 \text{ rad s}^{-1}$ . Calculate.

- Its angular momentum
- Torque to sustain this position

Given Data

Moment of inertia  $I = 0.80 \text{ kg m}^2$   
 Angular velocity  $\omega = 100 \text{ rad s}^{-1}$

Required

Angular momentum

Angular momentum  $L = I\omega$

### Solution

For  $L$  We use the equation  $L = I\omega$

$$L = 0.8 \times 10^{-3} \times 80 \text{ kg} \times \omega$$

As angular velocity is constant, so  $L$  is constant. If angular velocity is constant, then the net torque about the axis of rotation is zero. So

$$L = 0 = P \times \Delta P \times P = P$$

$$\omega = \frac{L}{I} = \frac{0}{10^{-3}} = 0 \text{ rad/s}$$

$$\tau = I\alpha = L\omega = 0$$

□

### Example 6.7

A child of mass 25 kg stands at the edge of a rotating platform of mass 150 kg and radius 4.0 m. The platform with the child on it rotates with an angular speed of 6.2 rad/s. The child jumps off in a radial direction. What happens to the angular speed of the platform? Treat the platform as a uniform disk.

#### Given Data

Mass of child  $m = 25 \text{ kg}$

mass of platform  $M = 150 \text{ kg}$ , radius  $r = 4.0 \text{ m}$

rotational speed  $\omega = 6.2 \text{ rad/s}$

#### Required

Final angular speed  $\omega = ?$

#### Solution

$$I_{\text{platform}} = \frac{1}{2}mr^2 = \frac{1}{2} \times 150 \text{ kg} \times (4 \text{ m})^2 = 1200 \text{ kgm}^2$$

From the observation of the child, we know that

The angular momentum is conserved as there is no external torque.

$$L_{\text{initial}} = L_{\text{final}} \Rightarrow I\omega = I'\omega'$$

$$1200 \times 6.2 = 400 \times \omega'$$

$$60 \times 6.2 = \omega'$$

$$\omega' = 372 \text{ rad/s}$$

$$\omega = 372 \text{ rad/s}$$

$$I_{\text{platform}} = 25 \text{ kg} \times (4 \text{ m})^2 = 400 \text{ kgm}^2$$

### Example 6.8

A 70 kg man is standing on a scale in an elevator which is accelerating, as it heads for the top floor of a building at  $4 \text{ ms}^{-2}$ . What apparent weight will show on the scale?

#### Given Data

Mass  $m = 70 \text{ kg}$

Acceleration  $a = 4 \text{ ms}^{-2}$

#### Required

Apparent weight  $F$

#### Solution

$$F = mg + ma$$

$$F = 70 \times 9.8 \text{ ms}^{-2} + 4 \text{ ms}^{-2}$$

$$F = 700 \text{ N}$$

### Example 6.9

Determine the orbital speed for the International Space Station (ISS). If its orbit  $4.0 \times 10^2 \text{ km}$  above the earth surface.

#### Given Data

The radius at which it orbits

$$r = R_e + 400 \times 10^3 \text{ m} = 6360 \times 10^3 + 400 \times 10^3 \text{ m}$$

$$r = 6760 \times 10^3 \text{ m} = 6.76 \times 10^6 \text{ m}$$

$$M_e = 6 \times 10^{24} \text{ kg}$$

Required

$$\text{Orbital speed } v_{\text{orb}} = ?$$

Solution:

$$v_{\text{orb}} = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} (6 \times 10^{24} \text{ kg})}{6.76 \times 10^6 \text{ m}}}$$

$$v_{\text{orb}} = 7.67 \times 10^3 \text{ m/s}$$

**Example 2.13**  
What should be the orbital speed to launch a satellite in a circular orbit 900 km above the surface of the earth?

Given Data

Height above the surface of earth is  $h = 900 \text{ km}$   
 Radius of earth  $= R_e = 64 \times 10^2 \text{ m}$   
 So radius of orbit will be  $r_o = R_e + h = 73 \times 10^4 \text{ m}$   
 Here  $G = 6.673 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$   
 And mass of earth  $= M_e = 6 \times 10^{24} \text{ kg}$

Required

$$\text{Orbital speed } v_o$$

Solution:

Using the formula  $v_o = \sqrt{\frac{GM}{r}}$

We get  $v_o = \sqrt{\frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{7.3 \times 10^4}}$

Or  $v_o = 7.4 \times 10^3 \text{ m/s}$   
 $v_o = 7.4 \times 10^3 \text{ m/s}$

## Text Book Exercises

Q. Select the correct answer of the following questions.

The angular speed in radians/hours for daily rotation of our earth is

- (a)  $2\pi$  (b)  $4\pi$  (c)  $\pi$  (d)  $\pi/2$

Linear acceleration

- (a)  $0^\circ$  (b)  $180^\circ$  (c)  $360^\circ$  (d)  $90^\circ$

What is moment of inertia of a sphere

- (a)  $MR$  (b)  $\frac{1}{2}MR^2$  (c)  $\frac{2}{5}MR^2$  (d)  $\frac{1}{2}M^2R$

A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of particle. The motion of the particle takes place in a horizontal plane. It follows

- (a) Linear momentum is constant (b) Velocity is constant  
 (c) Particle moves in a circular path (d) Particle moves in straight line

A body moving in a circular path with constant speed has

- (a) Constant acceleration (b) Constant retardation  
 (c) Variable acceleration (d) Variable speed and constant velocity

6. Astronauts appear weightless in space because  
 (a) there is no grav. in space (b) there is no force pushing upwards on them  
 (c) satellite is freely falling (d) there is no air in space
7. Which one is constant for a satellite in orbit?  
 (a) Velocity (b) K.E. (c) Angular Momentum (d) Centrifugal force
8. If the earth suddenly stops rotating the value of  $g'$  at equator would  
 (a) Decrease (b) Remain unchanged (c) Increase (d) Be zero
9. If solid sphere and solid cylinder of same mass and density rotate about their own axis, the moment of inertia will be greater for  
 (a) Solid sphere (b) Solid cylinder  
 (c) The one that has the largest mass arrives first (d) The one that has the largest radius arrives first
10. The gravitational force exerted on an astronaut on Earth's surface is 650 N down. When she is in the International Space Station, the gravitational force on her is  
 (a) larger, (b) exactly the same  
 (c) smaller (d) negative but not equal to zero
11. A solid cylinder of mass  $M$  and radius  $R$  rolls down an incline without slipping. Its moment of inertia about an axis through its center of mass is  $\frac{1}{2}MR^2$ . At any instant while in motion, its rotational kinetic energy about its center of mass is what fraction of its total kinetic energy  
 (a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{4}$  (d)  $\frac{1}{5}$

| No. | Option | ANSWER                         | EXPLANATION                                                                                                                                                                                             |
|-----|--------|--------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1   | (d)    | $\frac{\pi}{2}$                | $\frac{d}{t} = \frac{\pi R}{24\pi R} = \frac{1}{24}$ rev/s                                                                                                                                              |
| 2   | (d)    | 96                             | $\frac{4}{2} \times \frac{3}{2} \times \frac{4}{2} \times \frac{3}{2} \times \frac{4}{2} \times \frac{3}{2} = 96$<br>$\therefore$ 96 revolutions                                                        |
| 3   | (c)    | $\frac{7}{5}MR^2$              | For sphere $\frac{2}{5}MR^2$                                                                                                                                                                            |
| 4   | (c)    | It moves in a circular path    | Centrifugal force acts perpendicular to the motion of the ball<br>$\therefore$ it moves along a circle                                                                                                  |
| 5   | (a)    | constant acceleration          | When body moves in a circular path, it has a centripetal acceleration which has constant magnitude $a = \frac{v^2}{r}$ and is directed towards the centre of the circle                                 |
| 6   | (c)    | satellite is in a stable orbit |                                                                                                                                                                                                         |
| 7   | (c)    | Angular Momentum               | As no external torque is on satellite and it is isolated from outside<br>$\therefore$ conservation of angular momentum                                                                                  |
| 8   | (c)    | Increase                       | Rotation of earth causes centrifugal force. If earth stops rotating, centrifugal force becomes zero and the value of $g$ increases<br>$\therefore$ gravity increases                                    |
| 9   | (b)    | Solid cylinder                 | Moment of inertia of sphere $= \frac{2}{5}mr^2$<br>Moment of inertia of cylinder $= \frac{1}{2}mr^2$<br>From above equations, $m$ and $r$ are same for both<br>$\therefore$ $\frac{1}{2} < \frac{2}{5}$ |
| 10  | (c)    | Smaller                        | Gravitational force on her is a proportion to the distance from the centre of earth i.e. $F \propto \frac{1}{r^2}$                                                                                      |

|    |     |   |                                                                                                                                                                                                                                                                                                                                                                                                                 |
|----|-----|---|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 11 | (a) | 3 | $K.E_{\text{trans}} = \frac{1}{2}mv^2$ $K.E_{\text{rot}} = \frac{1}{2}I\omega^2$ $K.E_{\text{total}} = K.E_{\text{translational}} + K.E_{\text{rotational}}$ <p>For solid cylinder or disc</p> $K.E_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2} \left( \frac{1}{2}mr^2 \right) \omega^2$ $K.E_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$ |
|----|-----|---|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|



## SHORT ANSWERS OF THE EXERCISE

Q2 Write short answers of the following questions

1 Why is the fly wheel of an engine made heavy in the rim?

**Ans:** The rim of fly wheel of an engine is made heavy to concentrate more and more mass at great or perpendicular distance from axis of rotation i.e. at edges. Flywheel stores more and more inertia. It has large moment of inertia. It is wheel which rotates with a rotational speed.

**Reason:** As we know that moment of inertia of hoop is compared to disc. If reasoning be taken from middle and all mass is concentrated at the periphery. It increases its moment of inertia. It is in opposition to any change in uniform rotational motion. It is large due to which it maintains its uniform rotational motion.

2 Why is a rifle barrel 'rifled'?

**Ans:** Rifling of a rifle barrel means the spiral grooves in the interior of the barrel. The barrel of a rifle is rifled to keep the bullet along the required direction.

**Explanation:**

When a rifle is fired the bullet moves along the barrel and has translational motion as well as rotational motion about its own axis. This rotational motion helps the bullet to keep it along the required direction with aerodynamic shape. It is because kinetic energy of the bullet is a sum of translational K.E. and rotational K.E.

$$K.E = \sqrt{(K.E)_{\text{trans}} + (K.E)_{\text{rot}}}$$

$$K.E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

This much K.E. of the bullet gives the direction stability to the bullet so that it could preserve its direction for longer distances and hit the target with accuracy of missing the target.

3 Is it possible for a person to distinguish between a raw egg and a hard boiled one by spinning each on a table? Explain

**Ans:** Hard boiled egg will spin faster than raw egg when same torque is applied on both.

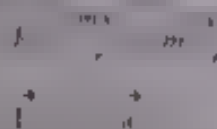
**Explanation:**

The raw egg contains liquid fluid and when you rotate it the centrifugal force will act on the fluid and pull it towards outer shell. Therefore moment of inertia of raw egg is greater and angular velocity is smaller. Hard boiled egg act as a rigid body while rotating. The moment of inertia of hard boiled egg is smaller and its angular velocity is greater. Hence hard boiled egg will spin faster than raw egg when same torque is applied on both the cases.



4 Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?

**ANS:** When a body is moving uniformly in a circle, the direction of the velocity is changing at each point due to which centripetal acceleration is produced. In a circle, the centripetal acceleration always points towards the centre. According to Newton's 2nd law, if motion the force on the body is directed towards the centre, the acceleration is also towards the centre of the circle or along the radius of the circle.



5 A ball is just supported by a string without breaking. If it is set swinging it breaks. Why?

**ANS:** When the mass is hanging vertically, the tension in the string balances the weight.

$$T = W = mg$$

But when the mass is set swinging, the tension in the string is reduced.

$$T - W = \text{centrifugal force}$$

Due to this increase in tension, the string breaks.

6 An insect is sitting close to the axis of a wheel. If the friction between the insect and the wheel is very small, describe the motion of the insect when the wheel starts rotating.

**ANS:** The insect will fly away from the wheel tangent to the circle at the point where it was sitting at the higher speed.

**Reason:**

When the wheel rotates, the centripetal force acts on the insect. But the insect keeps on moving with the wheel. As the speed of the wheel increases, the centripetal force required for the insect to move in a circle increases. As the friction is very small, the insect flies away from the wheel.

7 Explain how many minimum number of geo-stationary satellites are required for global coverage of TV transmission.

**ANS:** Minimum three satellites are required for global coverage of TV transmission.

**Reason:**

As each satellite can provide TV transmission over 1/3rd of the Earth, the global telecommunication system requires three correctly positioned geo-stationary satellites to provide the global coverage.

8 Explain the significance of moment of inertia in rotatory motion.

**ANS:** Moment of Inertia

- Moment of inertia is the property of body, which resists the change in its state of motion. It is called moment of inertia or rotational inertia of a body.
- It plays the same role in rotational motion as mass plays in linear motion.
- Greater the moment of inertia, greater will be the resistance to change in its state of motion.

9 Why does the coasting rotating system slow down as water drops into the beaker?

**ANS:** EXPLANATION

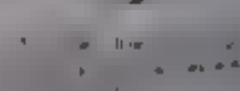
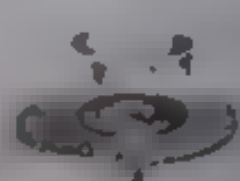
When water drops into the beaker, the mass of the system in the beaker increases, which increases the moment of inertia as  $I \propto mr^2$ .

$$\text{Angular momentum is given by } L = I\omega \Rightarrow \omega = \frac{L}{I}$$

$$\omega \propto \frac{1}{I}$$

According to the law of conservation of momentum, the angular momentum remains constant. There are when moment of inertia increases, the angular velocity decreases.

Point to Ponder



Q10 A body will be weightless when the elevator falls down just like a free falling body. Explain.

**Ans** When a body falls in free fall, its acceleration is  $a = g$ .  
Now we consider that the elevator falls in free fall with acceleration  $a = g$ .  
As  $T = W - ma$   
 $mg - ma$   
 $T = 0$  or  $mg - ma = 0$

### Result

So, the apparent weight of a body in a free falling elevator is zero.

Q11 When a tractor moves with uniform velocity its heavier wheel rotates slowly than its lighter wheel. Why? Explain.

**Ans** A heavy tyre rotates slowly than its lighter tyre.  
Reason: The moment of inertia is  $I = mr^2$

Where  $m$  = mass of the body

And  $r$  = Distance from the axis of the rotation

The mass and size of the heavier wheel is greater than the lighter wheel. So, when it rotates, it has a greater moment of inertia. So, when it rotates, it has a greater moment of inertia. So, when it rotates, it has a greater moment of inertia.



## Comprehensive Questions

Q1 Give a short response to the following questions.

What are centripetal acceleration and centripetal force? Derive the equations.

Q2 Show that angular momentum in magnitude is given by  $L = |\vec{r} \times \vec{p}| = m r^2 \omega = m r v$

Q3 Show that role playing by mass in linear motion is playing by moment of inertia in rotatory motion.

Q4 What do you mean by INTELSAT? At what frequency it operates? For how many TV stations this system is used?

Q5 State the Organization (INTELSAT)

INTELSAT is a satellite system managed by 126 countries. It is called INTELSAT (International Telecommunications Satellite Organization).

It works at the microwave frequencies of 4, 6, 11 and 12 GHz and uses the principle of communication through three TV channels.

Q6 Show that in angular form, centripetal acceleration  $\vec{a} = -\omega^2 \vec{r}$



We also know that  $\theta = \omega t$  (2)

Putting equation (2) in equation (1) we get,

$$S = r \omega t \quad (3)$$

Putting the values in equation (3) we get,

$$S = 10.5 \times 0.628 \times 120 \text{ m} \Rightarrow S = 7.83 \text{ m}$$

Or  $S = 11 \text{ m}$ , approximately

Distance travelled by fly =  $S = 11 \text{ m}$

1. A Circular drum of radius 40 cm is initially rotating at 400 revolutions/min. It is brought to stop after making 50 revolutions. What is angular acceleration and stopping time?

Solution

Given Data Radius of drum =  $r = 40 \text{ cm}$

$$\text{Initial angular velocity} = \omega_i = 400 \text{ rev/min} = \frac{400 \times 2\pi}{60} \text{ rad/sec}$$

$$\Rightarrow \omega_i = 41.866 \text{ rad/sec}$$

$$\text{Angular displacement } \theta = 50 \text{ revolution} = 50 \times 2\pi \text{ rad} = 314 \text{ rad}$$

$$\Rightarrow \text{Final angular velocity } \omega_f = 0$$

To Find:

a) Angular acceleration =  $\alpha$  ?

Drum comes to rest at  $\omega_f = 0$

b) Stopping time =  $t$  ?

Calculation.

a) We know that,

$$2\alpha\theta = \omega_f^2 - \omega_i^2 \quad (1)$$

Putting the values in equation (1), we get,

$$2 \times \alpha \times 314 = 0^2 - (41.866)^2 \Rightarrow 628\alpha = -1752.76$$

$$\Rightarrow \alpha = \frac{-1752.76}{628} \text{ rad/sec}^2 = -2.79 \text{ rad/sec}^2$$

$$\boxed{\alpha = -2.79 \text{ rad/sec}^2}$$

b) We know that,

$$\omega_f = \omega_i + \alpha t \quad (1)$$

Putting the values in equation (1), we get

$$0 = 41.866 - 2.79t$$

$$\Rightarrow 2.79t = 41.866$$

$$\Rightarrow t = \frac{41.866}{2.79} \text{ sec}$$

$$\Rightarrow t = 15.0 \text{ sec}$$

$$\boxed{t = 15.0 \text{ sec}}$$

Stopping time = 15.0 sec

2. A string 1m long is used to whirl a 100g stone in a horizontal circle at a speed of  $2 \text{ m s}^{-1}$ . Find the tension in the string

Solution.

Given Data: Length of string =  $r = 1 \text{ m}$ .

$$\text{Mass of stone} = m = 100 \text{ gm} = 0.1 \text{ kg}$$

To Find: Tension in the string =  $T$  = ?

Here the tension in the string must be equal to the centripetal force is

$$\text{Calculation } T = F_c = \frac{mv^2}{r}$$

$$\Rightarrow T = \frac{0.1 \times 2^2}{1} \Rightarrow \boxed{T = 0.4 \text{ N}}$$

- 4 The moon revolves around the earth in almost a circle of radius 382400 km in 27.3 days. What is the centripetal acceleration?

Solution

Given Data

$$r = 382400 \text{ km} = 3.824 \times 10^8 \text{ m}$$

$$T = 27.3 \text{ days} = 27.3 \times 24 \times 60 \times 60 \text{ s} = 2.36 \times 10^6 \text{ s}$$

To find

$$a_c = ?$$

Calculation

$$a_c = \frac{v^2}{r}$$

$$= \frac{\left( \frac{2\pi r}{T} \right)^2}{r}$$

$$= \frac{4\pi^2 r}{T^2}$$

$$= \frac{4 \times \pi^2 \times (3.824 \times 10^8)}{(2.36 \times 10^6)^2}$$

$$= 0.27 \text{ m/s}^2$$



- 5 A modern F1 car can accelerate from 0 to 62 mph (100 km/h) in 2.50 s. What is the angular acceleration of its 170 mm radius wheel?

Solution

Given Data

$$v = 100 \text{ km/h} = 27.78 \text{ m/s}$$

$$t = 2.50 \text{ s}$$

$$r = 170 \text{ mm} = 0.17 \text{ m}$$

Given Data

$$a_c = ?$$

Calculation

$$a_c = \frac{v^2}{r}$$

$$= \frac{(27.78)^2}{0.17}$$

$$= 4500 \text{ m/s}^2$$

$$a_c = 4500 \text{ m/s}^2$$

$$a_c = 4500 \text{ m/s}^2$$

$$a_c = 4500 \text{ m/s}^2$$

$$a_c = 4500 \text{ m/s}^2$$

$$a_c = 4500 \text{ m/s}^2$$

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$$a_c = 4500 \text{ m/s}^2$$

$$a_c = 4500 \text{ m/s}^2$$

- 6 An electric motor is running at 1800 rev/min. It comes to rest in 20 s. If the angular acceleration is uniform find the number of revolutions it made before stopping.

Solution

Given Data

$$\omega = 1800 \text{ rev/min} = 30 \text{ rev/s}$$

$$t = 20 \text{ s}$$

Final angular speed =  $\omega = 0$

Time taken =  $t = 20$  s

Number of revolutions =  $N = ?$

We have

$$N = \text{No. of rev. in } t = \frac{\text{Total angular displacement in } t}{2\pi}$$

$$N = \frac{\theta}{2\pi} \quad \text{--- (1)}$$

Now we have,

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\theta = \omega_0 t + \frac{1}{2} \frac{\omega_0 - \omega}{t} t^2$$

$$\theta = \omega_0 t + \frac{1}{2} (\omega_0 - \omega) t$$

$$\theta = 188.4 \times 20 + \frac{1}{2} (0 - 188.4) \times 20$$

$$\theta = 188.4 \times 20 + (-188.4) \times 10 = 3768 - 1884$$

$$\theta = 1884 \text{ rad}$$

Putting  $\theta = 1884$  rad in eq (1), we get,

$$N = \frac{1884}{2\pi} \quad \boxed{N = 300 \text{ rev}}$$

What is the moment of inertia of a 100 kg sphere whose radius is 50 cm

Solution

Given Data Mass of sphere =  $m = 100$  kg,

Radius of sphere =  $R = 50$  cm = 0.5 m,

To Find Moment of inertia =  $I = ?$

We know that moment of inertia of solid sphere is given by

$$I = \frac{2}{5} MR^2$$

$$I = \frac{2}{5} \times 100 \times (0.5)^2 = 10 \text{ kg m}^2$$

A rope is wrapped several times around a cylinder of radius 0.2 m and mass 30 kg. What is the angular acceleration of the cylinder if the tension in the rope is 40 N and it turns without friction.

Solution

Radius of cylinder = 0.2 m

Mass of cylinder = 30 kg

Angular acceleration =  $\alpha = ?$

Required

Calculation

We know that

$$\tau = I\alpha \quad \text{--- (1)}$$

We also know that

$$\tau = rF \quad \text{--- (2)}$$

Comparing equation (1) and equation (2), we get

$$I\alpha = rF \quad \Rightarrow \quad \alpha = \frac{rF}{I}$$

The moment of inertia of cylinder is given by

$$1 = \frac{1}{2} \omega^2 r^2 \quad (4)$$

By using equation (3) & (4) we get

$$\frac{1}{2} \omega^2 r^2 = \frac{1}{2} \omega^2 r^2 \quad (5)$$

By the values we get for  $\omega$  we get

$$\omega = 13.3 \text{ rad/sec} \quad \boxed{\omega = 13.3 \text{ rad/sec}}$$

9. What is the kinetic energy of a 5.0 kg solid ball whose diameter is 15m if it rolls across a surface with a speed of 2 ms<sup>-1</sup>?

**Solution**

**Given Data** Mass = 5 kg

Diameter = 15m

Radius = 7.5m

Speed = 2ms<sup>-1</sup>

**To Find** Kinetic Energy

We know that the kinetic energy of a body is given by the sum of its translational kinetic energy and its rotational kinetic energy.

$$K.E. = K.E._{trans} + K.E._{rot}$$

$$K.E. = \frac{1}{2} M V^2 + \frac{1}{2} I \omega^2$$

$$K.E. = \frac{1}{2} \times 5 \times 2^2 + \frac{1}{2} \times \frac{1}{2} M R^2 \omega^2$$

$$K.E. = 10 + \frac{1}{4} \times 5 \times 15^2 \times \omega^2$$

10. A cylinder of 50cm diameter at the top of an incline 29.4cm high and 10m long is released and rolls down the incline. Find its linear and angular speeds at the bottom. Neglect friction.

**Solution**

**Given Data** Diameter = 50cm

Radius = 25cm

Height = 29.4cm

**To Find**

Linear speed

Angular speed

**Solution**

$$V = \sqrt{2gh}$$

$$V = \sqrt{2 \times 9.8 \times 29.4}$$

$$\boxed{V = 24.2 \text{ m/s}}$$

$$\omega = \frac{V}{R}$$

$$\omega = \frac{24.2}{0.25}$$

$$\boxed{\omega = 96.8 \text{ rad/s}}$$

11. A disc without slipping rolls down a hill of vertical height 1000cm. If the disc starts from rest at the top of the hill, what is its magnitude of velocity at the bottom?

**Solution**

**Given Data** Height = 1000cm

Radius = 10cm

To Find Speed at the bottom =  $v$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 1}$$

$$v = 4.43 \text{ m/s}$$

12. A motor car is travelling at a speed of  $30 \text{ ms}^{-1}$  if the wheel has a diameter of  $1.5 \text{ m}$  find its angular speed in  $\text{rad s}^{-1}$  and  $\text{rev s}^{-1}$

Solution

Given Data  $v = 30 \text{ ms}^{-1}$   
 $d = 1.5 \text{ m}$   
 $r = \frac{d}{2} = \frac{1.5}{2} = 0.75 \text{ m}$

To Find  $\omega = ?$

As know that  $v = r\omega$

$$\omega = \frac{v}{r} = \frac{30}{0.75} = 40 \text{ rad s}^{-1}$$

$$\omega = 40 \text{ rad s}^{-1} \times \frac{1}{2\pi} \text{ rev s}^{-1} = 6.37 \text{ rev s}^{-1}$$



## Additional Conceptual Short Questions With Answers

1. Is there any work done by centripetal force?

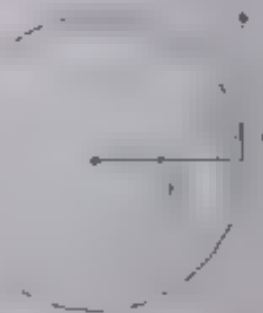
**Ans:** When a body is moving in circular path then centripetal force is perpendicular to the displacement. Work done by centripetal force is

$$W = \vec{F}_c \cdot \vec{d}$$

$$W = F_c d \cos 90^\circ$$

$$W = 0$$

∴ Work is done by centripetal force is zero. It is because it is perpendicular to the displacement.



2. The ratio of rotational K.E. of two bodies of moment of inertia  $9 \text{ kgm}^2$  and  $1 \text{ kgm}^2$  are same find ratio of their angular momentum?

**Ans:** For linear motion  $W = \frac{1}{2}mv^2$

$$\text{Rotational K.E.} = \frac{1}{2}I\omega^2$$

Similarly for rotational motion

$$\text{Rotational K.E.} = \frac{1}{2}I\omega^2$$

$$W = \frac{1}{2}I\omega^2$$

$$L = I\omega$$

$$L = I\omega$$

$$L = I\omega$$

$$L = I\omega$$

$$L = I\omega$$

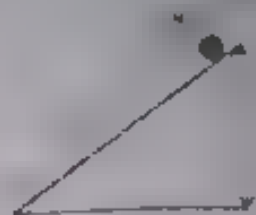
$$L = I\omega$$

$$L = I\omega$$

- 3 Find the rotational K.E. and speed of sphere at the bottom of inclined plane without slipping?

**Ans** Rotational K.E.

Rotational K.E.



- 4 Show that rotational K.E. of hoop and sphere have following relation:

$$K_{rot} = 2 K_{tr} \quad \text{or} \quad K_{rot} = \frac{2}{5} K_{tr}$$

**Ans**

$$(i) \quad (K.E.)_{\text{aphe}} = \frac{1}{2} m v^2$$

$$K.E._{\text{per}} = \frac{2}{5} \times \frac{1}{2} m v^2$$

$$K.E._{\text{aphe}} = \frac{2}{5} K.E._{\text{per}}$$

What is relation between  $(K.E.)_{\text{aphe}}$  and  $(K.E.)_{\text{per}}$ ?

$$K.E._{\text{aphe}} = \frac{2}{5} K.E._{\text{per}}$$

$$K.E._{\text{per}} = \frac{5}{2} K.E._{\text{aphe}}$$

$$K.E._{\text{per}} = \frac{5}{2} \times 4 \times 10^3 \text{ J}$$

$$K.E._{\text{per}} = \frac{5}{2} \times 4 \times 10^3 \text{ J}$$

The speed of a body moving in uniform circular motion is doubled and the radius of the circular path is halved. What happens to the centripetal force?

$$F_{\text{centripetal}} = \frac{mv^2}{r}$$

When  $v$  becomes  $2v$  and  $r$  becomes  $r/2$ , then

$$F_{\text{centripetal}} = \frac{m(2v)^2}{r/2} = \frac{4mv^2}{r/2} = 8 \times \frac{mv^2}{r}$$

$$F = 8F_0$$

The centripetal force becomes 8 times than its original value.

Find angular momentum of the earth about its own axis?

Let  $I$  be the moment of inertia of the earth about its axis.

$$I = \frac{2}{5} MR^2$$

$$I = \frac{2}{5} \times 6 \times 10^{24} \times (6.4 \times 10^6)^2$$

$$I = 9.81 \times 10^{41} \text{ kg m}^2$$

Angular velocity  $\omega$  is given by complete rotation in one day  $\theta = 2\pi$  and  $t = 86400 \text{ s}$

$$\omega = \frac{2\pi}{t} = \frac{2\pi}{86400}$$

$$\omega = 7.27 \times 10^{-5} \text{ rad/s}$$

So the angular momentum is

$$L = I\omega = 9.81 \times 10^{41} \times (7.27 \times 10^{-5})$$

$$L = 7.13 \times 10^{37} \text{ kg m}^2 \text{ s}^{-1}$$

What is relation between escape velocity and orbit velocity?

We know that

$$V_{\text{orb}} = \sqrt{\frac{GM}{R}}$$

$$\text{Or } V_{\text{orb}} = \sqrt{\frac{GM}{R}}$$

$$\text{So } V_{\text{orb}} = \sqrt{\frac{GM}{R}}$$

$$V_{\text{orb}} = \sqrt{2} \times \sqrt{\frac{GM}{R}}$$

$$V_{\text{orb}} = \sqrt{2} \times V_{\text{orb}}$$

$$\text{Putting } \sqrt{\frac{GM}{R}} = V_{\text{orb}}$$



# MCQ's From Past FBISE Exams FEDERAL BOARD

1. What is moment of inertia of a sphere  
 A  $\frac{1}{2} MR^2$  B  $\frac{1}{2} MR$  C  $\frac{2}{5} MR^2$  D  $\frac{1}{2} MR$
2. If the earth suddenly stops rotating the value of 'g' at equator would  
 A Decrease B Remain same C Increase D None
3. Time Period of circular motion is given by (FBISE 2016 supply)  
 A  $T = \frac{2\pi r}{v}$  B  $T = \frac{2\pi}{v}$  C  $T = 2\pi\omega$  D  $T = \frac{2\pi}{\omega}$
4. SI unit of Angular momentum is:  
 A  $N \cdot m$  B Radian C  $Ns$  D  $kg \cdot m^2/s$
5. Time period of pendulum in lift moving upward with constant velocity  
 A increases B decreases C remains constant D None
6. The relation between the orbital speed  $v_o$  of a planet and its orbital radius  $r_o$  is  
 A  $v_o \propto r_o$  B  $v_o \propto \frac{1}{r_o}$  C  $v_o \propto \sqrt{r_o}$  D  $v_o \propto \sqrt{\frac{1}{r_o}}$
7. A body of mass 2kg is suspended in a lift by means of a spring balance. The balance reads weight when the lift moves up with an acceleration of  $5 \text{ m/s}^2$  is  
 A 20 N B 24 N C 16 N D 18 N
8. The rotational kinetic energy of a hoop of mass m moving down an inclined plane with velocity v will be  
 A  $\frac{1}{2}mv^2$  B  $\frac{1}{4}mv^2$  C  $\frac{3}{4}mv^2$  D  $\frac{1}{2}mv^2$
9. Apparent Weight of freely falling body  
 A increases B decreases C remains same D None
10. Which of following is not axial vector  
 A linear momentum B angular momentum C torque D area vector
11. Point moves along arc of length "s" and radius "r" in time "t". Its angular velocity is given by  
 A  $\frac{s}{rt}$  B  $\frac{s}{t}$  C  $\frac{2\pi}{t}$  D  $\frac{2\pi s}{t}$
12. Minute hand of large clock is 2m long. What is its angular speed approximately  
 A  $0.001 \text{ rad/s}$  B  $0.002 \text{ rad/s}$  C  $0.003 \text{ rad/s}$  D  $0.004 \text{ rad/s}$
13. How many satellites form GPS in geostationary orbit  
 A 3 B 4 C 5 D 6
14. Height of geostationary satellites above equator is  
 A  $6.4 \times 10^6 \text{ m}$  B  $3.6 \times 10^6 \text{ m}$  C  $3.6 \times 10^7 \text{ m}$  D  $4.2 \times 10^7 \text{ m}$
15. Expression for orbital speed  
 A  $\sqrt{\frac{GM}{r}}$  B  $\sqrt{\frac{GM}{r^2}}$  C  $\frac{GM}{r}$  D  $\frac{GM}{r^2}$
16. Moment of inertia of solid disc is  
 A  $mr^2$  B  $\frac{1}{2}mr^2$  C  $\frac{3}{2}mr^2$  D  $\frac{1}{4}mr^2$
17. 100 kg motorcycle moving around a curved path of radius 100m with velocity 10 m/s. Centripetal force.  
 A 600N B 100N C 200N D 300N

18 SI Units of angular Momentum is

A  $\text{kg m}^2 \text{s}^{-1}$

B  $\text{kg m}^2 \text{s}$

C  $\text{kg m}^2 \text{s}^{-2}$

D  $\text{kg m}^2 \text{s}^{-1}$

19 Displacement covered by body during two revolutions on a circle of radius  $r$  is

A  $\pi r$

B  $2\pi r$

C  $4\pi r$

D  $8\pi r$

**Answers Key**

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| C  | B  | A  | C  | B  | A  | C  | B  | A  | C  |



## SELF ASSESSMENT PAPER

Total Marks 40

1 x 5 = 5

Question No 1 Choose the correct answer from the given options

## SECTION - A

- 1 A wheel of radius 50 cm having the angular speed of 5 rad/s will have linear speed in m/s  
(A) 1.5 (B) 2.5 (C) 3.5 (D) 4
- 2 The ratio of moment of inertia of disc and hoop is  
(A)  $\frac{1}{2}$  (B)  $\frac{1}{4}$  (C)  $\frac{3}{4}$  (D) 1
- 3 2 radian = \_\_\_\_\_  
(A)  $114.6^\circ$  (B)  $57.3^\circ$  (C)  $75.3^\circ$  (D)  $2^\circ$
- 4 A stone of mass 16 kg is attached to a string 144 m long and is whirled in a horizontal circle. The maximum tension the string can withstand is 16N. The maximum velocity of revolution that can be given to the stone without breaking it, will be,  
(A)  $12 \text{ ms}^{-1}$  (B)  $20 \text{ ms}^{-1}$  (C)  $10 \text{ ms}^{-1}$  (D)  $14 \text{ ms}^{-1}$
- 5 Which one is constant for a satellite in orbit?  
(A) Velocity (B) KE (C) Angular momentum (D) Total energy
- 6 If the earth suddenly stops rotating the value of  $g$  at equator would  
(A) Decrease (B) Remain same (C) Increase (D) None of these

Question No 2 Give short answers of following questions

1 x 2 = 2

## SECTION - B

- (i) Why is the fly wheel of an engine made heavy in the rim?
- (ii) A ball is supported by a string without breaking. If it is not supported it breaks. Why?
- (iii) Why does the ceiling rotating system slow down as water drops into the bucket?
- (iv) Explain why mud guards are used on the wheels of cycle.
- (v) Explain why is there weightlessness in space?
- (vi) At what speed in km/h is a bank angle of  $45^\circ$  required for a car to take a curve of radius 100 m?
- (vii) How artificial gravity is produced in space ships? Explain.

Question No 3 Extensive Questions

(4)

## SECTION - C

- (a) Define Centripetal force. Derive its formula.
- (b) Derive the formula for radius of geostationary orbit of geostationary satellites.
- (c) An electric motor is running at  $1800 \text{ rev min}^{-1}$ . It comes to rest in 20s. If the angular deceleration is uniform, Find the number of revolutions it made before stopping.

\*\*\* The End \*\*\*

# CHAPTER

# 6

## FLUID DYNAMICS

### Learning Objectives

- Define the terms velocity, streaming or laminar flow, incompressible flow and non viscous flow as applied to the motion of an ideal fluid
- Explain that at a section where velocity is low, the flow of an incompressible fluid does a transition from a state of laminar to turbulent
- Derive the equation of continuity with actual examples of fluid flow and its state of motion in fluids under the conditions of laminar conditions
- Derive the equation of continuity for a constant flow of an ideal and incompressible fluid moving through a pipe
- Explain that the equation of continuity is a form of the principle of conservation of mass
- Explain that the pressure difference can arise from different rates of flow of a fluid (Bernoulli effect)
- Derive the equation of Bernoulli in the form  $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$  for the case of horizontal tube
- Explain the application of Bernoulli's Effect in the lift pump, Venturi meter, carburetors, flow of air over an airfoil, etc.
- Explain the concept of viscosity in fluids
- Explain that viscous forces in a fluid cause a retarding force on an object moving through it
- Explain how the magnitude of the viscous force in fluid flow depends on the shape and velocity of the object
- Apply dimensional analysis to confirm the form of the equation  $F = A\eta v$  where  $A$  is a constant, cross-sectional area (Stokes Law) for the drag force under laminar conditions in a viscous fluid
- Apply Stokes law to derive an expression for terminal velocity of sphere falling through a viscous fluid



**Fluid** The fluid is the matter that flows from one place to other without any resistance.

**Examples** Liquids and gases are fluids as they flow.

**Fluid dynamics**

The study of motion of fluids with the study of fluids in motion is called fluid dynamics.

**Viscous Fluids**

The property of fluid which resists the flow is called viscosity.

▶ Viscosity is the resistance to resistance to flow.

▶ The greater the flow much force is required to slide one layer of fluid over another layer.

▶ Viscosity is the property of fluid. Honey has a high viscosity, it flows slowly and freely to water.

▶ The greater the cohesive force between particles of attraction, the more viscous is the liquid.

▶ Viscosity is the property of fluid which resists the flow because of internal energy.

▶ The greater the cohesive force between particles of attraction, the more viscous is the liquid.

▶ The greater the cohesive force between particles of attraction, the more viscous is the liquid.

▶ Substances like honey, Syrup, coconut oil and thick tar have high viscosity, so they can not flow easily.

▶ Substances like water and air has small coefficient of viscosity  $\eta$ . So they can flow easily.

**Viscosity depends upon**

▶ Nature of fluid

▶ Cohesive force

▶ Temperature of fluid

**Viscosity of liquids and gases**

Liquids and gases have non-zero viscosity.

▶ Viscosity of gases increases with increase in temperature (due to random motion of molecules).

▶ Viscosity of liquid decreases with rise in temperature (Cohesive force decreases with rise in temperature of liquid).

▶ The unit of coefficient of viscosity is kg m<sup>-1</sup> s<sup>-1</sup> or Pa s.

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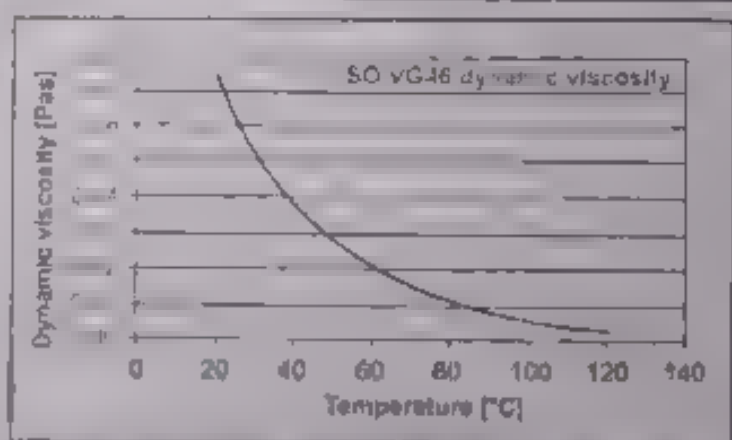
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**DO YOU KNOW?**

Drag force always opposes to the direction of motion of object, through fluid.



**Table: COEFFICIENT OF VISCOSITY OF VARIOUS SUBSTANCES**

| Material | Viscosity (Pa.s)      | Material                    | Viscosity (Pa.s)      |
|----------|-----------------------|-----------------------------|-----------------------|
| Air      | $1.8 \times 10^{-5}$  | Ethanol                     | $1.00 \times 10^{-3}$ |
| Acetone  | $2.9 \times 10^{-4}$  | Blood                       | $3.6 \times 10^{-3}$  |
| Methanol | $5.1 \times 10^{-4}$  | Honey                       | 1.42                  |
| Benzene  | $1.00 \times 10^{-3}$ | Blood (at body temperature) | $4.0 \times 10^{-3}$  |
| Water    | $8.91 \times 10^{-4}$ |                             |                       |



Q2 What is terminal velocity? Show that terminal velocity of fog droplet is directly proportional to the square its radius?

**Ans. TERMINAL VELOCITY**

The maximum constant velocity of a freely falling body for which drag force becomes equal to its weight is called terminal velocity.

- The constant maximum velocity attained and maintained by an object while falling through a resistive medium is called terminal velocity.
- We will be concerned the terminal velocity for the simplest case that is the uniform density spherical object falling through consistent medium as shown in Figure.
- Newton's laws apply for all objects whether free falling or falling in the presence of resistive forces. The accelerations, however, are quite different, due to difference in net force.
- In vacuum the net force is the weight because it is the only force.
- However, in the presence of air resistance, the net force is less than the weight, it is the weight minus drag force.

$$\text{Net force} = \text{Weight} - \text{Drag force}$$

$$F_{\text{net}} = mg - 6\pi\eta r v$$

$$ma = mg - 6\pi\eta r v \quad \text{--- (1)}$$

$$(F_G = W = mg)$$

- When drag force becomes equal to the weight of object, then it will start moving with uniform velocity, called terminal velocity ( $v_t$ ).
- Net force is zero i.e.  $F_{\text{net}} = 0$
- So its acceleration becomes zero i.e.  $a = 0$

Put  $a = 0$  in equation (1)

$$m(0) = mg - 6\pi\eta r v_t$$

$$0 = g - \frac{6\pi\eta r v_t}{m}$$

$$6\pi\eta r v_t = mg$$

$$v_t = \frac{mg}{6\pi\eta r} \quad \text{--- (2)}$$

Equation (2) represents terminal velocity of a spherical object of mass  $m$  and radius  $r$ .

Acceleration due to gravity  $g$  in a medium of coefficient of viscosity  $\eta$ .

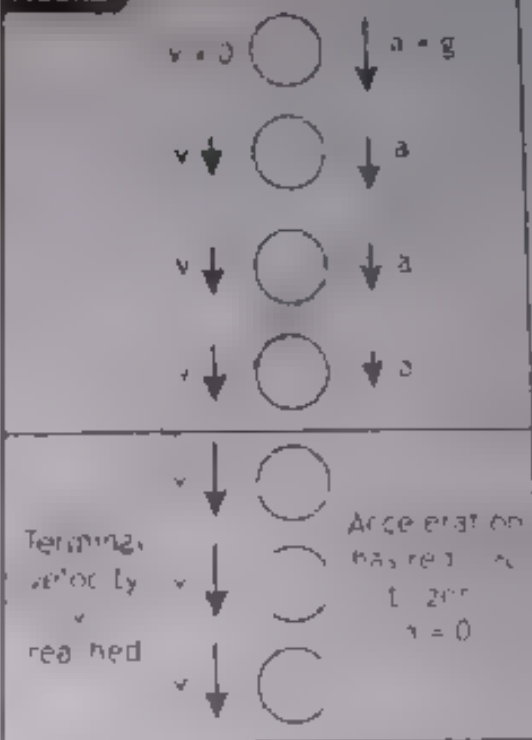
we know density =  $\frac{\text{mass}}{\text{volume}}$  (OR)  $\rho = \frac{m}{V}$

OR  $m = \rho V$

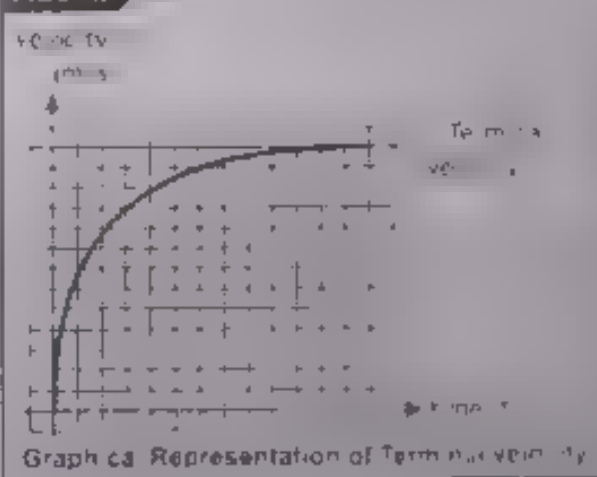
$$m = \frac{4}{3}\pi r^3 \rho \quad \text{--- (3)}$$

where  $V = \frac{4}{3}\pi r^3$  [for sphere]

FIGURE



FIGURE



## FIGURE

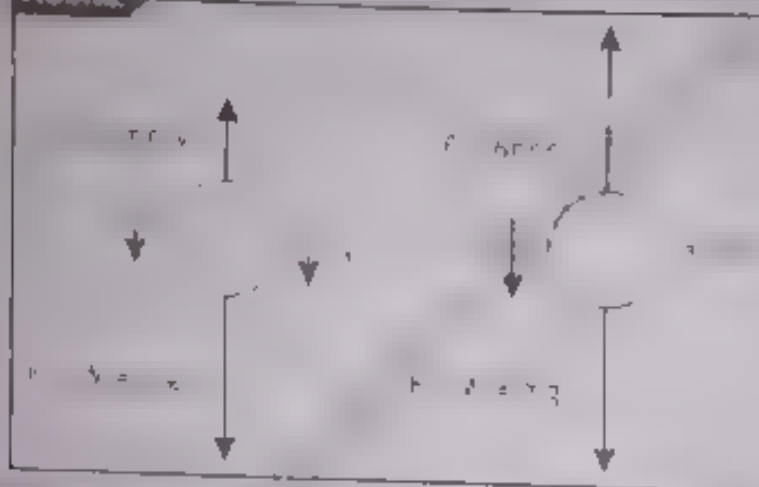


Diagram illustrating the forces acting on a falling object.

As the object falls, the drag force increases with the speed of the medium and speed, therefore there is no

terminal velocity. When an object falling through the air on Earth reaches terminal velocity after about 12 seconds, it has fallen about 450 meters. Table shows the terminal velocities of various objects.

**Table: TERMINAL SPEEDS FOR VARIOUS OBJECT FALLING THROUGH AIR**

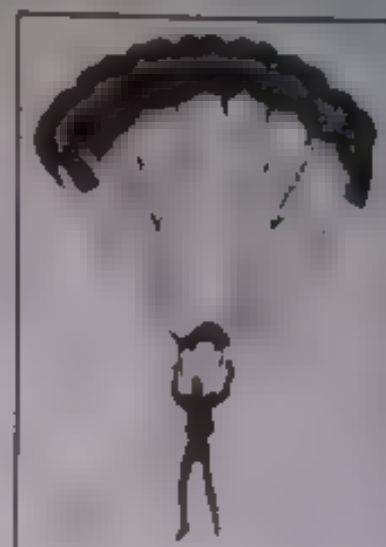
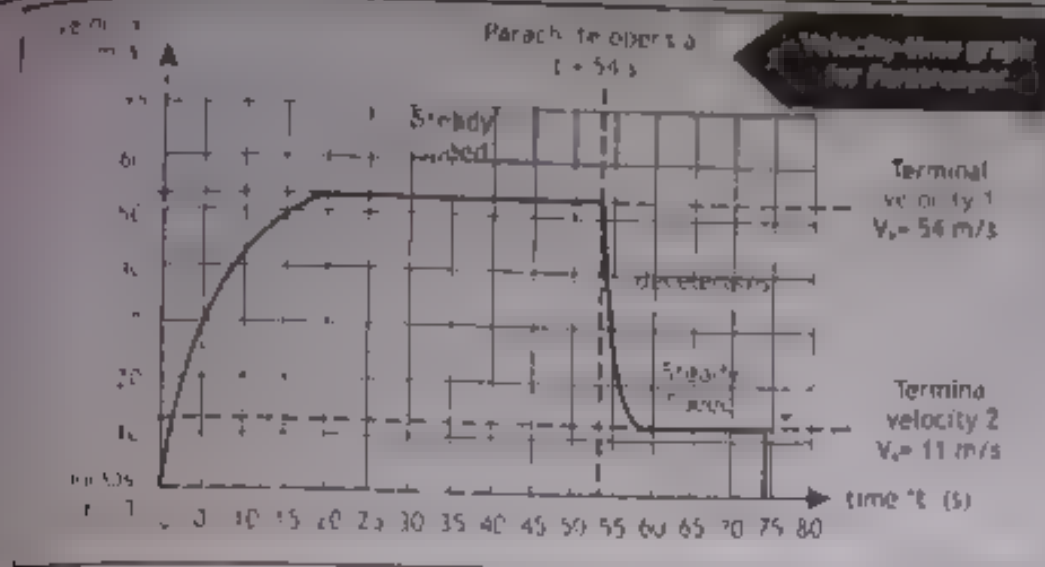
| Object                         | mass (kg)        | Cross-sectional Area ( $m^2$ ) | Terminal Speed (m/s) |
|--------------------------------|------------------|--------------------------------|----------------------|
| Human (average)                | 70               | $0.7$                          | 54                   |
| Human (average) (spread eagle) | 70               | $42 \times 10^{-3}$            | 41                   |
| Human (average) (spread eagle) | 70               | $1.4 \times 10$                | 44                   |
| Human (average) (spread eagle) | $48 \times 10^4$ | $79 \times 10^{-3}$            | 14                   |
| Human (average) (spread eagle) | $34 \times 10^4$ | $1.3 \times 10^{-3}$           | 6                    |

## DO YOU KNOW?

When an object falls through a fluid, it experiences a drag force that opposes its motion. The drag force is proportional to the square of the object's speed. As the object falls, the drag force increases until it equals the weight of the object. At this point, the object reaches terminal velocity and stops accelerating.

## Do you know?

The first person to reach terminal velocity was a skydiver who fell for 4 minutes and 35 seconds before parachuting.



Stage 1 at  $t = 0\text{ s}$  after starting moving from the ground the skydiver's motion is very fast with a small downward resistance so he accelerated downwards.

Stage 2 at  $t = 54\text{ s}$  eventually the force of the air resistance has increased so much that it balances the skydiver's weight. The forces are balanced and he speed remains constant. This is the first terminal velocity.

Stage 3 at  $t = 54\text{ s}$  when the parachute opens air resistance increases dramatically. The air resistance force is greater than the weight force so he speed slows down.

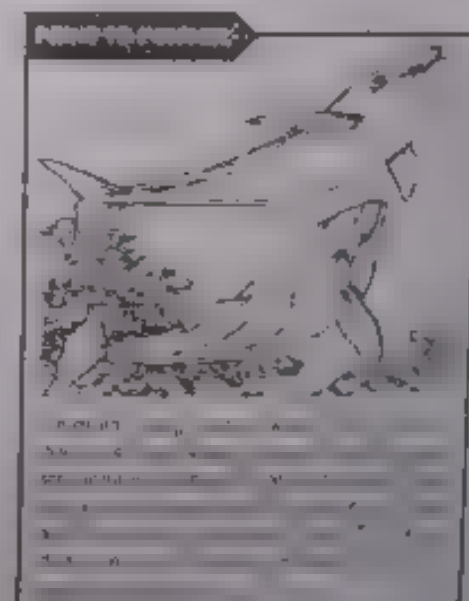
Stage 4 at  $t = 75\text{ s}$  as the skydiver slows the air resistance force from the parachute reduced until it is equal to the weight force. The forces are balanced and the speed remains constant. This is terminal velocity.

#### Problem 6.1:

A certain globular protein particle has a density of  $1246\text{ kg m}^{-3}$ . It falls through water (having coefficient of viscosity  $8.91 \times 10^{-4}\text{ Pa s}$ ) with a terminal speed of  $8.33 \times 10^{-6}\text{ ms}^{-1}$ . Find the radius of the particle.

Given Data: Density of protein particle =  $\rho = 1246\text{ kg m}^{-3}$   
 Co-efficient of viscosity =  $\eta = 8.91 \times 10^{-4}\text{ Pa s}$   
 Terminal velocity =  $8.33 \times 10^{-6}\text{ m/s}$   
 Radius of particle =  $r = ?$

Calculations: As  $\eta = \frac{2r^2 \Delta \rho g}{9v_t}$  Putting values, we get  
 $8.91 \times 10^{-4} = \frac{2r^2 \times 1246 \times 9.8}{9 \times 8.33 \times 10^{-6}}$   
 $r^2 = 2.735 \times 10^{-12}$   
 $r = 1.65 \times 10^{-6}\text{ m}$



Q3 What is the difference between steady and turbulent flow?

#### Fluid Flow

Consider the flow of the fluid through the pipes. It may be either stream line or turbulent.

##### Laminar Flow

The flow is said to be stream line steady or laminar if every particle of a fluid that passes through a point moves along exactly the same path as followed by particles that have passed that point earlier.



is the following

The fluid is incompressible

The flow is steady

The fluid is at rest

The fluid is at rest

The temperature does not vary! Phenomena

State and explain equation of continuity

### EQUATION OF CONTINUITY

For an ideal fluid flowing through a pipe, the mass of fluid that enters the pipe in a given time interval must be equal to the mass of fluid that leaves the pipe in the same time interval.

(10R)

For an ideal fluid, the volume of fluid that enters the pipe in a given time interval must be equal to the volume of fluid that leaves the pipe in the same time interval.

The mass of fluid that enters the pipe in a given time interval must be equal to the mass of fluid that leaves the pipe in the same time interval.

The volume of fluid that enters the pipe in a given time interval must be equal to the volume of fluid that leaves the pipe in the same time interval.

The mass of fluid that enters the pipe in a given time interval must be equal to the mass of fluid that leaves the pipe in the same time interval.

The volume of fluid that enters the pipe in a given time interval must be equal to the volume of fluid that leaves the pipe in the same time interval.

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The mass of fluid that enters the pipe in a given time interval must be equal to the mass of fluid that leaves the pipe in the same time interval.

The volume of fluid that enters the pipe in a given time interval must be equal to the volume of fluid that leaves the pipe in the same time interval.

The mass of fluid that enters the pipe in a given time interval must be equal to the mass of fluid that leaves the pipe in the same time interval.

44

$$r_2 = 4.0 \text{ m} = 0.01 \text{ m}$$

$$r_2 = r_1 = 0.21 \text{ cm} = 0.21 \times 10^{-2} \text{ m}$$

**statement**

It states that the sum of pressure,  $\frac{1}{2} \rho v^2$  per unit volume and  $\rho gh$  per unit volume of an ideal fluid flowing in steady state remains constant at each point along a stream line path in a pipe

**Mathematically**

$$P + \frac{1}{2} \rho v^2 + \rho gh_1 = P + \frac{1}{2} \rho v_2^2 + \rho gh_2$$

$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$$

- Bernoulli's equation is simply law of conservation of energy applied to fluid motion
- Consider an ideal flow through a pipe of non uniform size as illustrated in figure
- The work  $W$  is due to forces other than the conservative force of gravity so it equals the change in the total mechanical energy (kinetic energy  $\frac{1}{2} \rho v^2$  gravitational potential energy) associated with the fluid element

$$W = \Delta E$$

$$W = \Delta K + \Delta U$$

eq. 1 By definition of work

$$W = F \Delta x$$

$$W = \rho \Delta V \cos \theta$$

we know  $\theta = 0^\circ$  and  $\cos 0^\circ = 1$

$$\therefore W = F \Delta x$$

side on a cross section

$$F = PA$$

$$F = P_1 A_1 \quad \text{for section 1}$$

$$W_1 = F_1 \Delta x$$

and 2 By definition of work

$$W = F \Delta x$$

$$W_2 = F_2 \Delta x$$

we know  $\theta = 0^\circ$  and  $\cos 0^\circ = 1$

$$\therefore W_2 = F_2 \Delta x$$

eq. 2  $F_2 = P_2 A_2$  in the equation

$$W_2 = P_2 A_2 \Delta x \quad \text{..... (3)}$$

side on a cross section

$$\Delta V = A \Delta x \quad \text{..... (4)}$$

$$\Delta V = \frac{m}{\rho} \quad \text{..... (5)}$$

combining equation (4) and (5)

$$\frac{m}{\rho} = A \Delta x \quad \text{..... (6)}$$

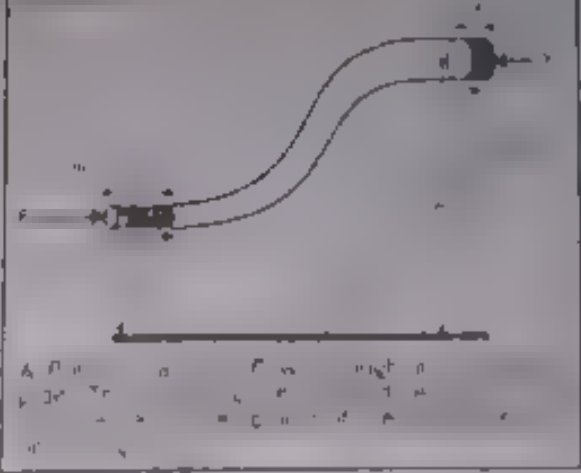
subst

$$A \Delta x = \frac{m}{\rho} \quad \text{in equation (3)}$$

subst

$$W = P \frac{m}{\rho} \quad \text{..... (7)}$$

**FIGURE**



**DO YOU KNOW?**

The Bernoulli's equation is a statement of the conservation of energy for a fluid in motion. It states that the sum of the pressure energy, kinetic energy, and potential energy per unit volume of a fluid is constant along a streamline. This equation is derived from the principle of conservation of energy and is applicable to ideal fluids in steady flow. The equation is used to explain various phenomena in fluid mechanics, such as the lift on an airfoil, the flow of water in a pipe, and the operation of a Venturi meter. The equation is also used in the design of aircraft, ships, and other structures that interact with fluids.

Step 1:  $\Delta p_1 \Delta x_1 = p_1 \Delta x_1$  in equation (3)

$$p_1 \Delta x_1 = p_1 \frac{\Delta m}{\rho} \quad \dots \dots (8)$$

► Total work done will be sum of all the individual work done

$$W = W_1 + W_2 + \dots \dots (9)$$

► Plug values from equation (7) and (8) in equation (9)

$$W = p_1 \frac{\Delta m}{\rho} + p_2 \frac{\Delta m}{\rho} + \dots \dots (10)$$

Net change in kinetic energy  $\Delta K$  is  $\Delta K = \frac{1}{2} \Delta m v_2^2 - \frac{1}{2} \Delta m v_1^2 \quad \dots \dots (11)$

Net change in potential energy  $\Delta U$  is  $\Delta U = \Delta m g h_2 - \Delta m g h_1 \quad \dots \dots (12)$

As we know that  $W = \Delta K + \Delta U \quad \dots \dots (13)$

► Plug values from equation (10), equation (11) and (12) in equation (13)

$$p_1 \frac{\Delta m}{\rho} + p_2 \frac{\Delta m}{\rho} + \dots = \frac{1}{2} \Delta m v_2^2 - \frac{1}{2} \Delta m v_1^2 + \Delta m g h_2 - \Delta m g h_1 \quad \dots \dots (14)$$

► As per law of conservation of mass should flow across both ends the same

$\Delta m_1 = \Delta m_2 = \Delta m$  in equation (14) we get

$$p_1 \frac{\Delta m}{\rho} + p_2 \frac{\Delta m}{\rho} = \frac{1}{2} \Delta m v_2^2 - \frac{1}{2} \Delta m v_1^2 + \Delta m g h_2 - \Delta m g h_1$$

Divide by  $\Delta m$  and multiply by  $\rho$  we get  $p_1 + p_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 + \rho g h_2 - \rho g h_1$

At point 1,  $p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \quad \dots \dots (15)$$

or  $p + \frac{1}{2} \rho v^2 + \rho g h = \text{constant} \quad \dots \dots (16)$

Equations (15) and (16) are termed as Bernoulli's equations. Bernoulli's equations is based on conservation of energy. For an incompressible and non-viscous fluid, the total mechanical energy of the fluid is constant.

### Example 8.3

Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3 ms<sup>-1</sup>, while at another point 3 m higher, the speed is 4 ms<sup>-1</sup>. At lower point the pressure is 80 kPa. Find the pressure at the upper point.

147.1 kPa

#### Given Data

Speed of water at one point =  $v_1 = 3 \text{ ms}^{-1}$

Speed of water at second point =  $v_2 = 4 \text{ ms}^{-1}$

Distance of height between two points  $h_2 - h_1 = 3 \text{ m}$

Pressure at lower point =  $p_1 = 80 \text{ kPa} = 80,000 \text{ Pa}$

#### To Find

Pressure at upper point =  $p_2 = ?$

For understanding statement of Bernoulli's equation

$$p + \frac{1}{2} \rho v^2 + \rho g h = \text{Constant}$$

Putting  $p = \frac{F}{A}$

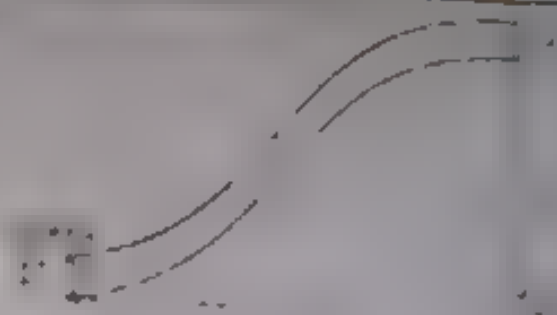
$$p = \frac{1}{2} \left( \frac{m}{\Delta t} \right) v^2 \quad \Delta t = \text{time interval}$$

$$p = \frac{\frac{1}{2} m v^2}{\Delta t} = \frac{\text{work done}}{\text{volume}}$$

$$\text{Pressure} = \frac{K-E}{\text{Volume}} = \frac{P-E}{\text{Volume}} \quad \text{Equation}$$

Calculation

1. A body of mass  $m$  is projected from the ground with an initial velocity  $u$  at an angle  $\theta$  to the horizontal. Find the maximum height reached by the body.



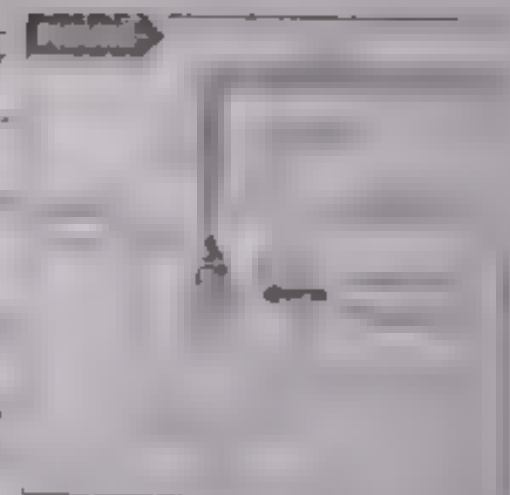
25 Explain application of Bernoulli's equation in atomizer and engine carburetor.

26 APPLICATIONS OF BERNOULLI'S EQUATION

from changes in a fluid's speed

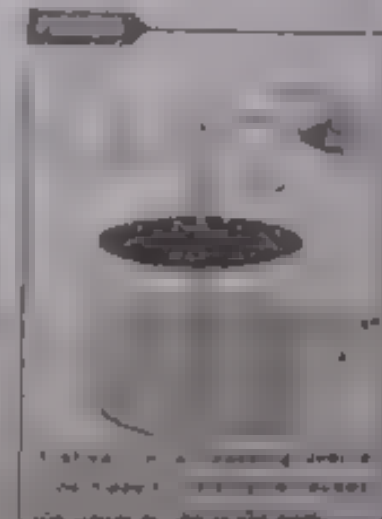
A. For Pipes

- ▶ Continuity equation:  $A_1 v_1 = A_2 v_2$  (where  $A$  is cross-sectional area and  $v$  is velocity)
- ▶ Bernoulli's equation:  $P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$  (where  $P$  is pressure,  $\rho$  is density,  $h$  is height)



B. Atomizers

- ▶ In an atomizer, a fast-moving fluid (like air) creates a region of low pressure, which draws in a liquid (like perfume) from a reservoir.
- ▶ The liquid is then carried away by the fast-moving fluid, creating a fine spray.
- ▶ This principle is used in carburetors to mix fuel with air for combustion in engines.



28 State and explain Torricelli's Theorem

29 Application of Bernoulli's Equation

C. Torricelli's theorem (Speed of efflux)

(Huygens)

Torricelli's theorem states that the speed of efflux is equal to the speed of a body falling from a height  $h$  under the action of gravity.

**Proof**

- Consider a large storage tank which develops a leak at the bottom as shown in the Figure 6.1. The pressure at both ends is the same ( $P_1 = P_2 = P$ ).  
 Area of cross section of upper end of tank =  $A$   
 Area of cross section of lower end =  $a$   
 Speed of the fluid at upper surface of tank =  $v_1$   
 Speed of the efflux =  $v$   
 Pressure at the upper end of tank =  $P$   
 Pressure at the lower end of tank =  $P$   
 Height of the free surface from bottom =  $h$   
 Height of hole from bottom =  $h_1$   
 Height of water above the hole =  $h - h_1$   
 Density of fluid =  $\rho$   
 Velocity at the top is considered as zero ( $v_1 = 0$ ).  
 While the bottom velocity is to be determined ( $v_2 = v$ ).

By Bernoulli's equation  $P_1 + \frac{1}{2}\rho v_1^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2$

Substituting appropriate values, we have

$$P + \frac{1}{2}\rho v_1^2 + \rho gh = P + \frac{1}{2}\rho v^2$$

or  $\cancel{P} + \cancel{P} + \rho gh = \cancel{P} + \frac{1}{2}\rho v^2$

hence  $\rho gh = \frac{1}{2}\rho v^2$  as  $\cancel{P} = \cancel{P}$

therefore  $v = \sqrt{2gh}$

Taking square root on both sides, we get

Therefore  $v = \sqrt{2gh}$

- The speed at the time as the vertical section of water is discharged after falling freely through a height 'h'. The equation (1) is termed as Torricelli's equation for the speed of fluid emerging from water storage.

**EXERCISES****EXTENSION EXERCISE**

If the speed of fluid emerging from a hole is given, how high is the water level?

**Assignment 8.4**

A tank full of water has a (small) hole near its bottom at a depth of 2 m which is open to air. What is the speed of the stream of water emerging?

Data:  $h = 2\text{ m}$

Speed of efflux =  $v = ?$

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 2} = \sqrt{39.2}$$

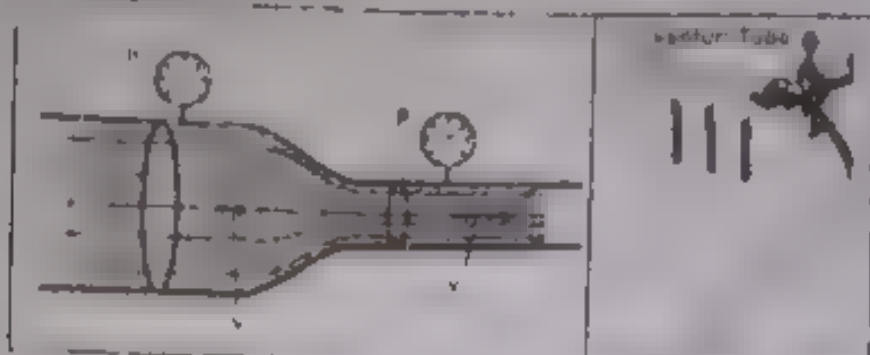
$$v = 6.26 \text{ m/s}$$

Q 8 What is flow meter or venturi meter? Explain

**Ans** Venturi Meter (Flow meter):

Venturi meter is a device used to measure the flow speed of fluid in a pipe.

- ▶ It works on the principle of pressure difference between restricted and full section.
- ▶ Consider the steady flow of the incompressible fluid and take two points 1 and 2.
- ▶ Hence we can use Bernoulli's equation.
- ▶ Consider the fig. 6.12. Let  $P_1$  and  $P_2$  be the pressure and  $A_1$  and  $A_2$  be the cross-sectional area and  $v_1$  and  $v_2$  be the velocity at the section 1 and 2 respectively.



Since  $P_1 = P_2$

Bernoulli's equation can be written as

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho gh = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh$$

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \quad \text{--- (1)}$$

Now we can use continuity equation

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad v_2 = \frac{A_1 v_1}{A_2} \quad \text{--- (2)}$$

Substituting eq. (2) in eq. (1) we get

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho \left( \frac{A_1 v_1}{A_2} \right)^2$$

$$\text{or } P_1 - P_2 = \frac{1}{2} \rho \left( \frac{A_1^2}{A_2^2} - 1 \right) v_1^2 \quad \text{rearranging we get } v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho \left( \left( \frac{A_1}{A_2} \right)^2 - 1 \right)}}$$

$$\text{If we take the square root of both sides } \sqrt{v_1^2} = \sqrt{\frac{2(P_1 - P_2)}{\rho \left( \left( \frac{A_1}{A_2} \right)^2 - 1 \right)}}$$

$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho \left( \left( \frac{A_1}{A_2} \right)^2 - 1 \right)}} \quad \text{(or)}$$

$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho \left( \frac{A_1^2}{A_2^2} - 1 \right)}}$$

$$\text{Solving } v_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho (A_1^2 - A_2^2)}} \quad \text{--- (3)}$$



Aerodynamic

Aerodynamic Fast & Acc. by Pressure m. o

5 m. air velocity



Figure 1 consists of two side-by-side diagrams of a normal artery. The left diagram shows a normal artery with a smooth lumen. The right diagram shows an artery with a stenosis (narrowing) in the lumen, labeled 'A'.

- represents the minimum pressure exerted when the heart contracts
- systolic pressure = 120 torr

## DIMENSIONAL PRESSION

- It represents the pressure in the arteries when the heart is relaxed. The value of low blood pressure (diastolic pressure) is about 75-80 torr

## Relation between torr and pascal

- 1 torr = 133.322 Pa

## MCQ's

1. Flow rate will be low where the speed of fluid is high [Medium]
2. Dimension of velocity is length/time [Easy]
3. The law of conservation of energy is the law of energy [Medium]
4. Blood pressure is measured by sphygmomanometer [Easy]
5. A 60 m high tank is full of water. A hole appears at the middle. What is the speed of efflux? 10 m/s [Easy]
6. The law of conservation of mass is related to continuity equation [Easy]
7. If the steam pipes of a boiler are forced closer together, then the pressure in the pipes will increase [Medium]
8. Venturi meter is a device used to measure velocity of flow [Easy]
9. Which of the following is a unit of pressure? 1 N/m<sup>2</sup> [Easy]
10. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
11. A horizontal pipe narrows from a diameter of 10 cm to 5 cm. For a fluid flowing from larger diameter to smaller diameter, the velocity increases [Medium]
12. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
13. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
14. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
15. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
16. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
17. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
18. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
19. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
20. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
21. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
22. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]
23. The velocity of a fluid in a pipe is directly proportional to the radius of the pipe [Medium]

The instrument which detects the instant at which the external pressure becomes equal to the systolic pressure is called a **Korotkoff's stethoscope**. The instrument used to measure the pressure of the blood is called a **sphygmomanometer**. The average for normal healthy person diastolic pressure is **80 mm Hg**.

|      |      |      |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|------|------|
| 3    | 2 C  | 1 C  | 1 C  | 5 A  | 6 B  | 7 A  | 8 B  | 9 A  | 10 C | 11 B | 12 C |
| 13 A | 14 C | 15 C | 16 B | 17 B | 18 B | 19 A | 20 B | 21 C | 22 C | 23 D | 24 B |

## FORMULAE

|   |                                        |                                                               |
|---|----------------------------------------|---------------------------------------------------------------|
| 1 | Drag force                             | $F_d = \frac{1}{2} C_d \rho A v^2$                            |
| 2 | Terminal velocity of fog droplet       | $v_t = \frac{2}{9} \frac{r^2 g (\rho - \sigma)}{\eta}$        |
| 3 | Equation of continuity                 | $A_1 v_1 = A_2 v_2$                                           |
| 4 | Bernoulli's equation                   | $P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$       |
| 5 | Torricelli's Theorem (speed of efflux) | $v = \sqrt{2gh}$                                              |
| 6 | Venturi's relation                     | $P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$ |

## Key Points

- **Fluid flow** - The motion of a fluid is called fluid flow. The fluid does not have any resistance to its flow.
- **Shape of a fluid** - The shape of a fluid is governed by the shape of its container.
- **Viscosity** - the resistance to flow of a fluid.
- **Drag force** - the force experienced by an object moving through a fluid.
- **Terminal velocity** - the maximum velocity attained and maintained by an object while falling through a fluid.
- **Streamline (steady, or laminar) flow** - every particle of a fluid moving along exactly the same path as the particles that have passed that point earlier.
- **Turbulent flow** - irregular flow characterized by small whirlpool-like regions.
- **Equation of Continuity** - The volume of an incompressible fluid passing any point in a pipe of non-uniform cross-section is the same in the steady flow.
- **Bernoulli's Equation** - Bernoulli's principle states that as we move along a streamline, the sum of the pressure, the kinetic energy per unit volume  $\frac{1}{2} \rho v^2$  and the potential energy per unit volume  $\rho g h$  remains a constant.
- **The equation is due to mass conservation for ideal flow.**
- **The equation is due to energy conservation for ideal flow.**



## Solved Examples

The radius of small fog droplet in air is found to be  $5.1 \times 10^{-6}$  m the coefficient of viscosity of air is  $1.9 \times 10^{-4}$  kg m<sup>-1</sup> s<sup>-1</sup>. Find out the settling speed of the droplet in air.

Given

Radius of droplet  $r = 5.1 \times 10^{-6}$  m  
 Coefficient of viscosity of air  $\eta = 1.9 \times 10^{-4}$  kg m<sup>-1</sup> s<sup>-1</sup>  
 Density of water  $\rho = 1000$  kg/m<sup>3</sup>  
 Acceleration due to gravity  $g = 9.8$  m/s<sup>2</sup>  
 Terminal velocity  $v_t = ?$

Solution

$$v_t = \frac{2}{9} \frac{r^2 g (\rho - \sigma)}{\eta}$$

$$= \frac{2}{9} \frac{(5.1 \times 10^{-6})^2 \times (9.8 \text{ m/s}^2)}{1.9 \times 10^{-4} \text{ kg m}^{-1} \text{ s}^{-1}}$$

$$= 1.1 \times 10^{-5} \text{ m/s} \quad \text{Answer}$$

A garden hose of inner radius 1.25 cm carries water at 2.60 m/s. The nozzle at the end has radius 0.30 cm. How fast does the water emerge out through the nozzle?

Radius of garden hose  $r_1 = 1.25$  cm  
 Radius of nozzle  $r_2 = 0.30$  cm

Speed through garden hose  $v_1 = 2.60$  m/s

Required:

Speed out of nozzle  $v_2 = ?$

Solution:

The equation of continuity is  $A_1 v_1 = A_2 v_2$

The area of circle is  $A = \pi r^2$

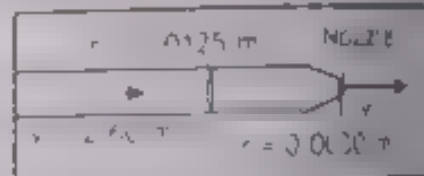
therefore, Equation of continuity can also be written as,  $\pi r_1^2 v_1 = \pi r_2^2 v_2$

$$r_1^2 v_1 = r_2^2 v_2 \quad \text{and} \quad v_2 = \frac{r_1^2 v_1}{r_2^2}$$

$$= \frac{(0.0125 \text{ m})^2 \times 2.60 \text{ m/s}}{(0.0030 \text{ m})^2}$$

$$= 45.14 \text{ m/s} \quad \text{Answer}$$

The speed of the water from the nozzle is 45.14 m/s



Water is flowing smoothly through a pipe. At one point the pressure is 33.2 kPa and the speed of water is 2 m/s. While at another point 2.3 m higher the pressure is 3.7 kPa. What speed is the water flowing through this point?

Pressure  $P_1 = 33.2 \text{ kPa} = 33.2 \times 10^3 \text{ Nm}^{-2}$

Pressure  $P_2 = 3.7 \text{ kPa} = 3.7 \times 10^3 \text{ Nm}^{-2}$

Speed of water  $v_1 = 2 \text{ m/s}$

Height  $h_1 = 0 \text{ m}$  Height  $h_2 = 2.3 \text{ m}$

Density of water  $\rho = 1000 \text{ kg m}^{-3}$

Find: speed

Speed of water  $v_2 = ?$

The Bernoulli's equation is

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

Or  $\frac{1}{2}\rho v_2^2 = P_1 - P_2 + \frac{1}{2}\rho v_1^2 + \rho gh_1 - \rho gh_2$

Multiplying both sides by  $\frac{2}{\rho}$  we get  $v_2^2 = \frac{2}{\rho}(P_1 - P_2) + v_1^2 + 2g(h_1 - h_2)$

Taking square root on both sides  $v_2 = \sqrt{\frac{2}{\rho}(P_1 - P_2) + v_1^2 + 2g(h_1 - h_2)}$

Or  $v_2 = \sqrt{\frac{2}{1000 \text{ kg/m}^3} (33.200 \text{ kg/m}^2 \text{ s}^2 - 3700 \text{ kg/m}^2 \text{ s}^2) + 2 \times 9.8 \text{ ms}^{-2} (2.3 \text{ m})}$

hence  $v_2 = 4 \text{ ms}^{-1}$  **Answer**

The water will flow at  $4 \text{ m/s}$  in the upper part of the pipe.

**Example 4:** A cylindrical water storage tank has a horizontal spigot near the bottom, at a depth of  $1.2 \text{ m}$  beneath the water surface. (a) When the spigot is opened, how fast does the water come out? (b) if the radius of spigot is  $6.0 \times 10^{-2} \text{ m}$ , what will be the volume flow rate?

**Given** Height of water in tank ' $h$ ' =  $1.2 \text{ m}$   
 Radius of spigot ' $A$ ' =  $6.0 \times 10^{-2} \text{ m}$   
 Acceleration due to gravity  $g = 9.8 \text{ ms}^{-2}$   
**Required** Speed of water  $v$   
 Volume flow rate  $\Delta V / \Delta t$

**Solution** (a) By Torricelli's theorem  $v = \sqrt{2gh}$

therefore  $v = \sqrt{2 \times 9.8 \text{ ms}^{-2} \times 1.2 \text{ m}}$

or  $v = 4.85 \text{ ms}^{-1}$  **Answer**

From equation of continuity the volume flow rate is  $\frac{\Delta V}{\Delta t} = Av$

or  $A = \pi r^2$  therefore  $\frac{\Delta V}{\Delta t} = \pi r^2 \times v$

put value  $\frac{\Delta V}{\Delta t} = 3.14 \times (6.0 \times 10^{-2} \text{ m})^2 \times 4.85 \text{ ms}^{-1}$

or  $\frac{\Delta V}{\Delta t} = 5.48 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}$  **Answer**

Water will come out  $0.548 \text{ m}^3$  will emerge out of spigot each second

**Example 5:** What is the aero-foils lift (in newtons) on a wing of area  $88 \text{ m}^2$  if the air passes at speed over its top surface at  $280 \text{ m/s}$  and bottom surface at  $150 \text{ m/s}$ ?

**Given** Surface area  $A = 88 \text{ m}^2$   
 Speed at top of wing  $v_1 = 280 \text{ m/s}$   
 Speed at bottom of wing  $v_2 = 150 \text{ m/s}$   
 density of the air  $\rho = 1.28 \text{ kg/m}^3$

**Required**  $\rho = 1000 \text{ kg m}^{-3}$

**Solution** Pressure  $\propto \text{depth}$   $\therefore P \propto h$

$$\therefore \frac{P_1}{P_2} = \frac{h_1}{h_2} \quad \therefore \frac{1}{2} = \frac{1}{h_2}$$

$$\therefore h_2 = 2 \text{ m} \quad \therefore P_2 = \rho gh_2 = 1000 \times 9.8 \times 2 = 19600 \text{ Pa}$$

$$\therefore \text{Pressure at } 2 \text{ m depth} = 19600 \text{ Pa}$$

$$\therefore \text{Pressure at } 1 \text{ m depth} = 9800 \text{ Pa}$$

$$\therefore \text{Pressure at } 0.5 \text{ m depth} = 4900 \text{ Pa}$$

$$\therefore \text{Pressure at } 0.25 \text{ m depth} = 2450 \text{ Pa}$$

$$\therefore \text{Pressure at } 0.125 \text{ m depth} = 1225 \text{ Pa}$$

$$\therefore \text{Pressure at } 0.0625 \text{ m depth} = 612.5 \text{ Pa}$$





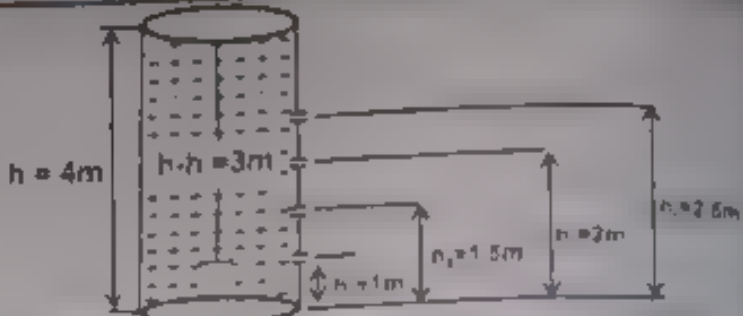
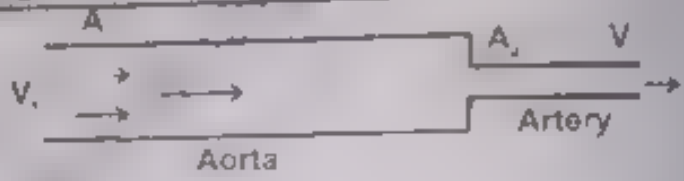
### Text Book Exercises

**Q 1** Select the correct answer of the following questions

- For viscous fluids that do not obey Newton's law of viscosity, the value for the coefficient of viscosity is  
 A positive B negative C zero D negative
- Value of viscosity of centipose is equal to what of poise?  
 A 100 B 10 C 1 D 1000
- Stokes law is applicable if a body is moving through a liquid with slow speed and has shape  
 A smooth B rough C a rough D any
- The net force that acts on a 10 N falling object, when it encounters 4 N of air resistance is  
 A 6 N B 4 N C 6 N D 10 N
- A skydiver jumps from a high flying helicopter. Before reaching terminal velocity, her acceleration  
 A increases B decreases C remains the same D is zero
- At terminal velocity the acceleration of a falling object is  
 A 0 B 9.8 m/s<sup>2</sup> C 9.8 m/s D 9.8
- According to equation of continuity  $Av = \text{constant}$ . This constant is equal to  
 A mass of fluid B mass of fluid C volume flow rate D volume flow rate
- At the constriction in the cross section for ideal flow, from equation of continuity it follows that the speed of fluid is  
 A less B less C same D more
- As water in a level pipe passes from a narrow cross section of pipe to a wider cross section, the pressure against the wall  
 A increases B decreases C remains the same D is zero
- A 4 m high tank filled with water is drilled with four identical small holes at 1 m, 1.5 m, 2 m and 2.5 m from the bottom of tank. The speed of efflux will be greatest from the hole at  
 A 1 m B 1.5 m C 2 m D 2.5 m
- Venturimeter is a device used to measure the  
 A mass B velocity C pressure D velocity

- A certain pipe has a cross-sectional area of  $0.001 \text{ m}^2$  in which water is flowing at  $10 \text{ m/s}$ . The volume flow rate is
- A  $0.001 \text{ m}^3/\text{s}$       B  $0.01 \text{ m}^3/\text{s}$       C  $0.1 \text{ m}^3/\text{s}$       D  $10.0001 \text{ m}^3/\text{s}$
- At sufficiently high speeds the flow of viscous fluid becomes
- A unexpected      B streamline      C non-viscous      D turbulent
- The water in the tank is  $10 \text{ m}$  above the leak point. The speed with which the water emerge from the leak is
- A  $10 \text{ m/s}$       B  $4 \text{ m/s}$       C  $194 \text{ m/s}$       D  $0.1 \text{ m/s}$
- When the radius of the artery is reduced, the blood pressure
- A increased      B decreased      C remains the same      D is zero

| Q.No. | Option | ANSWER | EXPLANATION                                                                                                                                                                                                                                                                                                      |
|-------|--------|--------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1     | A      | A      | $Q = \frac{1}{2} \rho v (1s)$ $Q = \frac{1 \times 10^{-5} \text{ N} \cdot \text{s}}{1 \times 10^{-2} \text{ m}^2} = \frac{10^{-5} \text{ N} \cdot \text{s}}{1 \times 10^{-2} \text{ m}^2}$ $Q = 10^{-3} \text{ N} \cdot \text{s} \cdot \text{m}^{-2}$ $Q = 10^{-3} \text{ N} \cdot \text{s} \cdot \text{m}^{-2}$ |
| 2     | A      | A      |                                                                                                                                                                                                                                                                                                                  |
| 3     | A      | A      |                                                                                                                                                                                                                                                                                                                  |
| 4     | A      | A      |                                                                                                                                                                                                                                                                                                                  |
| 5     | A      | A      | $F = \frac{F_0}{n}$ <p>When <math>n</math> increases, <math>F</math> decreases.</p>                                                                                                                                                                                                                              |
| 6     | A      | A      | $F = \frac{F_0}{n}$ <p>When <math>n</math> increases, <math>F</math> decreases.</p>                                                                                                                                                                                                                              |
| 7     | A      | A      | $Av = \text{constant}$ <p>When <math>A</math> increases, <math>v</math> decreases.</p>                                                                                                                                                                                                                           |
| 8     | A      | A      | $A_1 v_1 = A_2 v_2$ $A_2 < A_1$ $v_2 > v_1$  <p>Constriction</p>                                                                                                                                                            |
| 9     | A      | A      | $A_1 v_1 = A_2 v_2$ $v_2 = \frac{A_1 v_1}{A_2}$ $A_2 > A_1$ $v_2 < v_1$  <p>Constriction</p> <p>When area of cross section of pipe increases, the speed decreases.</p> <p>Where speed is high, the pressure is low.</p>      |

|    |   |              |                                                                                                                                                                                                                                                                                                                                                                             |
|----|---|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 10 | A | cm           |  <p> <math>V = \sqrt{2gh(h - h_1)} = \sqrt{2 \times 9.8 \times (4 - 1)} = 7.7 \text{ m/s}</math><br/>         The height of water level is maximum from the top when the height above from the base therefore speed is maximum.       </p>                                                |
| 11 | A | speed is 0.3 |                                                                                                                                                                                                                                                                                                                                                                             |
| 12 | A | speed is 0.3 | $V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.045} = 0.94 \text{ m/s}$                                                                                                                                                                                                                                                                                                      |
| 13 | A | speed is 0.3 |                                                                                                                                                                                                                                                                                                                                                                             |
| 14 | A | 4 m/s        | $V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1} = 4.43 \text{ m/s}$                                                                                                                                                                                                                                                                                                          |
| 15 | A | It increases |  <p> <math>A_1 V_1 = A_2 V_2</math><br/> <math>\pi r_1^2 V_1 = \pi r_2^2 V_2</math><br/> <math>V_2 = \frac{r_1^2}{r_2^2} V_1</math><br/>         When <math>r</math> decreases, then <math>V</math> increases.<br/>         Where speed of fluid is high the pressure is low.       </p> |



## Short Answers of the Exercise

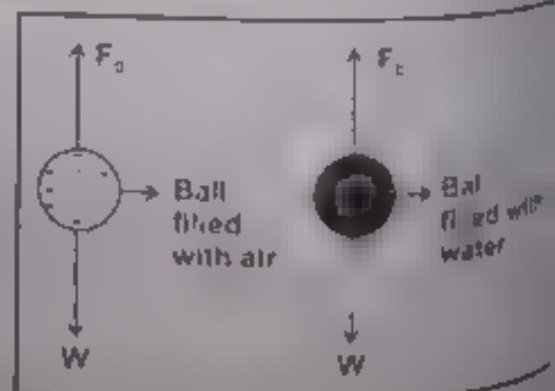
Q 2 Write short answers of the following questions

Q 1 From the top of a tall building you drop two table-tennis balls, one filled with air and the other with water. Which ball reaches terminal velocity first and why?

**Ans:** Terminal velocity is given by

$$v_t = \frac{mg}{6\pi\eta r}$$

- ▶ Above equation shows that the terminal velocity of the body is directly proportional to the weight of the object for same radius.
- ▶ The water filled table tennis ball reaches terminal velocity first as it is less for the same shape and size, so the friction force of upward air resistance becomes equal to the downward force of gravity.
- ▶ As the force of gravity on the water filled table tennis ball (more mass) is larger, its terminal velocity is larger and it strikes the ground first.

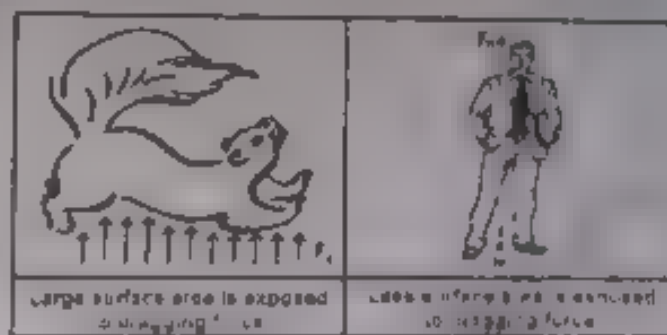


Q2 Why can a squirrel jump from a tree branch to the ground and run away undamaged while a human could break a bone in such a fall?

Ans A squirrel jumps from a tree branch to the ground and runs away undamaged while a human could break a bone in such a fall.

Reasons

- ▶ The squirrel lands from a tree with extended legs.
- ▶ As a result, his exposed surface area becomes maximum.
- ▶ The squirrel experiences large drag force and his terminal velocity is smaller.
- ▶ The squirrel falls on the ground with smaller momentum and runs away undamaged.
- ▶  $\therefore \frac{mv}{t} \propto \frac{1}{A}$
- ▶ The large heavier human being experiences smaller drag force and falls with high terminal velocity and larger momentum.
- ▶ So the bone of a human being may crack when he falls on the ground.

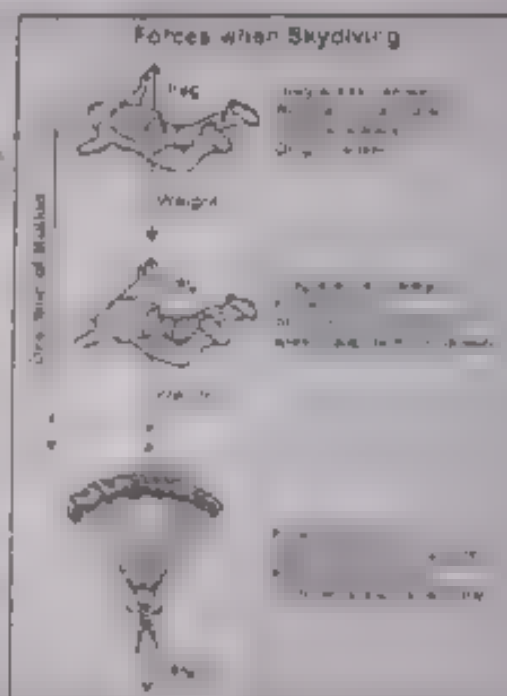


Q3 How does the terminal speed of a parachutist before opening parachute compare to the terminal speed afterward? Why is there a difference?

Ans The terminal velocity of a parachutist before opening the parachute is different from its terminal velocity after opening the parachute.

Reasons and Explanation

- ▶ The weight of the parachutist is constant but the surface area of the falling object is greater before opening the parachute and the drag force will be the dragging force and low will be its terminal velocity and vice versa.
- ▶ When the parachutist opens the parachute, the exposed surface area of the parachutist to air resistance is small. So he experiences less drag force on his body and falls with a higher terminal velocity.
- ▶ When the parachutist falls with a large surface area exposed to air resistance.
- ▶ As a result, the drag force becomes large and the terminal velocity is low.



Q4 you can squirt water over a greater distance by placing your thumb over the end of a garden hose, then by leaving it completely uncovered. Explain how this works?

Ans When you place your thumb over the end of a garden hose, the cross-sectional area is smaller and the speed of the water is higher. When you leave it uncovered, the cross-sectional area is larger and the speed of the water is lower.

Reasons

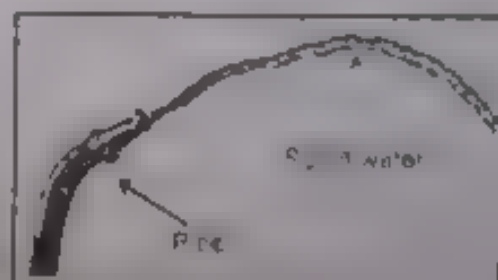
According to the equation of continuity, we have

$$A_1 v_1 = A_2 v_2$$

$$A v = \text{constant}$$

$$v \propto \frac{1}{A}$$

- ▶ The equation shows that the speed of liquid emerging out from an opening is inversely proportional to the area of cross-section of that opening. The smaller the area of cross-section of the opening, greater will be the speed of the outgoing fluid and vice versa.
- ▶ Now when we place our thumb over the end of a garden hose, its cross-sectional area decreases as a result, the speed of outgoing water increases.
- ▶ In this way, the water can be squirted over a larger distance.



**Q 5** Why does smoke rise faster in a chimney on a windy day?

**Ans** The smoke rises faster in a chimney on a windy day due to difference in pressure inside the chimney and outside the chimney.

**Reason**

- ▶ As the Bernoulli's principle pass a vital role in this process. According to Bernoulli's principle
- ▶ ~~When the speed of flow is high the pressure will be low and where the speed of flow is low the pressure will be high.~~
- ▶ As the wind blows with high speed across the top of a chimney the pressure will be low there than the pressure inside the chimney.
- ▶ ~~So the air and smoke are pushed upwards from high pressure (inside) to low pressure (outside) faster than in calm days.~~

**Q 6** Two boats moving in parallel paths close to one another risk colliding. Why?

**Ans** According to Bernoulli's relation

~~where  $v$  is a vector of  $\vec{v}$  is high, the pressure will be low and where  $v$  is low the pressure will be high.~~

- ▶ As the speed of water is high between the two boats ~~the pressure will be low.~~
- ▶ As the speed of water is low in water close to the boat ~~the pressure will be high.~~
- ▶ The difference in pressure will push the boats towards each other and they will collide.

**Q 7** A cricket ball moves past an observer from left to right, spinning counter clockwise. In which direction will the ball tend to deflect?

**Ans** Reason

As per Bernoulli's relation

▶ ~~where  $v$  is a vector of  $\vec{v}$  is high, the pressure will be low and where  $v$  is low the pressure will be high.~~

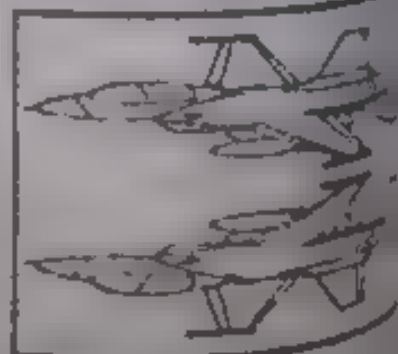
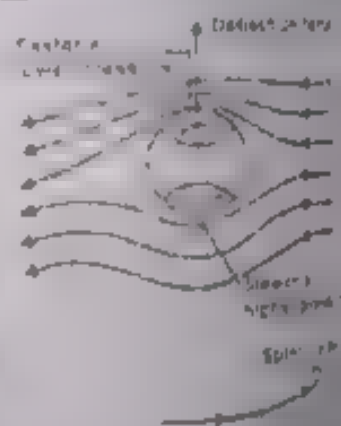
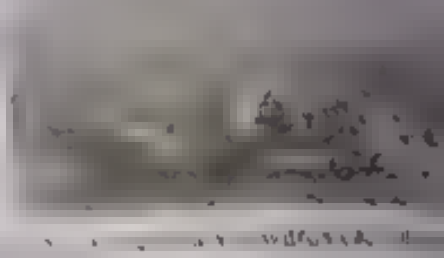
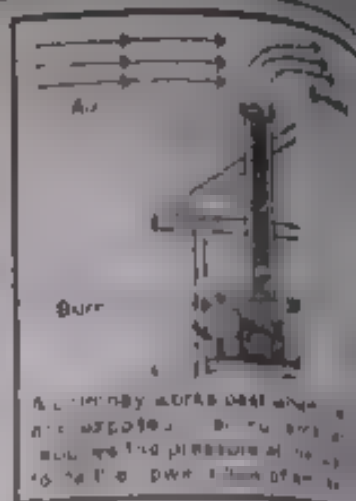
- ▶ The spinning ball the speed of air on one side becomes high and the pressure on that side becomes low.
- ▶ On the other side of the ball the speed of air is low and pressure will be high.
- ▶ The force acts on ball from high pressure towards low pressure which gives an extra curvature to the ball.

This force is given by  $F = \frac{1}{2} \rho A (v_1^2 - v_2^2)$

**Q 8** If aero-foil lift the aero-plane in upright position, how do the pilots make the aero-plane tips do down?

**Ans**

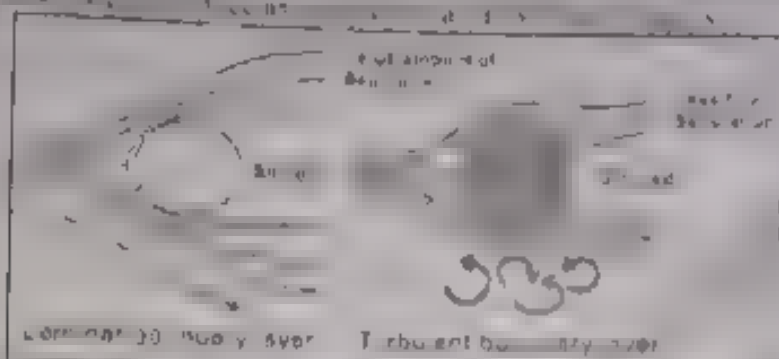
- ▶ As a plane down the wing is designed in such a way that it can make a steep descent when needed.
- ▶ As a result, the aerofoil is curved on the upper side and flat on the lower side. The air on the top of wing flows faster quickly as compared to the air below the wing.
- ▶ The difference in pressure will lift the wing upwards.
- ▶ The aerofoil is curved on both the upper and lower sides.
- ▶ As a result, the plane can fly either normally or inverted.
- ▶ The pilot can control the other by altering the angle of attack.
- ▶ The wing is designed in such a way that this design has a reduced parasite drag coefficient.



Q9 Why do the golf balls have dimples?

**Reasons and Explanation**

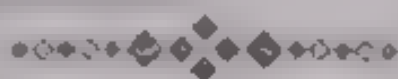
- ▶ The dimples on the surface of the golf ball create a rough surface that causes the air to flow in a more turbulent manner, reducing the drag force and allowing the ball to travel further.
- ▶ The dimples also create a boundary layer that separates from the surface of the ball at a later point than a smooth ball, further reducing the drag force.
- ▶ The dimples also create a small amount of lift, which helps the ball to stay in the air longer.



Q10 How by using wind deflector on the top truck cab can it reduce fuel consumption?

**Reason**

- ▶ The deflector on the top of the truck cab reduces the drag force by creating a more aerodynamic shape, which reduces the fuel consumption.
- ▶ The deflector also creates a low-pressure area behind the cab, which reduces the drag force.
- ▶ The deflector also creates a high-pressure area in front of the cab, which reduces the drag force.
- ▶ The deflector also creates a low-pressure area on the sides of the cab, which reduces the drag force.
- ▶ The deflector also creates a high-pressure area on the roof of the cab, which reduces the drag force.



## Comprehensive Q

Q1 Give a short response to the following questions.

What is viscous drag? State and explain Stokes Law.

What is terminal velocity? Derive mathematical relation for terminal velocity by using Stokes law.

Derive mathematically the equation of continuity and relate it to the time rate of volume flow. How equation of continuity is based on conservation of mass?

4. Derive mathematical expression for the Bernoulli's equation. How Bernoulli's equation is based on conservation of energy?

**Ans:** See Q from back

5. Using Bernoulli's equation what is the speed of efflux from a leak at the bottom of large storage tank?

**Ans:** See Q from back

6. By Bernoulli's equation how we can determine the speed of the fluid in a pipe?

**Ans:** See Q from back

7. What is aero-foli? Explain aero-foli lift on the wing of an aero-plane

**Ans:** See Q from back

8. Use Bernoulli's equation to explain the working of engine carburetor and perfume bottle spray

**Ans:** See Q from back



## Numerical Problems

1. Eight equal drops of oil are falling through air with steady velocity of  $0.1 \text{ ms}^{-1}$  the drops combine to form a single drop, what should be the new terminal velocity?

**Data:** Terminal velocity of each small droplet =  $v_t = 0.1 \text{ m/s}$   
 Radius of each small droplet =  $r$   
 Number of droplets =  $n = 8$

**To Find**

Eight droplets combine and terminal velocity of one big drop =  $v = ?$

**Calculation**

$$\text{Volume of one small droplet} = \frac{4}{3} \pi r^3$$

$$\text{Volume of } n \text{ small droplets} = V = n \times \frac{4}{3} \pi r^3$$

$$\text{Terminal velocity of each small droplet} = v_t = \frac{2r^2 g \rho}{9\eta} \quad \rightarrow (1)$$

When 8 droplets coalesce to form a big drop then volume of big drop is -

$$V = \frac{4}{3} \pi R^3 \quad (\text{where } R \text{ is radius of big drop})$$

Volume of one big drop = Volume of 8 small droplets

$$V = V$$

Putting values of  $V$  and  $V$

$$\frac{4}{3} \pi R^3 = 8 \times \frac{4}{3} \pi r^3$$

$$R = 2r$$

Taking cube root of both sides

$$R = 2r$$

$$v = \frac{2r^2 g \rho}{9\eta} \quad \text{putting } R = 2r$$

$$v = \frac{2r^2 g \rho (2r)^2}{9\eta}$$

$$v = \frac{2r^2 g \rho (2r)^2}{9\eta} = 4 \left( \frac{r^2 g \rho}{9\eta} \right) \quad \rightarrow (2)$$

Putting value from Eq (1) in (2)

$$v = 3v$$

$$v = 3(7) = 21$$

$$v = 21 \text{ m/s}$$

Water travels through a 9.6 cm diameter fire hose with a speed of 3 m/s. At the end of the hose the water flows out through a nozzle whose diameter is 2.5 cm. (a) What is the speed of the water coming out of the nozzle? (b) What diameter nozzle is required to give water speed of 21 m/s?

(a) 19 m/s (b) 2.4 cm

Given Data

$$\text{Diameter of fire hose} = D_1 = 9.6 \text{ cm} = 0.096 \text{ m}$$

$$\text{Speed of water at one end} = v_1 = 3 \text{ m/sec}$$

$$\text{Diameter of nozzle at other end } D_2 = 2.5 \text{ cm} = 0.025 \text{ m}$$

Required

$$(a) \text{ Speed of water at other end} = v_2 = ? \text{ m/s}$$

$$(b) \text{ Diameter of nozzle} = D_2' = ?$$

Calculation:

From equation of continuity, we have,  $A_1 v_1 = A_2 v_2$ 

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\therefore r_1^2 v_1 = r_2^2 v_2 \text{ and } r_1 = \frac{D_1}{2} \text{ so above equation becomes,}$$

$$\left(\frac{D_1}{2}\right)^2 v_1 = \left(\frac{D_2}{2}\right)^2 v_2$$

$$v_2 = \frac{D_1^2 v_1}{D_2^2} \quad \text{Putting value we get,}$$

$$v_2 = \frac{(9.6)^2 \times 3}{(2.5)^2}$$

$$v_2 = 19 \text{ m/s}$$

Hence,  $v_2 = 19 \text{ m/s}$ Diameter of nozzle  $D_2'$  is given by

$$D_2' = \frac{D_1^2 v_1}{v_2} \quad \text{Putting the values, we get,}$$

$$D_2' = \frac{(9.6)^2 \times 3}{21}$$

$$D_2' = 4.7 \times 10^{-2}$$

Taking square root of both sides

$$D_2' = 0.024 \text{ m}$$

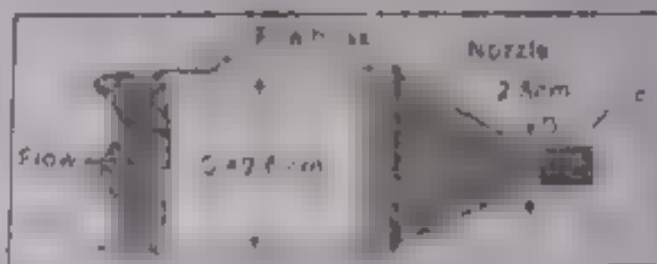
$$D_2' = 0.024 \text{ m} = 2.4 \text{ cm}$$

A fish tank has dimensions 0.30 m wide by 1.0 m long by 0.60 m high. If the filter should process all the water in the tank once every 3.0 h, what should the flow speed be in the 3.0 cm diameter input tube for the filter?

Given Data: Width of tank =  $w = 0.30 \text{ m}$ 

$$\text{Length of tank} = l = 1 \text{ m}$$

$$\text{Height} = h = 0.60 \text{ m}$$



Density of water =  $\rho = 1000 \text{ kg/m}^3$   
 Diameter of hole =  $D = 3 \text{ cm} = 0.03 \text{ m}$   
 Time =  $t = 3 \text{ s}$   
 $h = 3 \times 1600 = 4800 \text{ mm}$

To Find

Velocity of water =  $v = ?$

Calculation:

Volume of tank =  $AV = W/t$

$AV = 0.10 = \frac{4800}{1000} = 4.8 \text{ m}^3$

Area of cross section of hole =  $A = \pi r^2 = \pi \left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4} = \frac{3.14 \times (0.03)^2}{4} = 7.06 \times 10^{-4} \text{ m}^2$

Volume flow rate =  $\frac{AV}{t} = Av$

$v = \frac{AV}{A \cdot t} = \frac{4.8}{7.06 \times 10^{-4} \times 3} = 2.3 \times 10^3 \text{ m/s}$

$v = 2.3 \text{ m/s}$

Note: For hook answer put  $v = 2.3 \text{ m/s}$

4. A venturi meter is measuring the flow of water, it has a main diameter of 3.6 cm tapering down to a throat diameter of 1.0 cm. If the pressure difference is measured to be 18 mm Hg, what is the speed of the water entering the venturi throat?

Given Data

Diameter of main pipe =  $D_1 = 3.6 \text{ cm}$

Radius of pipe =  $r_1 = \frac{D_1}{2} = \frac{3.6}{2} \text{ cm} = 1.8 \text{ cm} = 0.018 \text{ m}$

Diameter of throat =  $D_2 = 1 \text{ cm}$

Radius of throat =  $r_2 = \frac{D_2}{2} = \frac{1}{2} \text{ cm} = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$

Pressure difference =  $P_1 - P_2 = 18 \text{ mm Hg}$  (1 mm of Hg = 133.3 Pa)

Pressure difference =  $P_1 - P_2 = 18 \times 133.3 \text{ Pa} = 2399.4 \text{ Pa}$

Speed of water in pipe =  $v = ?$

Density of water =  $\rho = 1000 \text{ kg/m}^3$

Calculation

Area of main pipe =  $A_1 = \pi r_1^2 = 3.14 \times (1.8 \times 10^{-2})^2 = 0.001 \text{ m}^2$

Area of throat =  $A_2 = \pi r_2^2 = 3.14 \times (0.5 \times 10^{-2})^2 = 7.85 \times 10^{-5} \text{ m}^2$

$v = A_1 \left( \frac{P_1 - P_2}{\rho A_2^2} \right)$  Putting the values

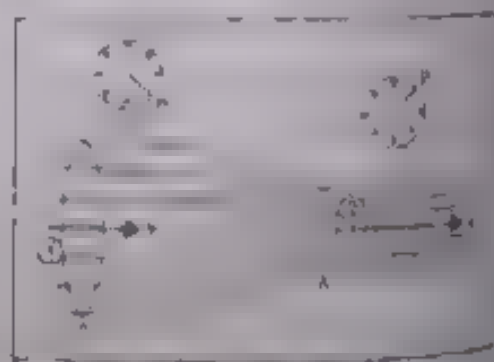
$v = 7.85 \times 10^{-5} \sqrt{\frac{2399.4}{1000 \times (0.001)^2 \times (7.85 \times 10^{-5})^2}}$

$v = 7.85 \times 10^{-5} \sqrt{\frac{2399.4}{9.2 \times 10^{-11}}}$

$v = 7.85 \times 10^{-5} \sqrt{26086.9}$

$v = 7.85 \times 10^{-5} \times 161.5$

$v = 0.127 \text{ m/s}$



5. A small circular hole 6.00 mm in diameter is cut in the side of a large water tank 14.0 m high. The water level in the tank is open to the air. Find (a) the speed of efflux the water and (b) the volume discharged per second

Given Data

Diameter of hole =  $D = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$

$r = \frac{D}{2} = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Height of tank =  $h = 14 \text{ m}$

Required:

(a) Speed of efflux of water =  $v = ?$

b) Volume discharge per second =  $\frac{\Delta V}{\Delta t} = ?$

Calculation.

a) We know that  $V = \sqrt{2gh}$

$V = \sqrt{2 \times 9.8 \times 14}$  m/s

$V = 6.6 \text{ m/s}$

As  $V = \frac{\Delta V}{\Delta t}$  then we have  $\frac{\Delta V}{\Delta t} = AV$

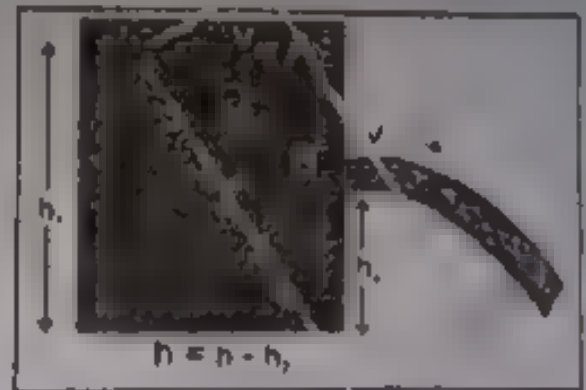
$\frac{\Delta V}{\Delta t} = \pi r^2 V$

$\frac{\Delta V}{\Delta t} = 3.142 (3 \times 10^{-2})^2 \times 6.6$

$\frac{\Delta V}{\Delta t} = 4.69 \times 10^{-4} \text{ m}^3/\text{s}$

$\frac{\Delta V}{\Delta t} = 4.69 \times 10^{-4} \text{ m}^3/\text{s}$

$\frac{\Delta V}{\Delta t} = 4.69 \times 10^{-4} \text{ m}^3/\text{s}$



4. What is the Aero-fol lift (in newton) due to Bernoulli's principle on a paper plane of wing area  $0.01 \text{ m}^2$  if the air passes over the top and bottom surfaces at speeds of  $9 \text{ m/s}$  and  $7 \text{ m/s}$  respectively? (Take the density of air as  $1.28 \text{ kg/m}^3$ ) (0.2 M)

Given Data: Area of wing =  $A = 0.01 \text{ m}^2$   
 Speed of air below the wing =  $v_2 = 7 \text{ m/s}$   
 Speed of air at top of wing =  $v_1 = 9 \text{ m/s}$   
 Density of air =  $\rho = 1.28 \text{ kg/m}^3$   
 $h_1 = h_2 = h$

To Find: Aero-fol lift =  $F = ?$

Calculation:

Pressure =  $\frac{\text{Force}}{\text{Area}}$

$P_1 - P_2 = \frac{F}{A} \Rightarrow F = A(P_1 - P_2) \quad \text{--- (1)}$

$P = \frac{1}{2} \rho v^2 + \rho gh \Rightarrow P_1 = \frac{1}{2} \rho v_1^2 + \rho gh$

$P_2 = P = \frac{1}{2} \rho (v_2^2 + v_1^2) \quad \text{--- (2)}$

Putting equation (2) in equation (1) we get

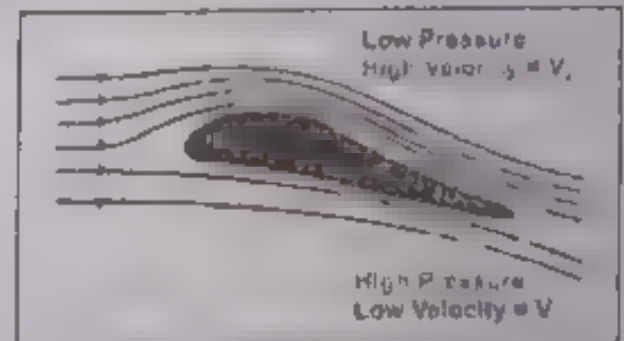
$F = A \times \frac{1}{2} \rho (v_1^2 - v_2^2) \quad \text{Putting the values}$

$F = 0.01 \times \frac{1}{2} \times 1.28 \times (9^2 - 7^2)$

$F = 6.4 \times 10^{-3} (81 - 49)$

$F = 6.4 \times 10^{-3} \times 32 \text{ N} = 204.8 \times 10^{-3}$

$F = 0.2 \text{ N}$



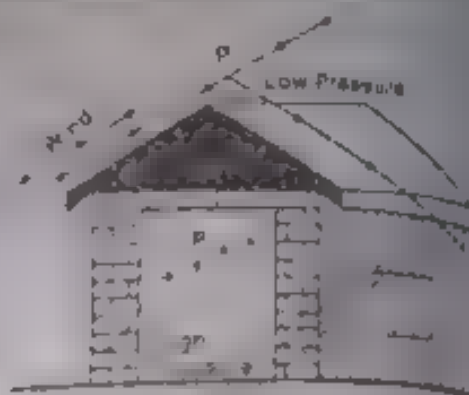
During a windstorm, a  $25 \text{ m/s}$  wind blows across the flat roof of a small home. Find the difference in pressure between the air inside the home and the air just above the roof, assuming the doors and windows of the house are closed. (The density of air is  $1.28 \text{ kg/m}^3$ )

Given Data: Speed of air at top of roof =  $v_2 = 25 \text{ m/s}$

Density of air =  $\rho = 1.28 \text{ kg/m}^3$

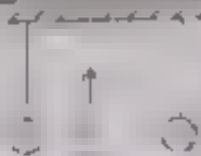
Required

Calculation



## Activity

1. Two light balls are suspended as shown in the figure. When a stream of air passes through the space between them what happens to the distance between the balls?



Answer: The balls move towards each other. This is because the air passing between them creates a region of low pressure, which pulls the balls together.

2. A paper aeroplane is balanced on a pivot. If a stream of air is blown over the top of its wings, what happens to its balance?

Answer: The aeroplane will rise. This is because the air flowing over the curved top surface of the wing moves faster, creating a region of low pressure. The higher pressure underneath the wing pushes the plane upwards.

3. Houses roofed with tiles often get damaged at high winds. Explain why? Why should we keep windows closed during these conditions?

Answer: At high winds, the pressure difference between the outside and inside of the house creates a large upward force on the roof tiles, which can lift them off. Closing windows helps maintain internal pressure and prevents further damage.

4. How do you use straw to suck water? Can you suck water with a straw on the moon?

Answer: When you suck through a straw, you reduce the pressure inside the straw. Atmospheric pressure outside the straw pushes the water up into the lower pressure area. On the moon, you cannot suck water with a straw because there is no atmosphere to exert this pressure.

5. It is often seen that leaves lying on the road start following the fast moving car when it passes through the road. Why?

Answer: As a car moves fast, the air closer to the car is pushed back, creating a region of high pressure. Further away, the air pressure is lower. This pressure difference pushes the leaves towards the fast-moving car.

6. Why are clouds appear to float in air?

Answer: Clouds are made of tiny droplets of water. They are kept aloft by the upward buoyant force of the air, which is less dense than the water droplets.

$$v = \frac{r\omega}{R}$$
$$\frac{v}{R} = \frac{r\omega}{R^2}$$

Since mass of the droplets is very small their terminal velocity is very small.

Why fog droplets appear to be suspended in air?

**Ans:** Reason  
Terminal velocity is very small.  
 $v = \frac{r\omega}{R}$   
 $\frac{v}{R} = \frac{r\omega}{R^2}$




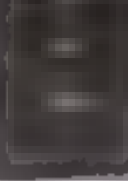

As we know that the radius of fog droplets is very small, the drag force becomes equal to its weight very soon. Therefore, fog droplets appear to be suspended in air.



- 1. Which of the following is equal to one torr?  
(a) 1 N (b) 1 mm Hg (c) 1 mm (d) 1 cm
- 2. Which of the following is the S.I. unit of viscosity?  
(a) N/m (b) Nsm<sup>2</sup> (c) Nsm<sup>-1</sup> (d) Nsm<sup>2</sup>s<sup>2</sup>
- 3. Sphere of mass 1 kg falls through fluid with maximum constant velocity then drag force is  
(a) 1 N (b) 49 N (c) 98 N (d) zero
- 4. Clouds float in atmosphere because of  
(a) low density (b) low viscosity (c) low temperature (d) low pressure
- 5. The radius at two ends of a pipe is in the ratio 2:3 then the speed of liquid at the two ends is in the ratio of  
(a) 2:3 (b) 2:9 (c) 4:9 (d) 9:4
- 6. Which of the following represent motion of freely falling water droplet?  
(a) (b) (c) (d)
- 7. Which of the following is the expression for terminal velocity?  
(a)  $v = \frac{2r^2g}{9\eta}$  (b)  $v = \frac{2r^2g}{9\eta}$  (c)  $v = \frac{2r^2g}{9\eta}$  (d)  $v = \frac{2r^2g}{9\eta}$
- 8. Venturi meter is used to find  
(a) speed of the fluid (b) density of the fluid  
(c) pressure of the fluid (d) viscosity of the fluid
- 9. When the streamlines are far apart from each other, the pressure will be  
(a) low (b) high (c) zero (d) maximum
- 10. Which of the following is the S.I. unit of flow rate?  
(a) m/s (b) m/s<sup>2</sup> (c) m/s<sup>3</sup> (d) m/s<sup>4</sup>
- 11. Which of the following is the unit of co-efficient of viscosity?  
(a) kg/m<sup>3</sup> (b) kg/m<sup>2</sup> (c) kg/m<sup>2</sup>s (d) kg/m<sup>2</sup>s<sup>2</sup>
- 12. Which of the following are the dimensions of co-efficient of viscosity?  
(a) M/L (b) M/L<sup>2</sup> (c) M/L<sup>2</sup>s (d) M/L<sup>2</sup>s<sup>2</sup>

13. If radius of the droplet is doubled at same density then its terminal velocity increases (P1128-307)  
 (a) Four times (b) Eight times (c) Six times (d) Three times
14. Which of the following is the device used to measure the speed of liquid flow? (P1128-307)  
 (a) Spectrometer (b) Spectrometer (c) Isotometer (d) Venturimeter
15. Which of the following are the dimensions of flow rate? (P1128-307)  
 (a)  $[L^3 T^{-1}]$  (b)  $[L^2 T^{-1}]$  (c)  $[L^3 T^{-1}]$  (d)  $[L^2 T^{-1}]$
16. The property of fluid by which its own molecules are attracted is said to be (P1128-307)  
 (a) Adhesion (b) Cohesion (c) Viscosity (d) Both A and B

## Answers Key

|                                                                                  |   |                                                                                   |   |                                                                                   |   |                                                                                     |   |                                                                                     |   |
|----------------------------------------------------------------------------------|---|-----------------------------------------------------------------------------------|---|-----------------------------------------------------------------------------------|---|-------------------------------------------------------------------------------------|---|-------------------------------------------------------------------------------------|---|
|  | a |  | a |  | c |  | d |  | c |
|                                                                                  | b |                                                                                   | b |                                                                                   | b |                                                                                     | b |                                                                                     | d |
|                                                                                  | c |                                                                                   | c |                                                                                   | c |                                                                                     | c |                                                                                     | c |
|                                                                                  | d |                                                                                   | d |                                                                                   | d |                                                                                     | d |                                                                                     | d |



# SELF ASSESSMENT PAPER

Total Mark 40

(1 x 5 = 5)

Question No 1

Choose the correct answer from the given options.

## SECTION - A

1. Which of the following is equal to one torr?

- (A)  $1.333 \text{ Nm}^{-2}$  (B)  $13.33 \text{ Nm}^{-2}$  (C)  $133.3 \text{ Nm}^{-2}$  (D)  $1333 \text{ Nm}^{-2}$

2. Which of the following is the SI unit of viscosity?

- (A)  $\text{Ns m}^{-2}$  (B)  $\text{Nm}^{-2}$  (C)  $\text{Ns m}^{-1}$  (D)  $\text{Nm}^{-2} \text{ s}^2$

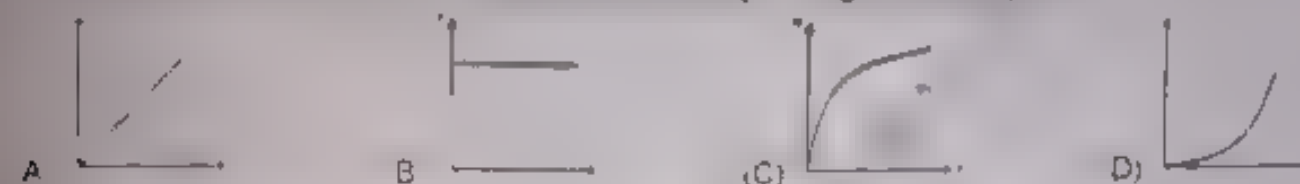
3. Sphere of mass 0.1 kg falls through fluid with maximum constant velocity then drag force is

- (A) 98N (B) 49N (C) 9.8N (D) 0.98N

4. The radius at two ends of a pipe is in the ratio 2:3, then the speed of liquid at the two ends is in the ratio of

- (A) 3:2 (B) 2:3 (C) 4:9 (D) 9:4

5. Which of the following represent motion of freely falling water droplet?



6. Bernoulli's equation is based upon law of conservation of

- (A) Mass (B) Momentum (C) Energy (D) None

Question No 2

Give short answers of following

(3 x 7 = 21)

## SECTION - B

- How does the terminal speed of a parachutist before opening parachute compare to the terminal speed afterward? Why is there a difference?
- Two boats moving in parallel paths close to one another risk colliding. Why?
- A cricket ball moves past an observer from left to right spinning counter clockwise. In which direction will the ball tend to deflect?
- Applying Bernoulli's equation explain the lift of an aero-plane
- Why do the golf balls have dimples?
- Differentiate between laminar and turbulent flow of fluid
- Water is flowing steadily through a closed pipe system. At one point the speed of water is  $3 \text{ ms}^{-1}$  while at another point 3m higher the speed is  $4 \text{ ms}^{-1}$ . At lower point the pressure is 80 kPa. Find the pressure at the upper end.

(13)

Question No 3

Extensive Questions.

## SECTION - C

- State and explain Bernoulli's Equation in detail. (06)
- What is the Aero lift (in newtons) due to Bernoulli's principle on a paper plane of wing area  $0.01 \text{ m}^2$  if the air passes over the top and bottom surfaces at speeds of  $9 \text{ m/s}$  and  $7 \text{ m/s}$  respectively? (Take the density of air as  $1.28 \text{ kg/m}^3$ ) (04)
- Applying Bernoulli's Equation Explain the working of Carburetor of car Engine? (03)

\*\*\* The End \*\*\*



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## CHAPTER

## 7

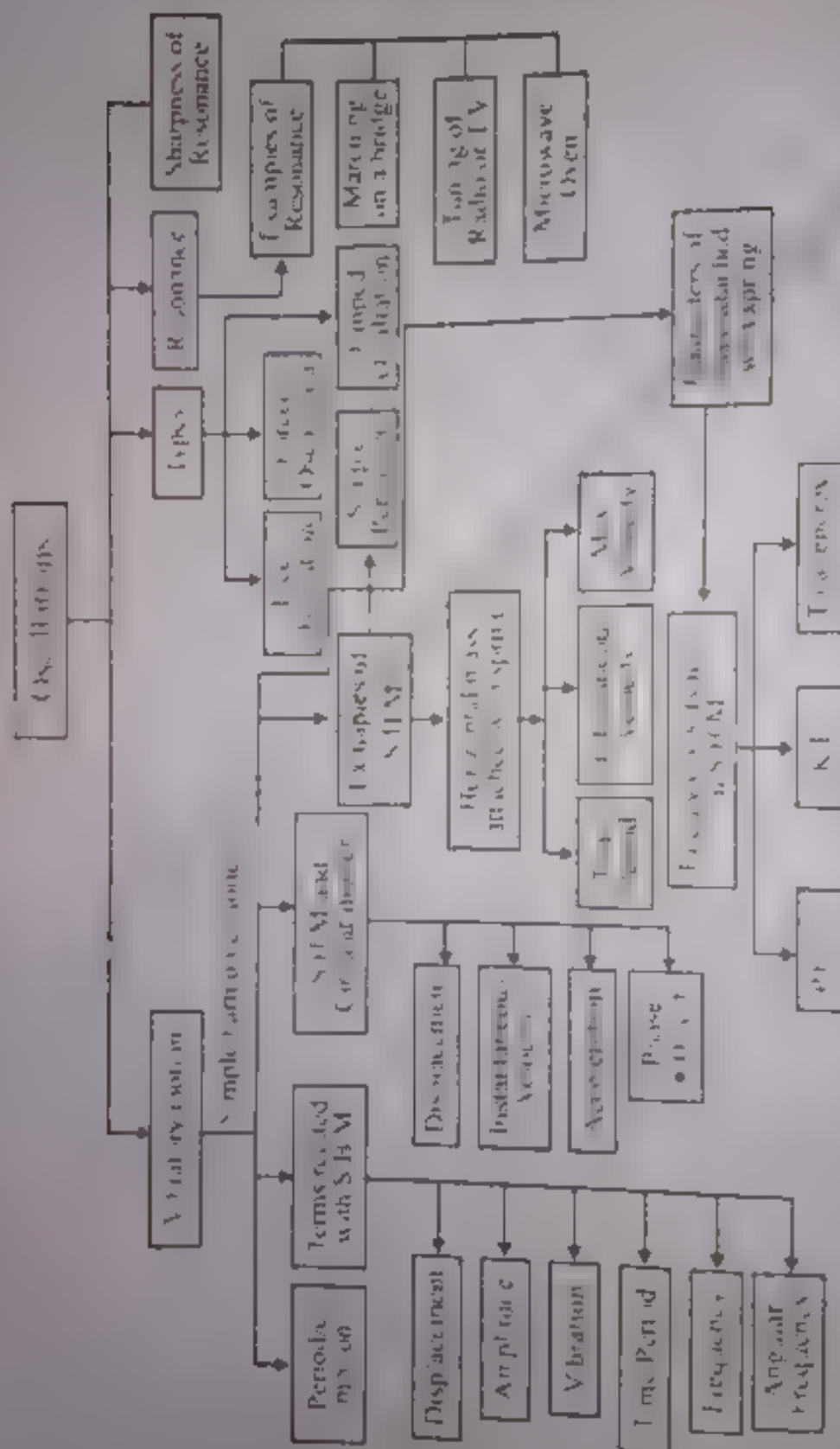
## OSCILLATIONS

Learning Objectives

- ✦ Describe simple examples of free oscillations
- ✦ Describe necessary conditions for execution of simple harmonic motions
- ✦ Describe that when an object moves in a circle the motion of its projection on the diameter of the circle is SHM
- ✦ Define the terms amplitude, period, frequency, angular frequency and phase difference and express the period in terms of both frequency and angular frequency
- ✦ Identify and use the equation,  $a = -\omega^2 x$  as the defining equation of SHM
- ✦ Prove that the motion of mass attached to a spring is SHM
- ✦ Describe the interchanging between kinetic energy and potential energy during SHM
- ✦ Analyze that the motion of a simple pendulum is SHM and calculate its time period
- ✦ Describe graphically how the amplitude of a forced oscillation changes with frequency near to the natural frequency of the system
- ✦ Describe practical examples of damped oscillations with particular reference to the effects of the degree of damping and the importance of critical damping in cases such as a car suspension system
- ✦ Describe qualitatively the factors which determine the frequency response and sharpness of the resonance

## Chapter No. 7

## CONCEPT MAP



## Oscillatory Motion

To and fro motion of a body about a mean position is called oscillatory or vibratory motion.

### Periodic Motion

The oscillatory motion that repeats itself after equal intervals of time is called periodic motion.

### Examples of vibrating bodies

1. The motion of mass suspended from a spring
2. The motion of bob of a simple pendulum
3. A steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways
4. A steel ball rolling in a curved dish

### Restoring force

The force which brings the system back to its stable equilibrium position is called elastic restoring force.

$$\text{Restoring force } F = -kx$$

It is equal and opposite to the applied force.

### Oscillation

An oscillation can be a periodic motion that repeats itself in a regular even

### Order to get oscillation

A body is pulled away on one side from its equilibrium position and then released. The body moves towards the equilibrium position due to restoring force. Under the action of this restoring force, the body accelerates and it passes over the equilibrium position due to inertia. The restoring force pulls it back. Since restoring force is always directed towards the mean position, so the acceleration is always towards the mean position.

### Note

The vibrating bodies produce waves e.g. A violin string produces waves.

There are many phenomena in nature, which are explained on the concept of vibrations and waves. There are many large structures such as skyscrapers and bridges, which appear to be rigid, but actually they vibrate. The architects and engineers take into account these vibrations while designing and building them.

### Terminology of Oscillatory Motion

#### Vibratory motion

To and fro (back and forth) motion of a body about the mean position is called vibratory motion.

#### Vibration

One complete round trip of a vibrating body about its mean position is called one vibration.

For example, motion of bob of simple pendulum from A to B and back from B to A is one vibration as shown in fig 7.1

#### Time Period

The time required by a body to complete one vibration is called time period.

It is represented by  $T$ . Its unit is second.

#### Frequency

The number of vibrations completed in one second by the body is called frequency. It is the reciprocal of the time period.

It is represented by  $f$ . The unit of frequency is hertz or vibrations/second or cycles/second.



Fig 7.1

$$f = \frac{1}{T}$$

OR

$$fT = 1$$

i.e. product of frequency and time period equals one.

### Instantaneous Displacement (x)

The shortest distance of the vibrating body at any instant from its mean position is called instantaneous displacement.

It is usually denoted by  $x$ . The value of instantaneous displacement is zero at mean position and is maximum value at the extreme positions.

### Amplitude ( $x_0$ )

The maximum displacement of the vibrating body on either sides from its mean position is called amplitude. It is denoted by  $x_0$ .

### Angular Frequency

The number of revolution per second of a body is called angular frequency.

If  $T$  is the time period of a body executing SHM, its angular frequency ( $\omega$ ) is given as

$$\omega = \frac{2\pi}{T}$$

$$\Rightarrow \omega = 2\pi f \quad (\text{Putting } \frac{1}{T} = f)$$

$$\Rightarrow \omega = 2\pi$$

### Note

Basically, angular frequency is the property of circular motion. In SHM, it provides an easy method for determining the instantaneous displacement and instantaneous velocity of body executing SHM.

#### For Your Information

The most relevant dynamic characteristics of a high rise building are its natural oscillatory period, mass, stiffness and damping coefficient. Tall buildings are characterized by low natural frequency, hence they can vibrate significantly under average dynamic earthquake loads.



**Q 1** Define simple harmonic motion (S H M) and give conditions of SHM

### **ANS** Simple Harmonic Motion

The type of oscillatory motion, in which acceleration of the body at any instant is directly proportional to displacement from the mean position and directed towards the mean position, is called simple harmonic motion (SHM).

#### Conditions for SHM

1. The system must have inertia
2. The system should have restoring force
3.  $a \propto -x$
4. The system should be frictionless

Q 2 Show that motion of mass attached with a spring is SHM

### Ans Motion of Mass attached to a spring

Consider a mass 'm' attached with one end of the spring which can move freely on a frictionless horizontal surface as shown in figure

When mass 'm' is displaced through a distance 'x' from mean position by a force 'F' then,

According to Hooke's law the extension in spring is proportional to the force within elastic limit

$$F = kx$$

Where 'k' is constant of proportionality, known as spring constant

$$\text{Spring Constant } K = \frac{F}{x}$$

► Spring constant is defined as the force per unit extension

► Its S.I. unit is  $\text{Nm}^{-1}$  and dimension is  $[\text{MT}^{-2}]$ .

Due to elasticity spring opposes the applied force. This opposing force is called restoring force.

### Restoring Force

The force which brings the body back towards its mean position is called elastic restoring force.

The restoring force represented  $F_r$  is

$$F_r = -kx \quad (1)$$

The negative sign shows that  $F_r$  is directed opposite to  $x$

When the mass is released, it begins to oscillate about the equilibrium position as shown in figure. Such type of oscillations are due to restoring force and inertia. This type of oscillatory motion is called simple harmonic motion.

### Expression for acceleration

The acceleration due to restoring force

$$F = ma \quad (2)$$

Comparing equations (1) and (2), we get

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

$$a = -\text{constant } x \text{ and } a \propto -x$$

### Angular Frequency

We know that  $a = -\omega^2 x$

Putting equations (3) and (4), we get

$$-\omega^2 x = -\frac{k}{m}x$$

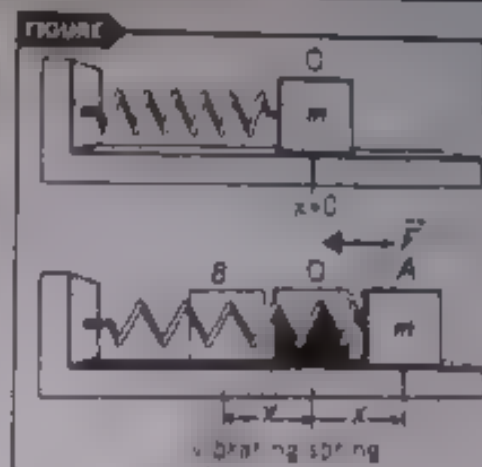
$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$\omega$  is the angular frequency

Time period

The time period of mass 'm' having SHM can be expressed as,



$$T = \frac{2\pi}{\omega}$$

Putting values of  $\omega$  from equation (5), we get

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

OR

$$T = 2\pi \sqrt{\frac{m}{k}} \quad \text{--- (6)}$$

**Frequency** As the reciprocal of the time period is called frequency. So,

$$f = \frac{1}{T}$$

Putting value of  $T$  from equation (6), we get  $f = \frac{1}{2\pi \sqrt{\frac{m}{k}}}$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

### Assignment 7.1

A 2.0 kg block hung from a vertical spring causes it to stretch by 20.0 cm. If the 2.0 kg block is replaced by a 0.60 kg mass, and the spring is stretched and released, what are the frequency and period of the oscillations?

**Given Data** Mass  $m = 2 \text{ kg}$   
 extension  $x = 20 \text{ cm} = 0.2 \text{ m}$

**Solution**

$$k = \frac{F}{x} = \frac{W}{x} = \frac{mg}{x} = \frac{2 \times 9.8}{0.2} = 98 \text{ N/m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$\text{Hence } f = \frac{1}{2 \times 3.142} \sqrt{\frac{98}{0.6}} = 2.21 \text{ Hz}$$

$$\text{b) } T = \frac{1}{f} = \frac{1}{2.21} = 0.45 \text{ s}$$

### MCQ's

- Distance covered during one vibration of an oscillating body in terms of amplitude  $A$  is  
 (A)  $A/2$  (B)  $A$  (C)  $2A$  (D)  $4A$
- The wave form of SHM is  
 (A) sine wave (B) square wave (C) Tangent (D) None
- The acceleration of a body performing the SHM depends upon  
 (A) Mass (B) Time period (C) Displacement (D) Amplitude
- Which of the following is the relation between linear frequency and angular frequency?  
 (A)  $\omega = 2\pi f$  (B)  $\omega = \frac{2\pi}{f}$  (C)  $\omega = \frac{2f}{\pi}$  (D)  $\omega = \frac{\pi}{2f}$
- The product of frequency and time period is always equal to  
 (A) 1 (B) speed (C) wavelength (D) energy
- Which of the following is true at mean position in SHM for P.E. and K.E?  
 (A) P.E. is max and K.E. is min (B) P.E. is zero and K.E. is max (C) Both are max (D) Both are min

### Answers Key

- |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1 D | 2 A | 3 C | 4 A | 5 A | 6 B |
|-----|-----|-----|-----|-----|-----|

23 Show that the motion of projection of body moving along a circular path is SHM and derive expression for acceleration and velocity of SHM

### Circular Motion and S.H.M

Let there be a particle of mass  $m$  whirling in a horizontal circle of radius  $r$  with angular velocity  $\omega$ . A distant light source & shadow of the stone on wall, the shadow executes simple harmonic motion. Similarly, when a body moves in vertical circle, its projection executes simple harmonic motion on the diameter of the circle having time-period, frequency and acceleration

#### Acceleration of Projection

Let  $AB$  is the diameter and  $O$  is the center of circle

- When the body moves in circle its projection  $Q$  vibrates along the diameter of circle about the mean position  $O$
- When the body is at point  $P$ , its projection  $Q$  is at distance  $x$  from mean position
- If  $a_c$  is the centripetal acceleration of body directed towards the mean position  $O$ . We have resolved the centripetal acceleration into its components.

From  $\Delta POQ$

$$\frac{x}{r} = \cos \theta \quad (\text{Putting } QO = a_x \text{ and } PO = a_c)$$

$$\frac{a_x}{a_c} = \cos \theta \quad (\text{cross multiplying})$$

$$a_x = a_c \cos \theta$$

$a_x$  is component of  $a_c$  along the diameter  $AB$

$$a_c = \frac{v^2}{r} = r\omega^2 \quad \text{where } \omega \text{ is the angular velocity of body}$$

Then equation (1) becomes

$$a_x = r\omega^2 \cos \theta$$

From Fig 7.2 we see,  $\cos \theta = \frac{x}{r}$

$$\frac{a_x}{r} = \cos \theta$$

$$a_x = r\omega^2 \cos \theta$$

- This is expression for instantaneous displacement of SHM

Putting value of  $\cos \theta$  from equation (1) in Eq (2)

$$a_x = r\omega^2 \left( \frac{x}{r} \right)$$

$$a_x = -\omega^2 x$$

- As  $a_x$  directed towards the mean position then we take its sign negative and above equation becomes,

$$a_x = -\omega^2 x$$

$$a_x = -\text{constant} \times x$$

Where  $\omega^2 = \text{constant}$

$$a_x \propto -x$$

Which is the equation of S.H.M

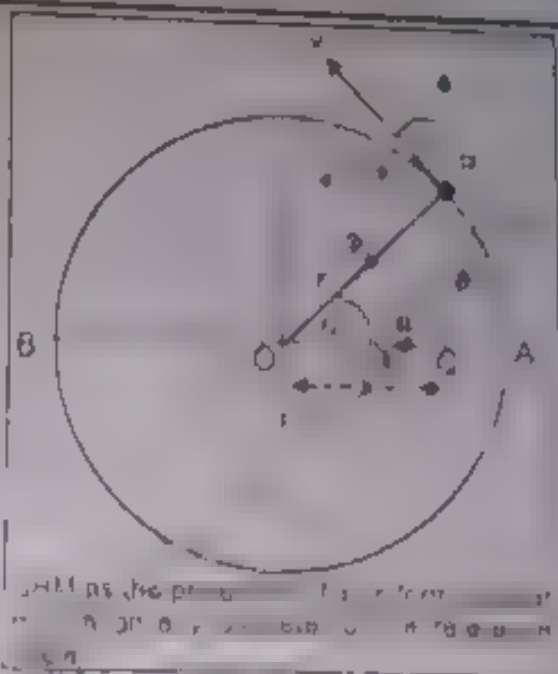


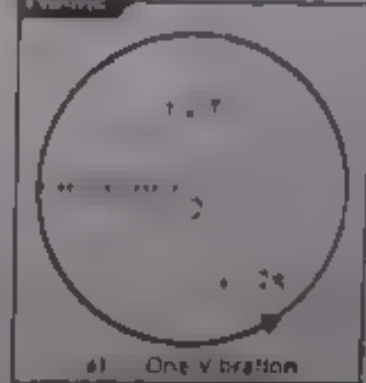
Fig 7.2

#### FIGURE



Projection of body moving in a vertical circle

#### FIGURE



- (2) **Time-period =  $T$**   
*The time required by a body to complete one vibration is called time-period.* In given diagram the vibration of a body is shown.

The time period is given by  $T = \frac{2\pi}{\omega}$

- (3) **Frequency =  $f = \frac{1}{T}$**

Putting for  $T$ , then  $f = \frac{\omega}{2\pi} \rightarrow \omega = 2\pi f$

which is the frequency of body in S.H.M

- (4) **Velocity of Projection**

- In figure, resolve the velocity in to rectangular components

$$v_y = v \cos \theta$$

$$v_x = v \sin \theta \rightarrow \dots$$

By trigonometry

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

From figure  $\cos \theta = \frac{x}{r}$

$$\text{So } \sin^2 \theta = 1 - \frac{x^2}{r^2}$$

$$\sin^2 \theta = \frac{r^2 - x^2}{r^2}$$

Taking square root of both sides

$$\sin \theta = \sqrt{\frac{r^2 - x^2}{r^2}} = \frac{1}{r} \sqrt{r^2 - x^2}$$

$$\text{putting } \sin \theta = \frac{\sqrt{r^2 - x^2}}{r} \text{ in equation (1)}$$

$$v_x = v \left( \frac{\sqrt{r^2 - x^2}}{r} \right)$$

Putting  $v = r\omega$

$$v_x = r\omega \left( \frac{\sqrt{r^2 - x^2}}{r} \right)$$

$$v_x = \omega \sqrt{r^2 - x^2} \rightarrow (2)$$

where ' $r$ ' is radius of circle ' $x$ ' is the instantaneous displacement of projection 'Q' from mean pos. 'O' and ' $\omega$ ' is the angular velocity of projection of particle

$$\omega = \frac{\theta}{t}$$

For one vibration

$$\omega = \frac{2\pi}{T}$$

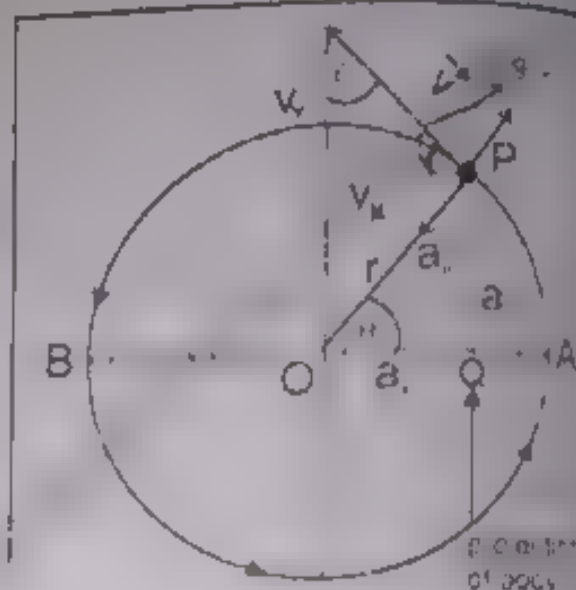


Fig 7.5.5 Circular motion of a mass

$$v_x = v \cos (90^\circ - \theta)$$

$$v_x = v \cos (90^\circ - \theta)$$

$$\text{Putting } \cos (90^\circ - \theta) = \sin \theta$$

$$v_x = v \sin \theta$$

$$v_y = v \sin (90^\circ - \theta)$$

$$v_y = v \sin (90^\circ - \theta)$$

$$\text{Putting } \sin (90^\circ - \theta) = \cos \theta$$

$$v_y = v \cos \theta$$

### MCQ's

- When a particle is moving along a circular path, its projection along the diameter executes  
 (A) Linear motion (B) Vibratory motion (C) Rotatory motion (D) SHM
- Acceleration of projection of a particle moving around a circle is given by relation  
 (A)  $a = g \sin \theta$  (B)  $a = -\omega^2 x$  (C)  $a = -\omega^2 r$  (D)  $a = -g \sin \theta$
- Which of the following is S.I. unit of plane angle?  
 (A) Radian (B) Degree (C) Steradian (D) Revolution



Comparing above two equations (1) and (2) we get

$$ma = -mg \sin \theta$$

OR  $a = -g \sin \theta$

For small value of angle  $\theta$ ,  $\sin \theta \approx \theta$

So,  $a = -g\theta$  (1)

From figure  $\theta = \frac{x}{l}$  [  $\Delta = r\theta \Rightarrow \theta = \frac{x}{r}$  ]

$$\theta = \frac{x}{l} \quad (\theta \text{ is small so arc } \Delta \approx x)$$

Putting  $\theta = \frac{x}{l}$  in eq. (1),

$$a = -g \frac{x}{l}$$

$$a = -\frac{g}{l} x \quad (4)$$

This is expression for acceleration of simple pendulum

$$a = -\omega^2 x \quad \text{So } \omega = \sqrt{\frac{g}{l}}$$

OR  $\omega \propto \frac{1}{l}$

This proves that the motion of simple pendulum is S.M.

### Angular Frequency

We know that acceleration for a body executing S.M is

$$a = -\omega^2 x \quad (5)$$

Comparing equation (4) and (5), we have

$$\omega^2 x = \frac{g}{l} x$$

OR  $\omega^2 = \frac{g}{l}$

OR  $\omega = \sqrt{\frac{g}{l}} \quad (6)$

This is expression for the angular frequency of simple pendulum

### Time period

Definition: Time required to complete one vibration is called time period.

As the time period for S.M can be expressed as,

$$T = \frac{2\pi}{\omega}$$

Putting value of  $\omega$  from equation (6) we get

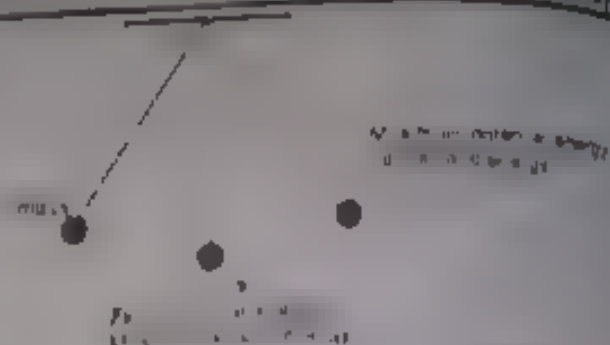
$$T = \frac{2\pi}{\sqrt{\frac{g}{l}}}$$

$$T = 2\pi \sqrt{\frac{l}{g}} \quad (7)$$

### Dependence of Time period

Eq. (7) shows that time period of the simple pendulum depends upon

- Length of pendulum
- Acceleration due to gravity ( $g$ )



- Ans
- The period of the simple pendulum is independent of
  - Mass of bob
  - Amplitude of oscillation

Frequency  
As  $T \propto \frac{1}{f}$ , hence  $f$  is independent of any of the above.

25. What is a second pendulum? Calculate its frequency.

Ans. Second pendulum

is a pendulum whose period is exactly 2 seconds.

Frequency of second pendulum



Q. 2

Ans

2.1.1. When the elevator is at rest, the period of the pendulum is measured. If the elevator moves with constant velocity, does the period increase, decrease, or remain the same? If the elevator accelerates upwards, does the period increase, decrease, or remain the same?

When the elevator is at rest, the period of the pendulum is measured. If the elevator moves with constant velocity, does the period increase, decrease, or remain the same? If the elevator accelerates upwards, does the period increase, decrease, or remain the same?

2.1.2. When the elevator is moving upwards with acceleration  $a$ , the period of the pendulum is  $T$ . What is the period when the elevator is moving downwards with acceleration  $a$ ?

From the equation  $T = 2\pi \sqrt{\frac{l}{g}}$ , so time period will decrease if  $g$  increases.

A pendulum extending from the ceiling almost touches the floor and that its period is 12 s. How tall is the tower?

2.1.3. A pendulum of length  $l$  is suspended from the ceiling of a tower of height  $H$ . The period of oscillation is  $T$ . Find the height of the tower.

2.1.4. A pendulum of length  $l$  is suspended from the ceiling of a tower of height  $H$ . The period of oscillation is  $T$ . Find the height of the tower.

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \text{and} \quad T = 2\pi \sqrt{\frac{H}{g}}$$

$$L = \frac{gT^2}{4\pi^2}$$

$$L = \frac{9.8 \times 1.4^2}{4 \times 3.14^2} = 36 \text{ cm}$$

### MCQ's From Past Board Papers

- When the bob of simple pendulum is at extreme position its kinetic energy is
  - Maximum
  - Minimum
  - Zero
  - 2g
- Which expression is correct for the time period of a simple pendulum?
  - $T \propto \sqrt{l}$
  - $T \propto l$
  - $T \propto m$
  - $T \propto \sqrt{g}$
- In case of oscillating simple pendulum its acceleration is
  - Zero
  - Proportional to length of pendulum
  - Inversely proportional to mass of bob
  - Proportional to displacement
- The frequency of Simple Pendulum is given by
  - $\frac{1}{2\pi} \sqrt{\frac{g}{l}}$
  - $2\pi \sqrt{\frac{g}{l}}$
  - $\frac{1}{2\pi} \sqrt{g}$
  - $2\pi \sqrt{g}$
- The frequency of second pendulum is
  - 1 Hz
  - 0.5 Hz
  - 5 Hz
  - 10 Hz
- If amplitude of a simple pendulum is increased by 4 times the time period will be
  - four times
  - half
  - Same
  - Twice
- If Mass of Pendulum becomes double, then its time period will be
  - Doubled
  - Half
  - Four times
  - Remains same
- If a simple pendulum vibrates with frequency 2.5 Hz then its length will be
  - 10 cm
  - 10 m
  - 10 cm
  - 100 cm
- Which of the following force is responsible for the vibratory motion of simple pendulum?
  - $mg \cos \theta$
  - $mg \sin \theta$
  - $mg$
  - $mg \tan \theta$
- If the time period of simple pendulum is 2 second its frequency will be
  - 1 Hz
  - 0.5 Hz
  - 5 Hz
  - 2 Hz
- A simple pendulum is moved from the Earth to the Moon. How does it change the period of oscillation? (Acceleration due to gravity on moon = 1.6 m/s<sup>2</sup>)
  - Time period remains the same
  - Time period is increased by factor 2
  - Time period is decreased by factor 2
  - Time period is decreased by factor  $\sqrt{5}$
- At which place the motion of a simple pendulum will be slowest?
  - Kashmir
  - Patna
  - Mumbai
  - Delhi

### Answers Key

|     |     |     |     |     |     |     |     |     |      |      |      |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|------|------|
| 1 C | 2 A | 3 A | 4 A | 5 B | 6 C | 7 D | 8 D | 9 C | 10 B | 11 B | 12 B |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|------|------|

**Q 5 Explain energy conservation in SHM. Prove that the total energy is conserved in SHM**

### ANSWER Energy Conservation in SHM

- Let us consider a mass 'm' attached with a spring of spring constant 'k' extended by 'x' from its mean position. (Fig 7.5) The mass oscillates with amplitude 'x<sub>0</sub>'.
  - I.e. oscillating mass is at displacement 'x' from mean position at any instant of time.
  - According to Hooke's law the applied force is directly proportional to displacement 'x'.
- Instantaneous K.E. in SHM**

$$KE = \frac{1}{2}mv^2$$

$$V = \frac{1}{2}kx_0^2 - x^2$$

$$KE = \frac{1}{2}m(\omega \sqrt{x_0^2 - x^2})^2$$

$$KE = \frac{1}{2}m\omega^2(x_0^2 - x^2) \quad (1)$$

$$E = \frac{1}{2}kx_0^2 = \frac{1}{2}m\omega^2x_0^2$$

Comparing eq (1) and (2)

$$E = \frac{1}{2}m\omega^2x_0^2 = \frac{1}{2}m\omega^2(x_0^2 - x^2) + \frac{1}{2}m\omega^2x^2$$

$$E = KE + V$$

Putting  $m \omega^2 = k$  in above equation

$$K.E = \frac{1}{2} k x_0^2 = \frac{1}{2} k x^2 \quad (2)$$

At extreme position  $x = x_0$  of mass spring system

Maximum K.E

► The K.E. is maximum at equilibrium where  $x = 0$  (putting  $x = 0$  in equation (2))

$$K.E = \frac{1}{2} k x^2 = 0$$

$$K.E = \frac{1}{2} k x^2$$

Minimum K.E

► The K.E. is minimum at extreme position where

$x = x_0$  putting  $x = x_0$  in equation (1)

$$K.E = \frac{1}{2} k x^2 = \frac{1}{2} k (x_0)^2$$

$$K.E = \frac{1}{2} k x^2$$

Instantaneous P.E. in SHM

► here  $F = -kx$  force of simple harmonic oscillator at  $x$  is  $F = -kx$

$$kx = m\omega^2 x \quad \text{putting } k = m\omega^2$$

$$F = -m\omega^2 x$$

$$F_{avg} = \frac{1}{T} \int_0^T F dt$$

$$F = -m\omega^2 x$$

$$= -m\omega^2 x$$

► If  $x = 0$  then displacement is zero then  $F = 0$

$$F = -m\omega^2 x = 0$$

► When  $x = x_0$  then  $F = -m\omega^2 x_0$

$$F = -m\omega^2 x_0 = -m\omega^2 x_0$$

$$F = -m\omega^2 x_0$$

$$F = -m\omega^2 x_0$$

► The work done by the restoring force for displacement  $x$  is

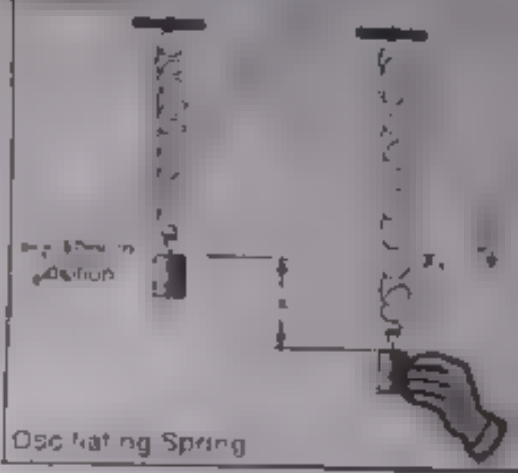
$$W = \int_0^x F dx \quad \text{Putting value of } F$$

$$W = \int_0^x -m\omega^2 x dx$$

$$W = -\frac{1}{2} m\omega^2 x^2$$

► This Work done is stored as elastic P.E. stored in oscillating mass spring system. So

FIGURE

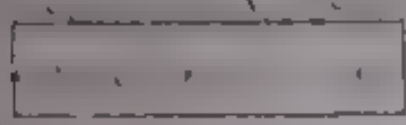


$$PE = \frac{1}{2} kx^2$$

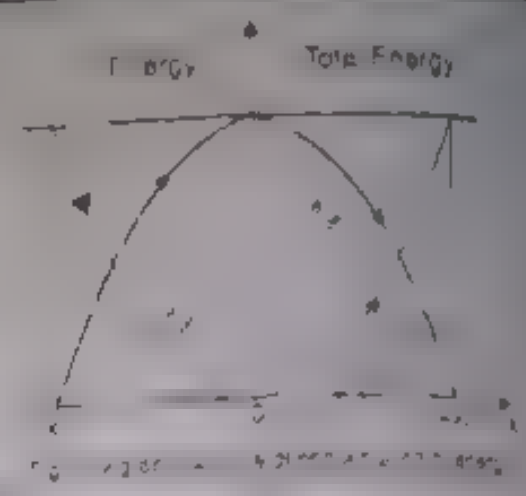
$$K.E = \frac{1}{2} mv^2$$

$$PE = \frac{1}{2} kx^2$$

$$K.E = \frac{1}{2} mv^2$$



Minimum P.E.



### Total Energy in SHM and Energy conservation

► Total Energy = P.E. + K.E. at any position.

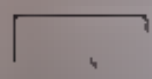
$$K.E = \frac{1}{2} mv^2 \quad (7)$$

and (4) in equation (5)

$$PE = \frac{1}{2} kx^2$$

$$K.E = \frac{1}{2} mv^2$$

$$PE = \frac{1}{2} kx^2$$



Note

At mean position

- 
- 

$$PE = \frac{1}{2} kx^2$$

At extreme position

- 
- 

$$PE = \frac{1}{2} kx^2$$

► At extreme position, the displacement is maximum.

### Activity 7.2:

Determine the period and frequency of a car whose mass is 1400 kg and whose shock absorbers have a spring constant of  $6.5 \times 10^4$  N/m after hitting a bump. Assume the shock absorbers are poor, so the car really oscillates up and down.

Data: Mass of car = 1400 kg, Spring constant =  $6.5 \times 10^4$  N/m

- (a) T
- (b) f

Solution

a) Since  $T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{14.0}{0.5 \times 10^4}} = 0.92s$

b)  $f = \frac{1}{T} = \frac{1}{0.92} = 1.08 \text{ Hz} \approx 1 \text{ Hz}$

IMPORTANT POINTS

| Physical Quantity | Mean Position        | Extreme Position     |
|-------------------|----------------------|----------------------|
| Displacement      | Zero                 | Maximum              |
| Velocity          | Maximum              | Zero                 |
| Acceleration      | Zero                 | Maximum              |
| Momentum          | Maximum              | Zero                 |
| K.E               | Maximum              | Zero                 |
| P.E               | Zero                 | Maximum              |
| Total Energy      | Equal to maximum K-E | Equal to maximum P-E |

MCQ's

- When the amplitude of vibration becomes double its energy becomes
- (A) Double (B) 4 times (C) One half (D) remains same
- Mass attached to spring is pulled slowly from mean position to  $x_0$ . Then the work done will be
- (A)  $\frac{1}{2} kx_0$  (B)  $\frac{1}{2} k(x_0)^2$  (C)  $kx_0$  (D)  $\omega^2 x^2$
- A body is executing the SHM. What fraction of its total energy will be kinetic energy when its displacement from the mean position is half of its amplitude?
- (A)  $\frac{1}{4}$  (B)  $\frac{3}{4}$  (C)  $\frac{3}{4}$  (D)  $\frac{1}{4}$
- The P.E stored by a mass spring system at an extension of 2cm is 10J. Which of the following is the P.E stored by the same system at an extension of 4cm?
- (A) 2J (B) 20J (C) 10J (D) 40J
- Which of the following type of energy is stored in compressed or stretched spring?
- (A) Elastic P.E (B) Gravitational P.E (C) K.E (D) Chemical P.E
- The velocity of a particle having SHM is  $v$  at mean position. If its amplitude is doubled then which of the following will be velocity at mean position?
- (A)  $\frac{v}{2}$  (B)  $3v$  (C)  $2v$  (D)  $4v$
- The total energy of horizontal mass spring system is independent of
- (A) Mass of the body (B) Amplitude (C) Spring constant (D) Nature of material of spring
- A spring of spring constant 10  $\frac{N}{m}$  after loading the amplitude is 2m. Which of the following is the maximum P.E?
- (A) 10 J (B) 20 J (C) 30 J (D) 40 J
- Which of the following is the expression for total energy of a particle executing SHM?
- (A)  $\frac{1}{2} kx$  (B)  $\frac{1}{2} k(x_0^2 - x^2)$  (C)  $\frac{1}{2} kx_0^2$  (D)  $\frac{1}{2} k(x^2 - x_0^2)$

Answers Key

- |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 B | 2 B | 3 C | 4 D | 5 A | 6 C | 7 A | 8 B | 9 C |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|

Q6 What are free and forced oscillations? Also define driven harmonic oscillator

Free Oscillations

A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force

For example

A simple pendulum vibrates freely with its natural frequency that depends only upon the length of the pendulum

## Forced Oscillations

Consider a mass-spring-damper system driven by a sinusoidal force  $F \cos(\omega t)$ .

For example:

- The mass is  $m$ .
- The spring constant is  $k$ .
- The damping coefficient is  $b$ .

Next, we consider the steady-state solution.

- The steady-state solution is of the form  $x(t) = A \cos(\omega t + \phi)$ .

Figure 7.10

The amplitude  $A$  and phase  $\phi$  depend on the driving frequency  $\omega$ . The resonance frequency is  $\omega_0 = \sqrt{k/m}$ .

- ▶ The amplitude  $A$  is maximum when  $\omega = \omega_0$ .
- ▶ The phase  $\phi$  is zero when  $\omega = \omega_0$ .
- ▶ The phase  $\phi$  is  $-\pi/2$  when  $\omega \ll \omega_0$ .

The resonance frequency  $\omega_0$  is the natural frequency of the system.

The amplitude  $A$  is maximum when  $\omega = \omega_0$ .

The phase  $\phi$  is zero when  $\omega = \omega_0$ .

The phase  $\phi$  is  $-\pi/2$  when  $\omega \ll \omega_0$ .

The phase  $\phi$  is  $\pi/2$  when  $\omega \gg \omega_0$ .

- ▶ The amplitude  $A$  is maximum when  $\omega = \omega_0$ .

Figure 7.11 shows the amplitude  $A$  and phase  $\phi$  as a function of the driving frequency  $\omega$ .

Case 1:  $\omega \ll \omega_0$ . The amplitude  $A$  is small and the phase  $\phi$  is  $-\pi/2$ .

Case 2:  $\omega = \omega_0$ . The amplitude  $A$  is maximum and the phase  $\phi$  is zero.

Case 3:  $\omega \gg \omega_0$ . The amplitude  $A$  is small and the phase  $\phi$  is  $\pi/2$ .

natural frequency  
equal to the transmission frequency

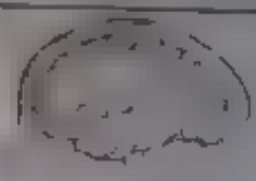


Fig. 7.11

- ▶ ... the ...
- ▶ ... the ...

### Magnetic Resonance Image (MRI)

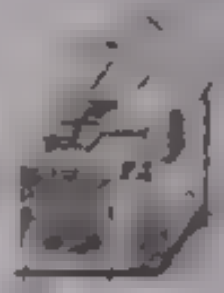
- ▶ ...
- ▶ ...



- ▶ A ... energy is absorbed by the molecules. The ...

### Cooking of Food and Resonance

- ▶ ... of water or fat molecules are used
- ▶ ... contains water molecules is placed in the oven, the water molecules ... absorbing energy ... gets heated up.

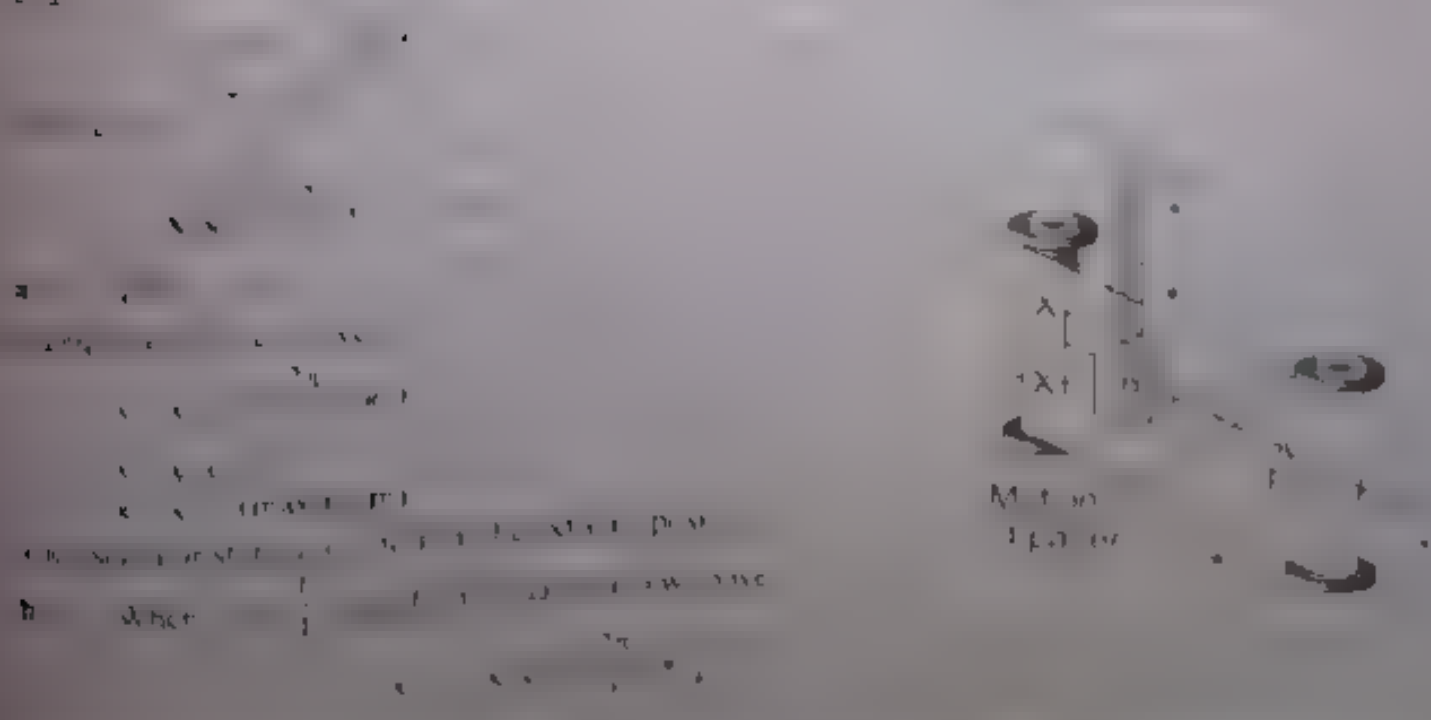
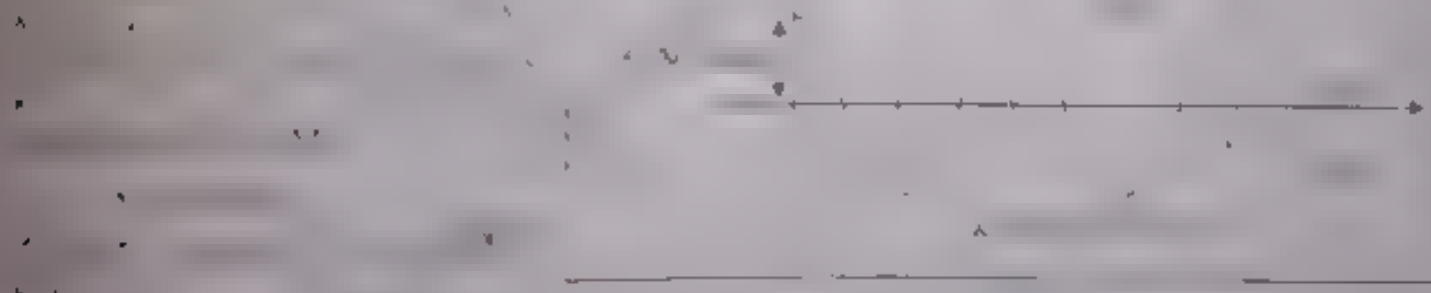


## CB Explain wave form of SHM

### Waveform of simple harmonic motion

#### Wave Form of SHM

... time ( $x-t$ ) graph ... harmonic oscillator is



$$x = x_0 \cos \frac{\pi}{2} = x_0 \cos 90^\circ = x_0 \times 0$$

$$x = 0$$

∴ the particle is passing through the mean position.

c) When  $t = \frac{3T}{4}$ , then from equation (2), we have

$$x = x_0 \cos \frac{2\pi}{T} \times \frac{3}{4}$$

$$x = x_0 \cos \pi = x_0 \cos 180^\circ$$

$$x = -x_0$$

$$x = -x_0$$

The particle is passing through the other extreme position.

d) When  $t = \frac{3T}{4}$ , then from equation (2), we have

$$x = x_0 \cos \frac{2\pi}{T} \times \frac{3}{4}$$

$$x = x_0 \cos \left(\frac{3\pi}{2}\right)$$

$$x = x_0 \cos 270^\circ = x_0 \times 0$$

$$x = 0$$

∴ the particle will be at the mean position.

e) When  $t = \frac{5T}{4}$ , then from equation (2), we have

$$x = x_0 \cos \frac{2\pi}{T} \times \frac{5}{4}$$

$$x = x_0 \cos \frac{5\pi}{2}$$

$$x = 0$$

$$x = x_0 \cos 450^\circ$$

$$x = x_0 \cos 90^\circ = x_0 \times 0$$

$$x = 0$$

$$x = 0$$

$$x = 0$$

$$x = 0$$

$$x = 0$$

$$x = 0$$

$$x = 0$$

$$x = 0$$

$$x = 0$$

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$$x = 0$$

### Experimental Tracing

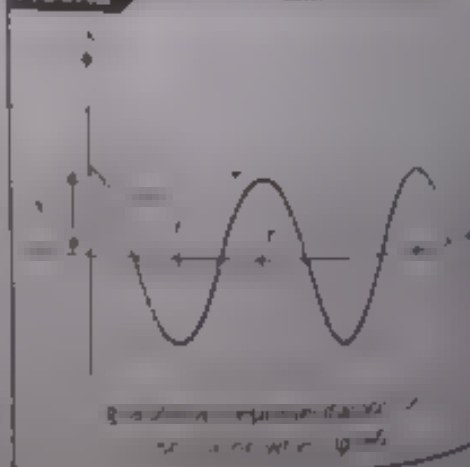
- ▶ A simple pendulum is set into oscillation. The displacement of the bob from the mean position is measured at regular intervals of time. The data is plotted on a graph of displacement versus time. The resulting curve is a sine wave, which represents the oscillatory motion of the pendulum.
- ▶ A spring-mass system is set into oscillation. The displacement of the mass from the mean position is measured at regular intervals of time. The data is plotted on a graph of displacement versus time. The resulting curve is a sine wave, which represents the oscillatory motion of the mass.

### Q 9 What is phase? Explain

#### ANS Phase

The phase of an oscillation is the fraction of one complete cycle that has elapsed since the start of the oscillation. It is denoted by the Greek letter  $\phi$  (phi). The phase is a measure of the displacement of the oscillating body from its mean position at any given time. The phase is a scalar quantity and is measured in radians. The phase is a periodic function of time and is given by  $\phi = \omega t$ , where  $\omega$  is the angular frequency and  $t$  is the time. The phase is a measure of the displacement of the oscillating body from its mean position at any given time. The phase is a scalar quantity and is measured in radians. The phase is a periodic function of time and is given by  $\phi = \omega t$ , where  $\omega$  is the angular frequency and  $t$  is the time.

FIGURE



Where  $\theta = (\omega t + \Phi)$  is the phase ang.<sup>o</sup>

- $\Phi$  is called starting or initial phase of an oscillator
- $\Phi$  also represents the phase difference between the states of motion of two oscillators. Let us explain this by the help of graph drawn between 'x' and 't'.
- If  $\Phi = 0$  then equation (2) become  $x = x_0 \cos \omega t$

Then putting different values of  $t = 0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, T$  we get a graph shown in Fig. 7.13

- If  $\Phi = 90^\circ$ , then from equation (2), we get

$$x = x_0 \cos(\omega t + 90^\circ) \quad (3)$$

Putting different values of  $t = 0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, T$  in equation (3) we get a graph shown in Fig. 7.14

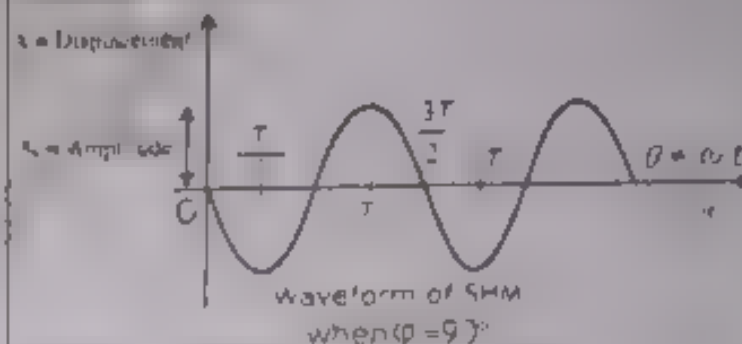
- Comparing curves in Fig. 7.16 and Fig. 7.17 we say that the curve in Fig. 7.16 leads in phase the curve in Fig. 7.17 by  $\lambda$ .
- Similarly if  $\Phi = -\pi$  then from equation (2) we get

$$x = x_0 \cos(\omega t - 180^\circ) \quad (4)$$

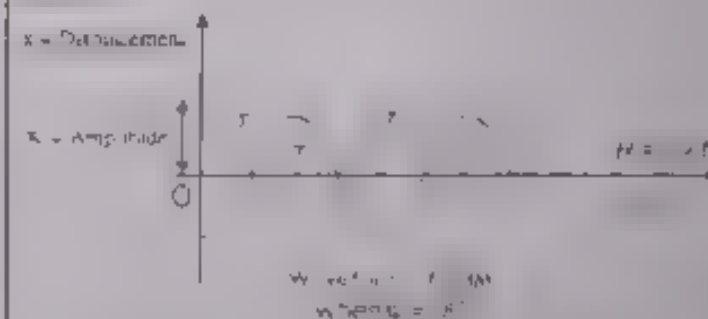
Putting different values of  $t = 0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, T$  in equation (4) we get a graph shown in Fig. 7.15

- Comparing Fig. 7.13 and Fig. 7.15 we see that in Fig. 7.13 the displacement of the oscillation reaches position maximum value  $x_0$  where as in the same time the other oscillation reaches a negative maximum value  $-x_0$ . Thus the two oscillations are said to be out of phase.

FIGURE



FIGURE



#### Assignment 7.4:

An object vibrates with an amplitude of 6cm and a frequency of 0.490 Hz. Starting from maximum displacement in the positive direction, when will be the first time that its displacement is 2cm?



## QUIZ

Consider a graphical representation of simple harmonic motion as described mathematically in Equation 7.22. If the particle is at point A on the graph, what can you say about its position and velocity?

- The position and velocity are both positive
- The position and velocity are both negative
- The position is positive and the velocity is zero
- The position is negative and the velocity is zero
- The position is positive and the velocity is negative
- The position is negative and the velocity is positive

An  $x-t$  graph for a particle undergoing simple harmonic motion. At a particular time, the particle's position is indicated by A on the graph.



### Ans. Conclusion about Position of Point "A"

It is clear from the figure that point A is in the negative x region. Therefore, the position of point A is negative.

### Conclusion about Velocity of Point "A"

The velocity of the particle is equal to the slope of the graph at any point. So we have

$$v = \frac{\Delta x}{\Delta t}$$

In Equation (1), " $\Delta x$ " represents the change in position and " $\Delta t$ " represents the time interval.

► Now at point A, the slope of the graph is positive. Therefore, the velocity at point A will be also positive.

► Thus option (f) is correct. As the position is negative and velocity is positive.

## MCQ's

- A quantity which indicates the state and direction of motion of a vibrating body is known as  
 (A) Time (B) Amplitude (C) Phase (D) Frequency
- The phase angle  $\theta = \omega t$  of a body performing the SHM indicates  
 (A) the magnitude of displacement (B) the magnitude of its velocity  
 (C) the magnitude of its acceleration (D) none
- If mass of the pendulum becomes double, its time period will  
 (A) be doubled (B) be halved (C) be the same (D) be four times

## Answers Key

1 C 2 C 3 D

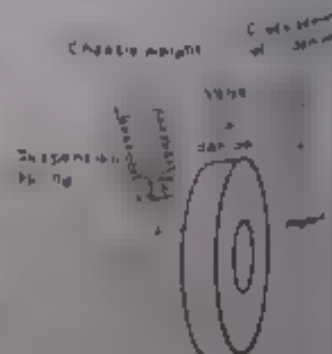
## Q 10 What are damped and un-damped oscillations? What is damping?

### Ans. Damped Oscillations

Oscillations in which amplitude decreases with time due to energy dissipation are called damped oscillations.

#### Explanation

- The amplitude of the oscillating body gradually becomes smaller and smaller because of friction and air resistance.
- As the energy of the oscillator is used up in doing work against the resistive forces, that is why the amplitude decreases with time till it becomes zero as shown in Figure 7.10.



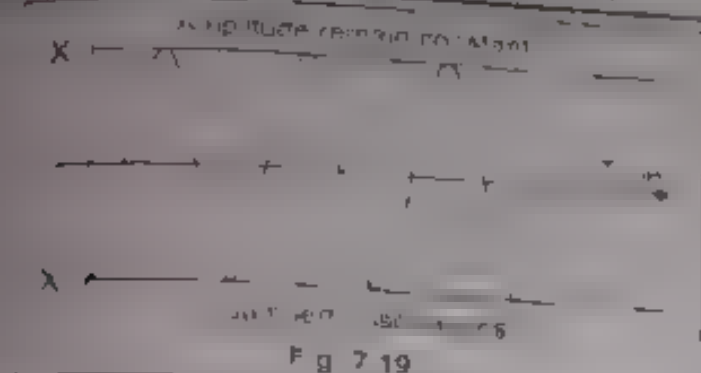
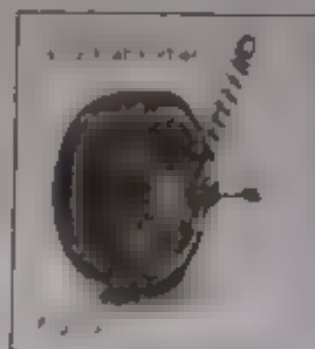


Fig 7.19



### Application

An application of damped oscillation is shock absorber. It absorbs the energy of the oscillation and stops the excessive oscillations.

### Damping

Damping is the process by which the energy of the oscillating system is dissipated.

### Un-damped oscillations

Oscillations in which the amplitude remains same with time are called un-damped oscillations.

In un-damped oscillation, the energy is not dissipated from the oscillating system as shown in Fig 7.20.

### Example

Oscillation of an ideal simple pendulum is the example of un-damped oscillation.



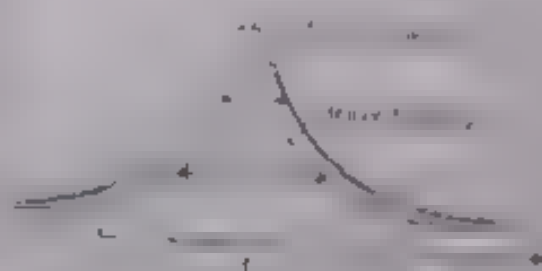
Fig 7.20

Q.11 What is the effect of damping on the sharpness of resonance?

### Sharpness of Resonance

- ▶ The amplitude of oscillation increases with the damping.
- ▶ Thus the presence of damping prevents the resonance from becoming very sharp.
- ▶ The amplitude decreases rapidly as the frequency of the driving force deviates from the resonant frequency.
- ▶ The amplitude as well as sharpness depends upon damping.
- ▶ A heavily damped system has fairly flat resonance curve.

Fig 7.22



### Example to see the effect of damping

- Attach a pendulum having very light mass such as pith ball and another of same length with a heavy mass of equal size such as lead ball.
- Set them in vibrations by third pendulum of equal length and attached to the same end.
- It is observed that the amplitude of the heavy ball is much greater than the light ball.
- So the sharpness of the resonance curve of resonating system depends on energy loss due to friction.



Pith ball Lead ball Lead ball

## MCQs

- The frequency of waves produced in microwave oven is  
 (A) 2250 MHz (B) 2400 MHz (C) 2550 MHz (D) 2650 MHz
- The wavelength of waves produced by microwave oven is  
 (A) 0.12 cm (B) 1 cm (C) 6 cm (D) 12 cm
- Turning a radio is the best example of \_\_\_\_\_.  
 (A) Mechanical resonance (B) Electrical resonance (C) Magnetic resonance (D) Acoustic resonance
- The process by which the energy is dissipated from an oscillating system is called \_\_\_\_\_.  
 (A) Resonance (B) Damping (C) Forced vibrations (D) None of these
- Oscillation of shock absorber of a car is a practical example of \_\_\_\_\_.  
 (A) Simple harmonic motion (B) Forced oscillation (C) Damped oscillation (D) None of these

## Answers Key

|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| 1 B | 2 D | 3 B | 4 D | 5 C |
|-----|-----|-----|-----|-----|

## FORMULAE

|                                                                |                                         |                   |
|----------------------------------------------------------------|-----------------------------------------|-------------------|
| Hook's law                                                     | $F = -kx$                               |                   |
| Restoring force                                                | $F = -kx$                               |                   |
| Acceleration of mass-spring system                             | $a = -\frac{k}{m}x$                     |                   |
| Angular frequency                                              | $\omega = \sqrt{\frac{k}{m}}$           | $2\pi$            |
| Instantaneous displacement of body executing SHM               | $x = A \sin(\omega t + \phi)$           |                   |
| Instantaneous velocity of body executing SHM                   | $v = \omega \sqrt{A^2 - x^2}$           |                   |
| Instantaneous acceleration of body executing SHM               | $a = -\omega^2 x$                       |                   |
| Time period and frequency of body executing SHM                | $T = \frac{2\pi}{\omega}$               | $f = \frac{1}{T}$ |
| Value of $\omega$ for mass-spring system                       | $\omega = \sqrt{\frac{k}{m}}$           |                   |
| Time period of mass-spring system executing SHM                | $T = 2\pi \sqrt{\frac{m}{k}}$           |                   |
| Frequency of mass-spring system executing SHM                  | $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ |                   |
| Instantaneous displacement of mass-spring system executing SHM | $x = A \sin(\omega t + \phi)$           |                   |
| Instantaneous velocity of mass-spring system executing SHM     | $v = \omega \sqrt{A^2 - x^2}$           |                   |
| Maximum velocity of mass-spring system executing SHM           | $v_{\max} = \omega A$                   |                   |

|                                                                                         |                                              |  |
|-----------------------------------------------------------------------------------------|----------------------------------------------|--|
| Instantaneous velocity of mass spring system in terms of maximum velocity executing SHM | $v = v_m \sqrt{1 - \frac{x^2}{A^2}}$         |  |
| Restoring force for simple pendulum                                                     | $F = -mg \sin \theta$                        |  |
| Acceleration of simple pendulum                                                         | $a = -\frac{g}{l} x$                         |  |
| Value of $\omega$ for simple pendulum                                                   | $\omega = \sqrt{\frac{g}{l}}$                |  |
| Time period of simple pendulum                                                          | $T = 2\pi \sqrt{\frac{l}{g}}$                |  |
| Frequency of simple pendulum                                                            | $f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$      |  |
| Instantaneous P.E. of mass spring system                                                | $P.E. = U = \frac{1}{2} kx^2$                |  |
| Maximum P.E. of mass spring system                                                      | $P.E._{max} = \frac{1}{2} kA^2$              |  |
| Instantaneous K.E. of mass spring system                                                | $K.E. = \frac{1}{2} kx^2 - \frac{1}{2} kx^2$ |  |
| Maximum K.E. of mass spring system                                                      | $(K.E.)_{max} = \frac{1}{2} kA^2$            |  |
| Total energy of mass spring system                                                      | $E = \frac{1}{2} kA^2$                       |  |

- ◆ The to and fro motion of a body about its mean position is called oscillation.
- ◆ Amplitude is the maximum displacement of particles from their normal position.
- ◆ Damping is the dissipation of energy during oscillation, which prevents an object from continuing in simple harmonic motion and will eventually force it to stop oscillating altogether. Damping is usually caused by friction.
- ◆ The angle ( $\theta = \frac{x}{A}$ ) which specifies the displacement  $x$  as well as the direction of motion of SHM is called phase.
- ◆ For a particle experiencing oscillation, frequency is the number of cycles that take place during one second. Frequency is measured in hertz.
- ◆ The repeated movement of a particle about a position of equilibrium is called Harmonic Motion.
- ◆ hertz is unit of frequency. It may be expressed in units of cycle/s.
- ◆ Period is the amount of time required for one cycle in oscillating motion.
- ◆ Motion that is repeated at regular intervals is called periodic motion.
- ◆ Resonance is the vibrations of a body under the action of a force having frequency equal to the natural frequency of body.



## Solved Examples

## Example 7.1

A mass of 0.5 kg is suspended from a spring. The spring is stretched by 0.098 m. Calculate the spring constant of the mass when it is given a small displacement.

**Solution:**

$$\text{Mass} = 0.5 \text{ kg}$$

$$\text{Force} = \text{weight} = mg = 0.5 \times 9.8 = 4.9 \text{ N}$$

$$\text{Extension} = x = 0.098 \text{ m}$$

$$\text{In case of spring: } k = \frac{F}{x}$$

$$k = \frac{4.9}{0.098} = 50 \text{ N/m}$$

## Example 7.2

A spider swings in the breeze from a silk thread with a period of 0.6 s. How long is the spider's strand of silk?

**Given:**

$$T = 0.6 \text{ s}$$

$$\text{Gravitational acceleration, } g = 9.8 \text{ m/s}^2$$

**Required:**

$$\text{length } l = ?$$

**Solution:**

$$T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2 \frac{l}{g}$$

$$l = \frac{gT^2}{4\pi^2}$$

$$= \frac{9.8 \times (0.6)^2}{4 \times 3.14^2}$$

$$= 0.17 \text{ m}$$

## Example 7.3

A mass at the end of a spring vibrates up and down with a frequency of 0.600 Hz and an amplitude of 5 cm. What is its displacement 2.56 s after it starts at a maximum?

**Given:**

$$A = 5 \text{ cm}$$

$$\text{Frequency, } f = 0.600 \text{ Hz}$$

**Required:**

$$\text{displacement } x = ?$$

**Solution:**

$$x = A \cos 2\pi ft$$

$$= (5 \text{ cm}) \cos 2\pi (0.6)(2.56)$$

$$= 1.8 \text{ cm}$$



## Text Book Exercises

Q 1 Select the correct answer of the following questions.

- Tuning of a radio set is an example of  
 (a) Mechanical resonance (b) Magnetic resonance (c) Electrical resonance (d) None of these
- The heating and cooking of food evenly by microwave oven is an example of  
 (a) S.H.M (b) Resonance (c) Damped Oscillation (d) None of these
- The time period of the same pendulum at Karachi and Murree are related as (CBSE-2019)  
 (a)  $T_1 = T_2$  (b)  $T_1 > T_2$  (c)  $T_1 < T_2$  (d)  $T_1 = 2T_2$
- In an isolated system the total energy of vibrating mass and spring is  
 (a) Variable (b) Low (c) High (d) constant

- 4) While deriving the equation of time period for simple pendulum which quantity should be kept small  
 a) Length of simple pendulum  
 b) Amplitude  
 c) Mass of simple pendulum  
 d) Gravitational acceleration  $g$
- 5) If the period of oscillation of mass  $M$  suspended from a spring is  $2s$ , then the period of mass  $4M$  will be (FIIT JEE (ON) 2019)  
 a)  $s$   
 b)  $2s$   
 c)  $3s$   
 d)  $4s$
- 6) The time period of a simple pendulum is 2 seconds. If its length is increased by 4 times, then its period becomes  
 a)  $6s$   
 b)  $2s$   
 c)  $8s$   
 d)  $4s$
- 7) To make the frequency double of a spring oscillation, we have to  
 a) Reduce the mass to one fourth  
 b) Quadruple the mass  
 c) Double the mass  
 d) Half the mass
- 8) The restoring force of SHM is maximum when particle  
 a) Displacement is maximum  
 b) Half way between  $-A$  and  $A$   
 c) Crosses the mean position  
 d) At rest
- 9) Two springs of spring constants  $k_1$  and  $k_2$  are joined in series. The effective spring constant of the combination is given by  
 a)  $k_1 + k_2$   
 b)  $k_1 k_2 / (k_1 + k_2)$   
 c)  $\sqrt{k_1 k_2}$   
 d)  $\sqrt{k_1 + k_2}$

| No. | Option | ANSWER                    | EXPLANATION                                                                                                                                                                                                                                                                                                      |
|-----|--------|---------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1   | a      | Length of simple pendulum |                                                                                                                                                                                                                                                                                                                  |
| 2   | b      | $2s$                      |                                                                                                                                                                                                                                                                                                                  |
| 3   | c      | $4s$                      | $T = 2\pi \sqrt{\frac{L}{g}}$ $T \propto \sqrt{L}$ $\frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}}$ $\frac{4s}{2s} = \sqrt{\frac{L_2}{L_1}}$ $2 = \sqrt{\frac{L_2}{L_1}}$ $4 = \frac{L_2}{L_1}$ $L_2 = 4L_1$                                                                                                           |
| 4   | a      | Displacement is maximum   |                                                                                                                                                                                                                                                                                                                  |
| 5   | b      | Amplitude                 | When amplitude is small then restoring force is smaller and time period will be more or less same.                                                                                                                                                                                                               |
| 6   | d      | $4s$                      | $T = 2\pi \sqrt{\frac{M}{k}} \quad \text{--- (1)}$ $T = 2\pi \sqrt{\frac{4M}{k}} \quad \text{--- (2)}$ $\text{Putting } M = 4M$ $T = 2\pi \sqrt{\frac{4M}{k}}$ $T = 2 \left[ 2\pi \sqrt{\frac{M}{k}} \right] \quad \text{--- (3)}$ $\text{Putting value from equation (1) in (2)}$ $T = 2T$ $T = 2(2s)$ $T = 4s$ |
| 7   | d      | $4s$                      | $T = 2\pi \sqrt{\frac{L}{g}} \quad \text{--- (1)}$ $T = 2\pi \sqrt{\frac{4L}{g}} \quad \text{--- (2)}$ $\text{Putting } g = 4g$                                                                                                                                                                                  |

|    |   |                                   |                                                                                                                                                                                                                                                                                                                        |
|----|---|-----------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|    |   |                                   | $I = \frac{2}{\pi} \sqrt{\frac{eE}{R}}$ $I = \frac{2}{\pi} \frac{1}{2R} \sqrt{\frac{eE}{R}} \quad \text{--- (2)}$ <p>Putting value from eq. (1) in (2)</p> $I = \frac{2}{\pi} \sqrt{\frac{eE}{R}}$                                                                                                                     |
| 8  | 1 | Resonance frequency and bandwidth | $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad \text{--- (1)}$ $\Delta f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad \text{--- (2)}$ <p>Putting value from eq. (1) in (2)</p> $\Delta f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ $\Delta f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ $\Delta f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ |
| 9  | 1 | Resonance frequency and bandwidth | $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ $\Delta f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$                                                                                                                                                                                                                                 |
| 10 | 1 | Resonance frequency and bandwidth | $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ $\Delta f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$                                                                                                                                                                                                                                 |



## Short Answers of the Exercise

Q 2 Write short answers of the following questions

Q 1 Give two applications in which resonance plays an important role

Ans (1) Tuning a radio (Electrical resonance)

- Tuning of radio is a good example of electrical resonance. To tune a radio
- We turn the knob of a radio
- It changes the natural frequency of electrical circuit of receiver until it becomes equal to the frequency of transmitter
- Now the resonance is produced and energy absorption is maximum
- The radio begins to receive that waves which produce resonance in it
- Hence a station is tuned



(2) Cooking by microwave oven

- Resonance plays an important role in heating and cooking food by microwave oven.
- The microwaves produced by oven are absorbed due to resonance by water and fat molecules. This food gets heated.
- Thus it is important to understand resonance.



Q2 What happens to the time period of a simple pendulum if its length is doubled?

Ans: The time period of a simple pendulum is given by

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Explanation

► When length is  $L$ , time period is  $T$

► Now when length is  $2L$ , time period is  $T'$

$$T = 2\pi \sqrt{\frac{L}{g}} \quad \text{and} \quad T' = 2\pi \sqrt{\frac{2L}{g}}$$

$$\frac{T'}{T} = \sqrt{\frac{2L}{L}} = \sqrt{2}$$

$$T' = \sqrt{2} T = 1.414 T$$

∴ Time period becomes 1.414 times.

$$T' = 1.414 T$$

► ∴ Time period becomes 1.414 times.

Q3 What will be the frequency of a simple pendulum if its length is 1 m

Ans: The frequency of a simple pendulum is given by

$$f = \frac{1}{T} = \frac{1}{2\pi \sqrt{\frac{L}{g}}}$$

$$f = \frac{1}{2\pi \sqrt{\frac{1}{9.8}}}$$

$$f = 0.5 \text{ Hz}$$

$$f = 0.5 \text{ Hz}$$

∴ Frequency is 0.5 Hz.

Q4 Give one practical example each of free and forced oscillation

Ans: Free Oscillations

A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force.

For example

- A simple pendulum vibrates freely with its natural frequency that depends only on the length of the pendulum.

### Forced Oscillations

A body is said to be executing forced vibrations if it oscillates with the interference of an external force.

#### For example

- ▶ A mass of a vibrating pendulum system.
- ▶ A body of mass of factory floor.

**Q 5** How can you compare the masses of two bodies by observing their frequencies of oscillation when suspended by a spring.

**Ans** Frequency of mass spring system is

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Let  $f_1$  and  $f_2$  be the frequencies of masses  $m_1$  and  $m_2$  respectively.

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{k}{m_1}}$$

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{k}{m_2}}$$

Dividing eq. (1) by (2)

$$\frac{f_1}{f_2} = \frac{\frac{1}{2\pi} \sqrt{\frac{k}{m_1}}}{\frac{1}{2\pi} \sqrt{\frac{k}{m_2}}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{m_2}{m_1}}$$

Squaring both sides

$$\frac{f_1^2}{f_2^2} = \frac{m_2}{m_1}$$

From eq. (3) masses of two bodies can be compared by observing their frequencies of oscillation and  $f_1$  and  $f_2$  are compared.

**Q 6** A wire hangs from the top of dark high tower so that the top of tower is not visible. How would be able to determine the height of that tower?

**Ans** In order to find the height of tower we have to determine the value of  $g$ .

- ▶ First we attach a bob to the tower and find the value of  $g$ .
- ▶ Now find the time period of simple pendulum as we know value of  $g$  we can find the time period of the bob is

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T = 4\pi \sqrt{\frac{l}{g}}$$

$$l = \frac{gT^2}{4\pi^2}$$

Putting the values of  $T$  and  $g$ ,  $l$  can be calculated and hence height of tower can be determined.

Why in S.H.M the acceleration is zero when the velocity is greatest

Acceleration is zero when the velocity is greatest

The velocity is maximum at a point where the displacement is zero

$$v = \omega \sqrt{A^2 - x^2}$$

$$v = \omega \sqrt{A^2 - x^2}$$

At mean position  $x = 0$

►  $v = \omega \sqrt{A^2 - 0} = \omega A$

$$v = \omega \sqrt{A^2 - x^2}$$

►  $v = \omega \sqrt{A^2 - x^2}$

$$v = \omega \sqrt{A^2 - x^2}$$

► In S.H.M at mean position the acceleration is zero but velocity is maximum

What is the total distance traveled by a simple harmonic oscillator in a time equal to its period. The of amplitude is A?

$$v = \omega \sqrt{A^2 - x^2}$$

Explanation

- In one complete cycle, the oscillator travels a distance of  $4A$ .
- In one complete cycle, the oscillator travels a distance of  $4A$ .
- In one complete cycle, the oscillator travels a distance of  $4A$ .



What happens to the frequency of a simple pendulum as its oscillations die down from large amplitude to small?

- The frequency of a simple pendulum is independent of its amplitude.
- The frequency of a simple pendulum is independent of its amplitude.
- For larger amplitude, the frequency is slightly less than the above value or predicted.

A singer holding a note of right frequency, can shatter a glass. Explain

- The frequency of the note is equal to the natural frequency of the glass.
- When a singer sings a note of the same frequency as the natural frequency of a glass, the frequencies match in resonance and the glass vibrates.
- The resonance amplitude of vibration increases, the glass vibrates with maximum amplitude and the glass can be shattered.
- To break the glass, we need to broadcast not only a sound that is just the right frequency, but also has a high enough amplitude.



## Comprehensive Questions

Q1 Give a short response to the following questions

1 Show that motion of a mass attached with a spring executes S.H.M

**Sol:** See problem 1

2 Prove that the projection of a body motion in a circle describes S.H.M

**Sol:** See problem 2

3 Show that energy is conserved in case of S.H.M

**Sol:** See problem 3

4 Differentiate free and forced oscillations

**Sol:** See problem 4

5 What is resonance give three of its applications in our daily life

**Sol:** See problem 5

6 Derive equations for kinetic and potential energy of a body of mass  $m$  executing S.H.M

**Sol:** See problem 6

7 Explain what is meant by damped oscillations

**Sol:** See problem 7



## Numerical Problems

1 A force of  $0.4\text{ N}$  is required to displace a body attached to a spring through  $0.1\text{ m}$  from its mean position. Calculate the spring constant of spring

**Data:** Force  $F = 0.4\text{ N}$   
Displacement  $x = 0.1\text{ m}$

**To Find:** Spring constant  $k$

**Solution:**

$$F = kx$$

$$k = \frac{F}{x} = \frac{0.4}{0.1} = 4\text{ N/m}$$

2 A pendulum clock keeps perfect time at a location where the acceleration due to gravity is exactly  $9.8\text{ m/s}^2$ . When the clock is moved to higher altitude, it loses  $80\text{ s}$  per day. Find the value of  $g$  at new location

**Data:** Acceleration due to gravity  $g = 9.8\text{ m/s}^2$   
Time lost per day  $t = 80\text{ s}$   
No. of seconds in one day  $24 \times 60 \times 60 = 86400\text{ s}$   
New value of acceleration due to gravity  $g'$

**Solution:** Let the new period be  $T'$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Since clock is losing  $80\text{ s}$  per day, it is slow, i.e.,  $T' > T$ . Thus after one day, since the new period is  $T' = T + 80\text{ s} = 86480\text{ s} = 24 \times 60 \times 60 + 80\text{ s}$

$$\text{The new period is } T' = 2\pi \sqrt{\frac{l}{g'}}$$

Ther

$$\frac{\partial \mathcal{L}}{\partial \mathbf{y}} = \mathbf{y}$$

**Figure 1**

1988 978 1 85196 118 8

Time per rd = 1.5 s.

1987

2

1.  $\pi_1$

Squaring the circle is also

1. 1. 1.

4.  $\frac{1}{2}$

96

74

Find out  $f, f_2$

1993

$$\forall x \in \mathbb{N} \quad \exists y \in \mathbb{N} \quad x + y = 1$$

regard to the

Displacement + complete overlap = 1.0

1.  $f(x) = x^2 + 2x + 1$

Displacement + spring =  $\lambda$ 

f f



Frequency of bus spring system =

$$1 - \frac{1}{24} \sqrt{\frac{N}{n}}$$

$P = 10^{-3} K$

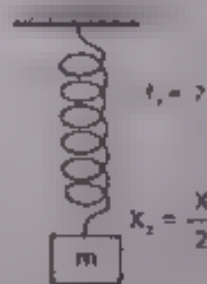
$$I = \frac{1}{\pi} \sqrt{\frac{2M}{1 + 2M}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

Frequency of complete spring  $s$ 

$$r = \frac{1}{2\pi} \sqrt{\frac{2}{\lambda}}$$

When spring is cut into two equal parts and same mass is suspended with both spring, then



Frequency of half spring is

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{k}{\lambda}}$$

Putting  $\lambda = \frac{\lambda_1}{2}$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{\frac{\lambda_1}{2}}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{2k}{\lambda_1}}$$

$$f = \sqrt{2} \left[ \frac{1}{2\pi} \sqrt{\frac{k}{\lambda_1}} \right] \quad \dots (2)$$

Putting value eq (1) in (2)

$$f = \sqrt{2} \cdot f_2$$

$$\frac{f_1}{2} = \frac{1}{\sqrt{2}}$$

$$\left[ \frac{f_1}{f} \right] = 0.707$$

### A ternate method

Frequency of half spring

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{\lambda}} \quad (1)$$

If the length of spring is cut in half, then the spring constant becomes  $\lambda = \frac{\lambda_1}{2}$

So  $k = F/\lambda$

$$k = \frac{F}{\lambda_1/2}$$

$$k' = 2k \quad \text{or } k = \frac{k'}{2}$$

So its frequency

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{k'}{m}}$$

using eq (1)

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{k'}{m}}$$

(2)

Divide equation (1) by (2)

$$\frac{f_1}{f_2} = \frac{1}{\sqrt{2}}$$

$$\frac{f_1}{f_2} = \frac{1}{\sqrt{2}}$$

$$\frac{f_1}{f_2} = \frac{1}{\sqrt{2}} \quad \dots (3)$$

5. A mass at the end of spring describes S.H.M with  $T = 0.40 \text{ s}$ . Find out  $a$  when the displacement is  $0.04 \text{ m}$ .

Date Displacement =  $x = 0.04 \text{ m}$

Time period =  $T = 0.40 \text{ s}$

To Find Acceleration =  $a = ?$

Solution

$$a = -\omega^2 x$$

Putting  $\omega = \frac{2\pi}{T}$

$$a = -\left(\frac{2\pi}{T}\right)^2 x$$

$$B = -\frac{1}{T} \times$$

$$B = -\frac{4 \times \frac{1}{2} \times 0.61}{(1.4)}$$

$$B = 0.86 \text{ m/s}$$

A block weighing 4.0 kg extends a spring by 0.16 m from its un stretched position. The block is removed and a 0.50 kg body is hung from same spring. If the spring is now stretched and then released. What is its period of vibration?

Data mass of block  $m = 4 \text{ kg}$

Extension  $x = 0.16 \text{ m}$

To Find Spring constant  $k = ?$

Data mass of body  $m = 0.50 \text{ kg}$

To Find time period  $T = ?$

Solution:

$$a) F = kx$$

$$k = \frac{F}{x}$$

Putting  $F = W = mg$

$$k = \frac{mg}{x}$$

$$k = \frac{4 \times 9.8}{0.16}$$

$$k = 245 \text{ N/m}$$

b) Time period  $T = ?$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$T = 2 \times 3.14 \sqrt{\frac{0.5}{245}}$$

$$T = 0.63 \text{ s}$$

What should be the length of simple pendulum whose time period is one second? What is its frequency?

Time period  $T = 1 \text{ s}$

$g = 9.8 \text{ m/s}^2$

Length  $l = ?$

Frequency  $f = ?$

Solution:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Squaring both sides

$$T^2 = \frac{4\pi^2 l}{g}$$

$$l = \frac{g T^2}{4\pi^2}$$

$$l = \frac{9.8 \times (1)^2}{4 \times (3.14)^2}$$

$$l = 0.25 \text{ m}$$

$$f = \frac{1}{T} = \frac{1}{1} = 1 \text{ Hz}$$

- 8 A spring, whose spring constant is  $80.0 \text{ Nm}^{-1}$  vertically supports a mass of  $1.0 \text{ kg}$  is at rest position. Find the distance by which the mass must be pulled down so that on being released, it may pass the mean position with velocity of one meter per second.

**Data** Spring constant  $k = 80 \text{ N/m}$   
 Mass  $m = 1 \text{ kg}$   
 Max. velocity  $v = 1 \text{ m/s}$   
**To Find** Max. displacement  $x$   
**Solution**

$$v = x \omega$$

$$1 = x \sqrt{\frac{k}{m}}$$

$$1 = x \sqrt{\frac{80}{1}}$$

$$x = \frac{1}{\sqrt{80}}$$

$$x = 0.1118 \text{ m}$$

- 9 A  $8000 \text{ g}$  body vibrates S.H.M with amplitude  $0.30 \text{ m}$ . The restoring force is  $60 \text{ N}$  and the displacement is  $0.12 \text{ m}$ . Find out (i)  $T$ , (ii)  $a$ , (iii)  $v$ , (iv)  $KE$  vs  $PE$  when the displacement is  $12 \text{ cm}$ .

**Given Data:** Mass of body  $m = 8 \text{ kg}$   
 Amplitude  $x = 0.30 \text{ m}$   
 Restoring force  $F = 60 \text{ N}$   
 Displacement  $x = 0.12 \text{ m}$

**To Find**

- Period
- Acceleration
- Speed
- $KE$
- $PE$

When the displacement is  $0.12 \text{ m}$

**Calculation**

(i) Time period

$$\text{According to Hook's law } F = kx$$

$$60 = k \times 0.12$$

$$k = \frac{60}{0.12}$$

$$k = 500 \text{ N/m}$$

$$\therefore k = 500 \text{ N/m}$$

Now using the formula for time period of mass-spring system

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Putting values we get

$$T = 2\pi \sqrt{\frac{8}{500}}$$

$$T = 0.568 \times \sqrt{0.016}$$

$$T = 0.568 \times 0.126$$

$$T = 0.0717 \text{ s}$$

$$T = 1.3 \text{ s}$$

$$\therefore \text{According to } \omega = \frac{2\pi}{T}$$

$$F = m\omega^2 x$$

$$60 = 8 \times \left(\frac{2\pi}{T}\right)^2 \times 0.12$$

$$60 = \frac{4 \times 4 \times \pi^2 \times 0.12}{T^2}$$

$$60 = \frac{1.2 \times \pi^2}{T^2}$$

$$T = 1.3 \text{ s}$$

## 2. Acceleration.

$$a = -\omega^2 x \quad \text{As } \omega = \sqrt{\frac{k}{m}}$$

$$a = -\frac{k}{m} x$$

Putting values we get

$$a = -\frac{200}{8} \times 0.12$$

$$a = -3 \text{ ms}^{-2}$$

∴ Negative sign shows that acceleration is directed towards the mean position.

## ii) Speed

The speed of the body executing SHM is given by

$$v = \omega \sqrt{x^2 - x_0^2}$$

$$v = \sqrt{\frac{k}{m}} \sqrt{x^2 - x_0^2} \quad \text{As } \omega = \sqrt{\frac{k}{m}}$$

Putting values we get

$$v = \sqrt{\frac{200}{8}} \sqrt{(0.12)^2 - (0.08)^2}$$

$$v = 5 \sqrt{0.008}$$

$$v = 5 \sqrt{0.008}$$

$$v = 1.414 \text{ m/s}$$

$$\frac{1}{4} \times 5.6568 \times 10^{-2} \text{ m/s}$$

## iii) K.E

Using the formula for kinetic energy

$$K.E = \frac{1}{2} mv^2$$

Putting values we get

$$K.E = \frac{1}{2} \times 8 \times (1.414)^2$$

$$K.E = 4 \times 1.80$$

$$K.E = 7.2 \text{ J}$$

$$K.E = 7.2 \text{ J}$$

## iv) P.E

The formula for P.E is

$$P.E = \frac{1}{2} kx^2$$

$$P.E = \frac{1}{2} \times 200 \times (0.12)^2$$

$$P.E = 1.80 \times 0.0144$$

$$P.E = 0.02592 \text{ J}$$

- 10 Find the amplitude, frequency and time period of an object oscillating at the end of a spring, if the equation for its position at any instant  $t$  is given by  $x = 0.25 \cos \frac{\pi}{8} t$ . Find the displacement of the object after 2.0 s.

Given data  $x = 0.25 \cos \frac{\pi}{8} t$

To Find  
Amplitude =  $x$

To Find  
Frequency =  $f$   
Time period =  $T$

Calculation

Amplitude  $x = 0.25 \cos \frac{\pi}{8} t$

$$x = 0.25 \cos \frac{\pi}{8} t$$

Amplitude =  $0.25 \cos \frac{\pi}{8} t$

$$f = \frac{1}{T} = \frac{1}{2\pi} \times \frac{\pi}{8} = \frac{1}{16} \text{ Hz}$$

Time period  $T = \frac{1}{f} = 16 \text{ s}$

Frequency  $f = \frac{1}{T} = \frac{1}{16} \text{ Hz}$

$$T = \frac{1}{f} = \frac{1}{\frac{1}{16}} = 16 \text{ s}$$

Time period

$T = 16 \text{ s}$

Amplitude =  $0.25 \cos \frac{\pi}{8} t$

$$x = 0.25 \cos \frac{\pi}{8} t$$

Amplitude =  $0.25 \cos \frac{\pi}{8} t$

$$x = 0.25 \cos \frac{\pi}{8} t$$

$$x = 0.25 \cos \frac{\pi}{8} t$$

$$x = 0.25 \cos \frac{\pi}{8} t$$

Amplitude =  $0.25 \cos \frac{\pi}{8} t$



## Additional Conceptual Short Questions With Answers

1. If a pendulum clock keeps perfect time at the base of mountain, will it keep perfect time when moved to the top of the mountain?

Ans:

$$T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T \propto \frac{1}{\sqrt{g}}$$

2. If  $x_0$  is amplitude of vibration then at which displacement K.E. and P.E. will be equal in SHM?

Ans:

$$K.E. = P.E.$$

$$\frac{1}{2} kx^2 = \frac{1}{2} kx_0^2$$

$$kx^2 = kx_0^2$$

$$x^2 = x_0^2$$

$$x = x_0$$

3. If the time period of a pendulum is 2 second on the earth surface. Then what is its time period at the center of earth? What is its frequency?

Ans:

Time period of a pendulum is

$$T = 2\pi \sqrt{\frac{l}{g}}$$

At the center of earth

$$g = 0$$

So

$$T = \infty$$

Time period will be infinite

We know that

$$f = \frac{1}{T}$$

Frequency is 0 Hz

4. If pendulum of time period  $T$  is taken from earth to moon, then what will be its time period on moon?

**Ans.** A second pendulum on the earth has time period is 2 second. Since value of  $g$  on moon is 6 times less than its value on earth. And

$$T = 2\pi \sqrt{\frac{l}{g}}$$

• Time period of a simple pendulum on moon is calculated as

$$T' = 2\pi \sqrt{\frac{l}{g_m}}$$

where

$$g_m = \frac{g}{6}$$

$$T' = 2\pi \sqrt{\frac{l}{\frac{g}{6}}} = 2\pi \sqrt{\frac{6l}{g}} = \sqrt{6} \cdot 2\pi \sqrt{\frac{l}{g}} = \sqrt{6} \cdot T$$

∴ Time period of pendulum on moon is  $\sqrt{6}$  times of its time period on earth.

$$T' = \sqrt{6} T$$

5. A simple harmonic motion has an amplitude  $A$  and time period  $T$ . Then how much time will take to travel from  $x = A$  to  $x = \frac{A}{2}$ ?

**Ans.** Let the displacement of a particle executing S.H.M. is

$$x = A \cos \omega t$$

$$x = A \cos \omega t$$

where

$$x = A \text{ amplitude}$$

$$x = \frac{A}{2}$$

$$x = A \cos \omega t$$

$$\frac{A}{2} = A \cos \omega t$$

$$\frac{1}{2} = \cos \omega t$$

$$\cos \omega t = \frac{1}{2}$$

$$\omega t = \cos^{-1} \left( \frac{1}{2} \right)$$

$$t = \frac{\cos^{-1} \left( \frac{1}{2} \right)}{\omega}$$

$$\omega = \frac{2\pi}{T}$$

$$t = \frac{\cos^{-1} \left( \frac{1}{2} \right)}{\frac{2\pi}{T}}$$

$$t = \frac{T}{2\pi} \cos^{-1} \left( \frac{1}{2} \right)$$

$$t = \frac{T}{2\pi} \cdot \frac{\pi}{3}$$

$$t = \frac{T}{4}$$

$$t = \frac{T}{4}$$

$$t = \frac{T}{4}$$

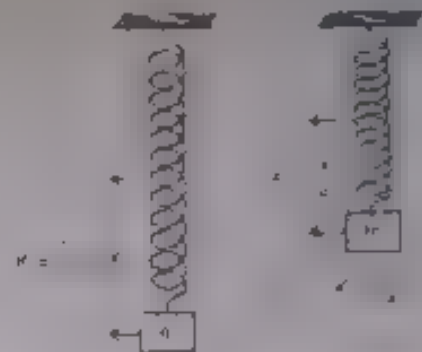
$$t = \frac{T}{4}$$

$$t = \frac{T}{4}$$

$$t = \frac{T}{4}$$

Q. A spring of spring constant  $k$  is cut into two pieces of equal lengths then what will be the spring constant of each part?

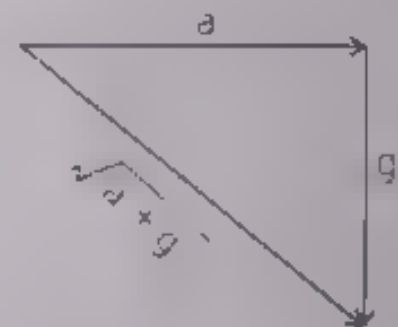
Ans. According to Hooke's law  
 $F = kx$   
 $k = \frac{F}{x}$   
 When the spring is cut into two equal parts, the length of each part is  $\frac{l}{2}$ .  
 Let the spring constant of each part be  $k'$ .  
 Then,  $F = k'x$   
 $k' = \frac{F}{x}$   
 Since the force  $F$  and displacement  $x$  are the same for both parts, the spring constant  $k'$  of each part is equal to the original spring constant  $k$ .



Q. A simple pendulum of length  $l$  is suspended from the roof of a train which moves in a horizontal direction with acceleration  $a$ . Then what will be its time period?

Ans. The time period of pendulum  $T$  when the train is at rest is,

When the train is moving with acceleration  $a$ , the effective acceleration is  $\sqrt{g^2 + a^2}$ .  
 The time period  $T$  is given by  
 $T = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$

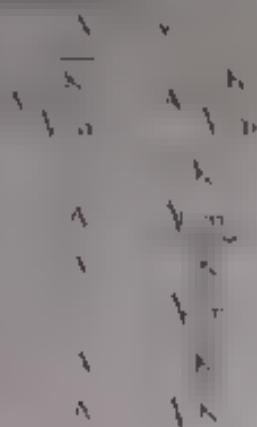


Therefore, the time period of the pendulum is  $T = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$ .

Q. Two particles A and B of equal masses are suspended from two massless springs of spring constant  $K_1$  and  $K_2$ . If maximum velocity during oscillation is equal, find the ratio of amplitude.

Ans. Maximum velocity  $v = A\omega$   
 For particle A,  $v = A_1 \sqrt{\frac{K_1}{m}}$   
 For particle B,  $v = A_2 \sqrt{\frac{K_2}{m}}$   
 Since  $v$  is equal for both,  $A_1 \sqrt{\frac{K_1}{m}} = A_2 \sqrt{\frac{K_2}{m}}$   
 $A_1 \sqrt{K_1} = A_2 \sqrt{K_2}$   
 $\frac{A_1}{A_2} = \sqrt{\frac{K_2}{K_1}}$

1. and up to  $g \sin \theta$  or  $g \cos \theta$



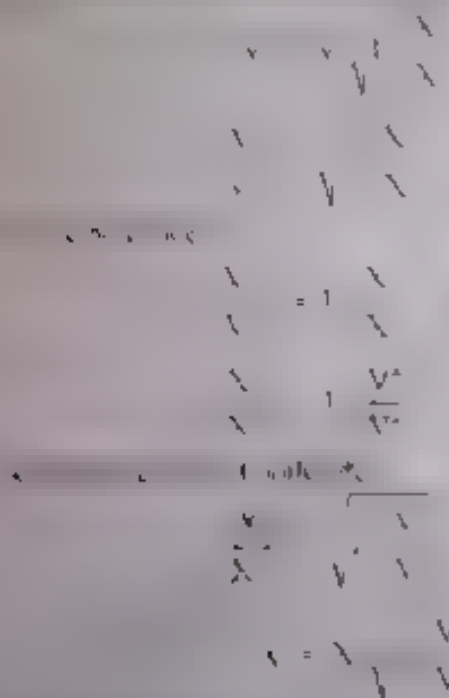
9. If  $x$  is instantaneous displacement and  $x_0$  is amplitude of vibration of a mass-spring system

then prove that  $X = X_0$



**Q10**

A mass  $m$  is suspended from a spring with constant  $k$ . The mass is displaced downwards by a distance  $x$  from its equilibrium position. Find the work done by the spring force.



10. The work done by the spring force is



### MCQ's From Past FBISE Papers (FEDERAL BOARD)

- The component of weight  $mg \sin \theta$  in a simple pendulum is.
  - along the string
  - perpendicular to string
  - along the string
  - none of these
- If the amplitude of vibrating body is doubled, its velocity at mean position
  - becomes double
  - becomes triple
  - becomes four times
  - becomes five times
- Keeping length constant if mass of bob of simple pendulum is doubled then its time period becomes
  - double
  - half
  - remain same
  - four times

4. Which of the following is the expression for instantaneous velocity of a body executing SHM?  
(a)  $v = \sqrt{a^2 - \omega^2 x^2}$  (b)  $v = \sqrt{a^2 + \omega^2 x^2}$  (c)  $v = \sqrt{a^2 - \omega^2 x}$  (d)  $v = \sqrt{a^2 + \omega^2 x}$
5. The unit of spring constant is same as that of  
(a) force (b) pressure (c) energy (d) work
6. If mass of mass spring system becomes four times, then its time period 'T' becomes  
(a) 4T (b) 2T (c) T/2 (d) T/4
7. Which of the following is the length of second pendulum is approximately?  
(a) 1m (b) 10cm (c) 2m (d) 1cm
8. Which of the following is the length of pendulum having time period of 1s?  
(a) 1m (b) 0.5m (c) 0.25m (d) 0.1m
9. When length of pendulum is doubled, ratio of new frequency to old frequency is  
(a)  $\frac{1}{4}$  (b)  $\frac{1}{\sqrt{2}}$  (c)  $\sqrt{2}$  (d)  $\frac{1}{2}$
10. Which of the following is the frequency of SHM?  
(a)  $\frac{2\pi}{T}$  (b)  $\frac{T}{2\pi}$  (c)  $\frac{2\pi}{T^2}$  (d)  $\frac{T^2}{2\pi}$
11. The waves produced in a microwave oven have frequency.  
(a) 2450 MHz (b) 2.45 kHz (c) 2.45 MHz (d) 2.45 GHz
12. Tuning a radio Set is the best example of  
(a) Mechanical resonance (b) Electromagnetic resonance (c) Magnetic resonance (d) Acoustic resonance
13. A simple pendulum is moved from the Earth to the Moon. How does it change the period of oscillations? (Acceleration due to gravity on moon = 1.6 m/s<sup>2</sup>)  
(a) The period is increased by factor  $\sqrt{6}$  (b) The period is decreased by factor  $\sqrt{6}$  (c) The period is increased by factor 6 (d) The period is decreased by factor 6
14. A simple pendulum on earth has period of 6.0s. What is the approximate period of this pendulum on the moon where the acceleration due to gravity is roughly  $\frac{1}{6}$ th of earth's gravity?  
(a) 15s (b) 1.0s (c) 36s (d) 2.4s
15. To double the period of simple pendulum, its length must be:  
(a) Increased four times (b) Increased four times (c) Decreased by 4 (d) Decreased by 4

Answers Key

|   |   |   |   |
|---|---|---|---|
| b | b | c | d |
| b | d | d | b |
| c | b | a | b |



## SELF-ASSESSMENT PAPER

Total Mark 40

Question No 1 Choose the correct answer from the given options.

11 x 5 = 55

## SECTION - A

1. The maximum velocity of 1 kg mass attached to a spring of force constant of  $1 \text{ Nm}^{-1}$  up to the displacement of 5 cm is  
 A.  $1 \text{ ms}^{-1}$  B.  $0.5 \text{ ms}^{-1}$  C.  $5 \text{ ms}^{-1}$  D.  $0.1 \text{ ms}^{-1}$
2. Which of the following is the wave form of S.H.M?  
 A. Sine wave B. Square wave C. Sawtooth wave D. Sawtooth wave
3. Which of the following is the length of the second pendulum?  
 A. 99.3 cm B. 99.1 cm C. 99.2 cm D. 99.4 cm
4. A process whereby energy is dissipated from the oscillating system is called  
 A. resonance B. damping C. decay D. decay
5. For a mass spring system placed on a smooth horizontal surface oscillating with amplitude  $x_0$ . At what displacement from the mean position is kinetic energy is equal to its elastic potential energy?  
 A.  $\frac{x_0}{2}$  (B)  $\frac{x_0}{\sqrt{2}}$  C.  $\frac{x_0}{3}$  D.  $\frac{x_0}{4}$
6. A simple pendulum has a period  $T$  what will be the percentage change in period if the amplitude is decreased by 50%?  
 A. 0% B. 100% C. 500% D. 1000%

Question No 2 Answer 10 questions of 2 marks each.

2 x 10 = 20

## SECTION - B

- (i) Find the ratio of the periods of two simple pendulums if its length is doubled?
- (ii) Show that the acceleration is zero when the velocity is greatest.
- (iii) How can you distinguish between a simple harmonic motion and a periodic motion?
- (iv) A simple pendulum is suspended from the ceiling of a train moving with a constant velocity. How will it oscillate?
- (v) A simple pendulum is suspended from the ceiling of a train moving with a constant velocity. How will it oscillate?
- (vi) A pendulum is suspended from the ceiling of a train moving with a constant velocity. How will it oscillate?
- (vii) A pendulum is suspended from the ceiling of a train moving with a constant velocity. How will it oscillate?

Question No 3 Extended Questions

3

## SECTION - C

- (a) What is simple harmonic motion? Show that motion of simple pendulum is S.H.M. for small angular displacement.
- (b) A simple pendulum keeps perfect time at a location where the acceleration due to gravity is  $9.8 \text{ ms}^{-2}$ . When it is moved to higher altitude it loses 80 seconds per day. Find the height of the new location.
- (c) Give two examples of simple harmonic motion.

The End

## CHAPTER

## 8

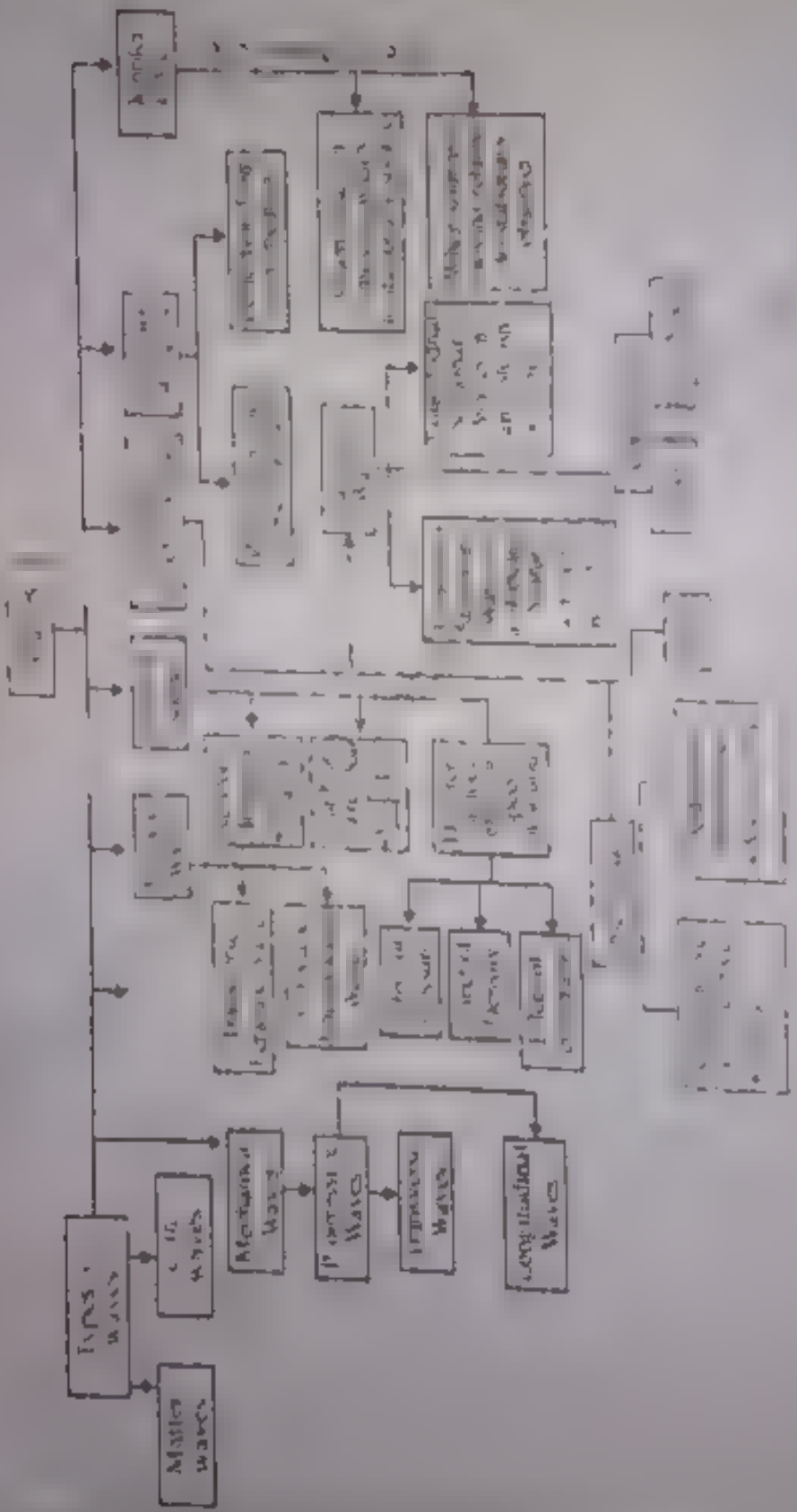
## WAVES

Learning Objectives

- ◆ Describe what is meant by wave motion as illustrated by vibrations in ropes, springs and tank?
- ◆ Demonstrate that mechanical waves require a medium for their propagation while electromagnetic waves do not.
- ◆ Define and apply the following terms to the wave model: medium, displacement, amplitude, period, compression, rarefaction, crest, trough, wavelength, velocity.
- ◆ Solve problems by using the equation,  $v = \lambda f$
- ◆ Describe the energy is transferred due to a progressive wave.
- ◆ Identify that sound waves are vibrations of particles in a medium.
- ◆ Compare transverse and longitudinal waves.
- ◆ Explain that speed of sound depends on the properties of the medium in which it propagates and describe Newton's formula of speed of waves.
- ◆ Describe the Laplace correction in Newton's formula for speed of sound in air.
- ◆ Identify the factors on which speed of sound in air depends.
- ◆ Describe the principle of superposition of two waves from coherent sources.
- ◆ Describe the phenomenon of interference of sound waves.
- ◆ Describe the phenomenon of formation of beats due to interference of non-coherent sources.
- ◆ Explain the formation of stationary waves using graphical method.
- ◆ Define the terms: node and antinodes.
- ◆ Describe modes of vibration of strings.
- ◆ Describe formation of stationary waves in vibrating air columns.
- ◆ Explain the observed change in frequency of a mechanical wave coming from a moving object as it approaches and moves away (i.e. Doppler Effect).
- ◆ Explain that Doppler Effect is also applicable to E.M. waves.
- ◆ Explain the main principles behind the use of ultrasound to obtain diagnostic information about internal structures.

# Chapter No. 8

## CONCEPT MAP



Q 1 What are waves? Describe its types

Ans: Waves

A wave is the mechanism by which energy is transferred from one place to another. The mechanism is called transport of energy without transporting matter.

- A wave is a disturbance in a medium which causes the particles of the medium to oscillate. The nature of wave may be different, but the mechanism by which it transports energy is the same.

Types of Waves

Waves are of three types.

### 1) Mechanical waves

Waves which require a material medium for their propagation are called mechanical waves. (R)

The waves in which there is oscillation of material particles are called mechanical waves.

For example

Water waves, sound waves, string waves etc.

### 2) Electromagnetic waves

Waves which do not require the oscillations of electric and magnetic fields and they require no medium are called electromagnetic waves.

► These waves consist of oscillations of electric and magnetic field.

For example

Radio waves, light waves, micro waves, x-rays etc.

### 3) Matter waves

Waves which consist of motion of particles in a medium are called matter waves or de Broglie waves.

For example

Waves associated with the motion of electron.

Q 2 What are periodic waves? How they are produced

Ans: Periodic Waves

The waves which are produced by continuous and rhythmic disturbances in a medium are called periodic waves.

Continuous waves can be produced by a source which produces a disturbance in a medium. As the source oscillates continuously, the particles of the medium also oscillate with the same frequency and amplitude. This is called a continuous wave motion.

- A string which is fixed at one end to a fixed support and the other end is moved up and down periodically produces a transverse wave. This wave is produced in a string.

- Each part of the rope moves up and down periodically as shown in Fig 8.1

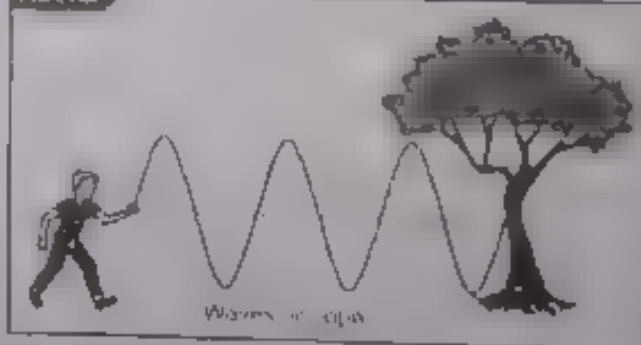
- ii) When the frequency of the electromagnetic vibrator is increased then transverse pulses are produced in the string.

- These waves move down the cord from the vibrator to the clamped end as shown in Fig 8.2

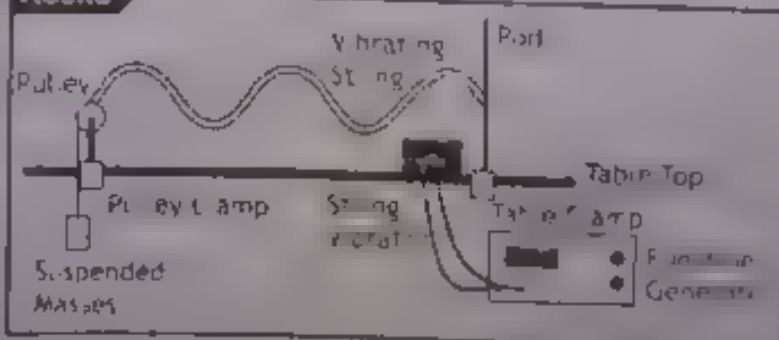
- iii) The vibrating spherical droppers of ripple tank just touching the water surface, driven by a small electric motor produce periodic waves.

The pattern of waves obtained at any instant of time is shown in Fig 8.3

FIGURE



FIGURE





- ▶ After the pulse has reached point A and it has completed exactly one oscillation and the rest of the pulse has moved a distance equal to wavelength  $\lambda$ .
- ▶ The particles of the medium vibrate with the same frequency as the wave and in phase with it.
- ▶ No net displacement of the particles takes place in the direction of wave propagation.
- ▶ The particles of the cord simply oscillate about their mean position to which a wave moves through.

### Necessary Conditions for Wave Motion

- ▶ The following conditions are necessary for the propagation of waves.
  - ▶ The medium must be elastic.
  - ▶ The particles of the medium should not be rigid with each other, so that they could exert force on each other. Transverse and longitudinal waves can be setup in solid. In fluids, however, transverse wave die out very quickly and usually cannot be produced.

### Classification of progressive waves

There are two kinds of progressive wave

### 24 What are transverse waves? Explain with examples

#### Transverse Waves

The waves, in which particles of the medium vibrate perpendicular to the direction of propagation of waves, are called as transverse waves.

- ▶ Slinky spring is the soft spring which has small initial length but relatively large extended length.
- ▶ Consider a slinky spring system with its one end fixed. When the free end is moved from side to side, a pulse of wave moves along the spring. A wave pulse pattern, as shown in figure, will move along the spring.
- ▶ The direction of vibration of particles is perpendicular to the direction of wave. Hence transverse wave.

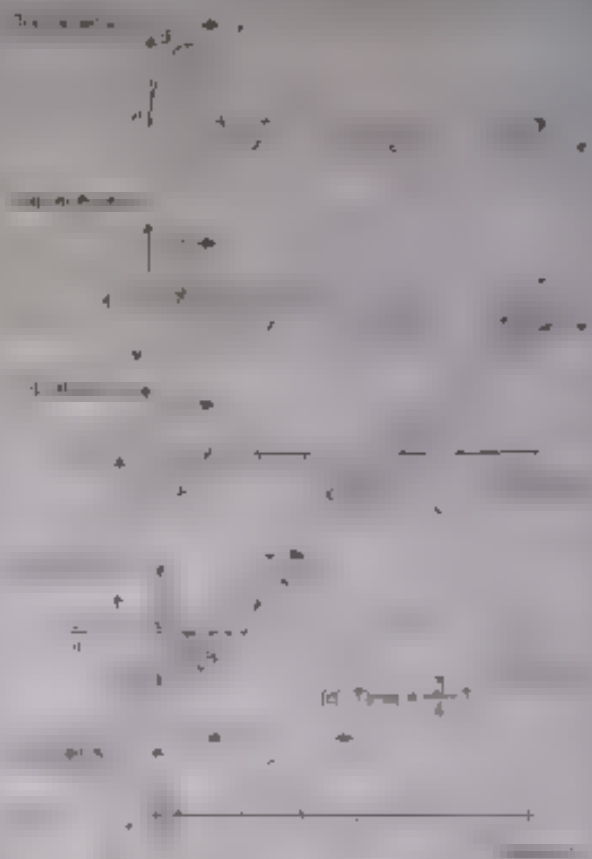
#### Wave Crest

The part of the wave where the medium is raised to its maximum displacement is called crest.

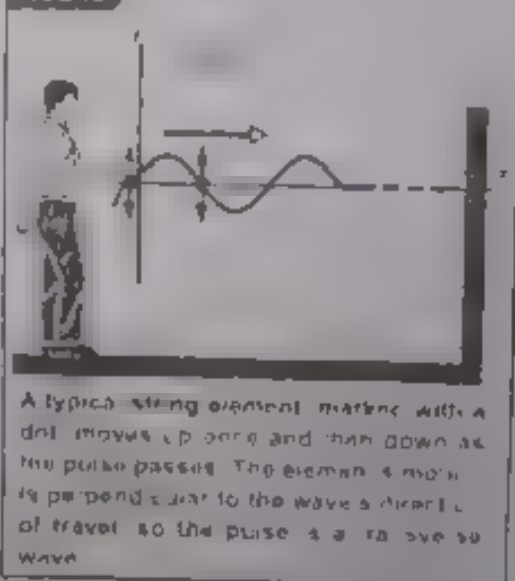
#### Trough

The part of the wave where the medium is lowered to its minimum displacement is called trough.

Example: Consider a rope whose one end is attached to a strong support and the other end is moved up and down periodically. A transverse wave pulse is produced as shown in figure.

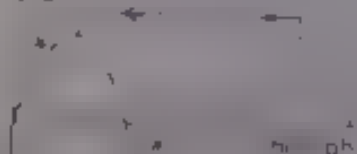


FIGURE

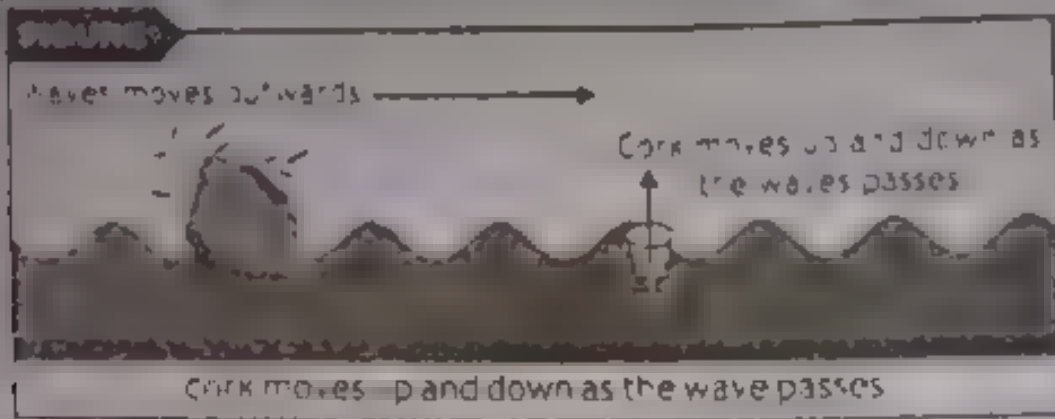


A typical string element marked with a dot moves up and down as the pulse passes. The element's motion is perpendicular to the wave's direction of travel so the pulse is a transverse wave.

Crest



Trough



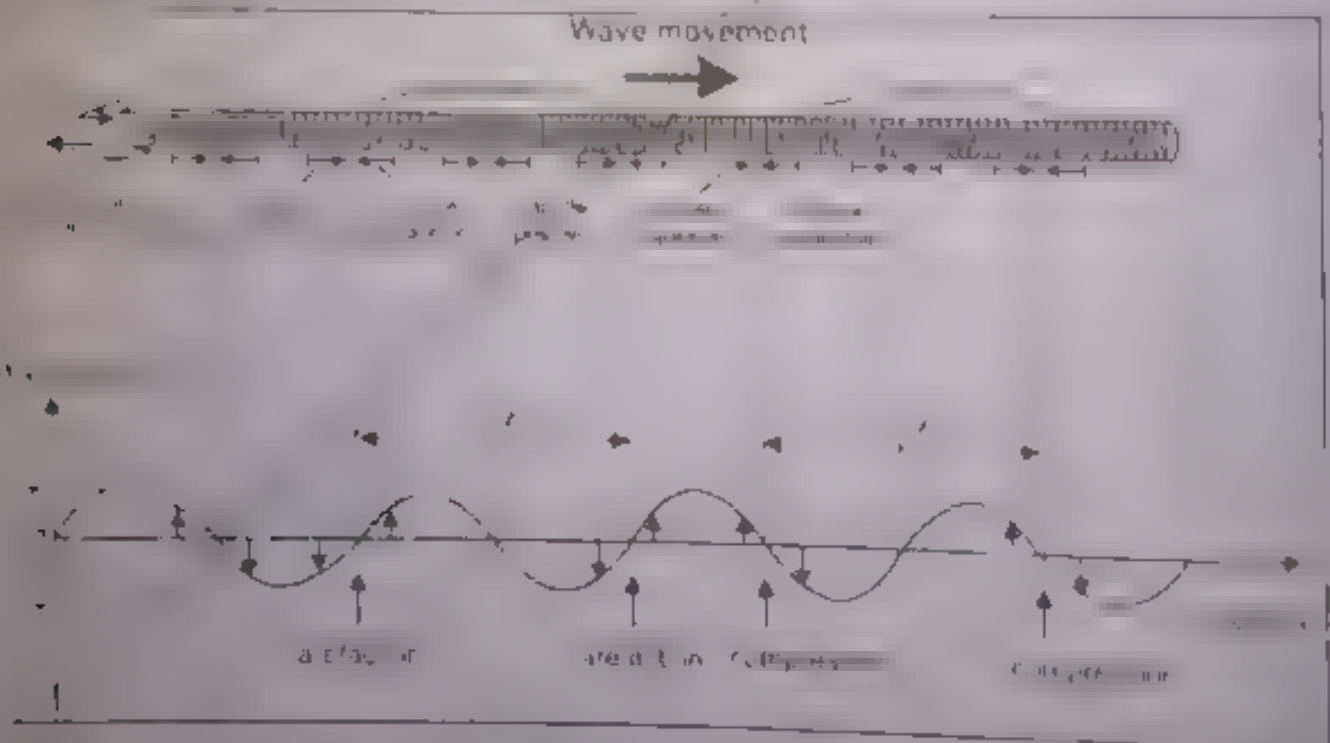
**Q 5** What are longitudinal waves? Explain with example

### 9.1.1 Longitudinal Waves (Compressional waves)

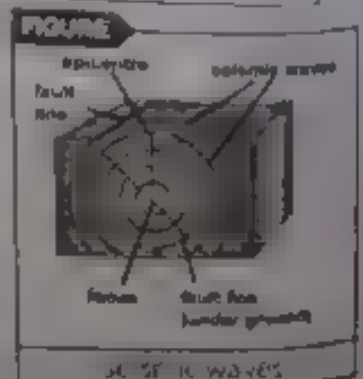
The waves in which particles of the medium vibrate along the direction of propagation of the waves are called as longitudinal wave.

**Examples:** sound waves, shock waves.

If a wave pulse travels towards a cork and it vibrates along the direction of propagation of the wave, then the wave is called longitudinal waves, as shown in figure 9.2.



- ▶ When a wave is passed towards right side then turn of spring coils compress and rarefaction produces.
- ▶ When a wave is passed towards left side then distance between turn of spring coils and rarefaction produces.
- ▶ Longitudinal waves are sound waves and shock waves and seismic waves which are also called seismic waves.



MCQ's

- Longitudinal waves are also known as
- A. Stationary waves      B. Transverse waves      C. Compression waves      D. Electromagnetic waves
- The distance covered by wave in 1 second is
- A. Wave speed      B. Wave number      C. Frequency      D. Wave speed
- The waves associated with the particles in motion are called
- A. Light waves      B. Electromagnetic waves      C. Matter waves      D. Stationary waves
- Distance between the crest and adjacent trough is
- A.  $\lambda$       B.  $\frac{\lambda}{2}$       C.  $\frac{\lambda}{4}$       D.  $\frac{\lambda}{8}$
- A wave which transfers energy is moving away from the source of disturbance is called
- A. Progressive wave      B. Standing wave      C. Both a & b      D. None of these
- The time period of wave is 0.2 s. Which of the following is its frequency?
- A. 2 Hz      B. 3 Hz      C. 4 Hz      D. 5 Hz
- Two waves can interfere only if they have
- A. Same nature      B. Same wave length      C. Different frequencies      D. Different wave length
- The louder the sound, the greater will be its
- A. Amplitude      B. Wave length      C. Speed      D. Frequency
- If 20 waves pass through a medium in 1 sec with a speed of 20 m/s then the wavelength is
- A. 2 m      B. 400 m      C. 40 m      D. 1 m
- The waves that do not need any medium for their propagation are
- A. Mechanical waves      B. Light waves      C. Sound waves      D. Compressional waves
- Half wavelength corresponds to
- A.  $0^\circ$       B.  $90^\circ$       C.  $180^\circ$       D.  $360^\circ$
- Transverse waves are distinguished from longitudinal waves by the property
- A. Reflection      B. Diffraction      C. Refraction      D. Polarization
- The profile of periodic waves generated by a source executing S.H.M. is represented by a
- A. Sine wave      B. Cosine wave      C. Both A & B      D. None of these
- If 332 waves pass through a medium in one second with speed of 332 m/s then wavelength will be
- A. 1 m      B. 332 m      C. 664 m      D. 996 m

Answer Key

|      |      |     |     |     |     |     |     |     |      |      |      |
|------|------|-----|-----|-----|-----|-----|-----|-----|------|------|------|
| 1 C  | 2 D  | 3 C | 4 B | 5 C | 6 D | 7 A | 8 A | 9 D | 10 C | 11 C | 12 D |
| 13 B | 14 D |     |     |     |     |     |     |     |      |      |      |

Q8 Explain the terms wave speed, frequency of waves, time period, wave length, amplitude and intensity

Ans Characteristics of Wave

A wave is described by a set of following parameters  
Wave speed, frequency, time period, phase, wave length, amplitude, intensity

Wave Speed

Speed of a wave is the distance travelled by a wave per unit time  
It is denoted by  $v$ . The speed of wave is related to the frequency  $f$  and wavelength  $\lambda$  as  
distance = speed  $\times$  time or  $\lambda = v \times T$  where  $T$  is time period

$$v = \frac{\lambda}{T} \quad \text{--- (1)}$$

$$v = \frac{f}{M} \quad \text{--- (2)}$$

Where  $M$  is the mass,  $f$  is the length of the string,  $\mu$  is the mass per unit length

► If  $m$  is the mass per unit length of the string then  $\mu = \frac{m}{l}$

$$= \sqrt{\frac{1}{\rho}}$$

- The speed of sound is greater as compared to a wave in a medium.
- The speed of sound depends on the properties of the medium L and density.
- The speed of sound is given by

$$v = \sqrt{\frac{E}{\rho}}$$

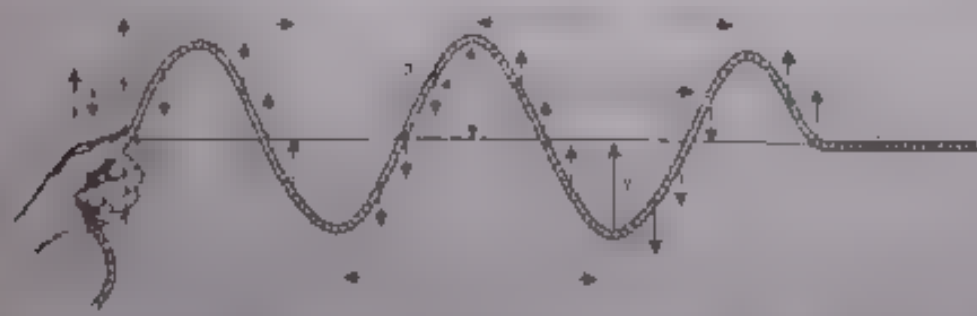
where E is the modulus of elasticity and  $\rho$  is the density of the medium.

### Frequency of Waves

$$f = \frac{1}{T} \text{ (Hz)}$$

### Time Period of Wave

### Wave Length $\lambda$



- The distance between two consecutive crests or troughs is called the wavelength.
- The frequency of a wave is the number of waves sent per second.
- The time taken for one complete wave to pass a point is called the time period.
- The speed of a wave is

$$v = \frac{\text{Distance covered}}{\text{Time Taken}}$$

Here

$$v = \frac{\lambda}{T}$$

$$v = (1.4 \times 10^3) \times (1.5)$$

$$v = 2.1 \times 10^3 \text{ m/s}$$

For a wave

$$v = \frac{\lambda}{T}$$

$$T = \frac{\lambda}{v} = \frac{1.5}{2.1 \times 10^3} \text{ s}$$

There are two types of waves

### Amplitude of Wave

The maximum displacement of a particle from its mean position is called the amplitude.

$$A$$

The maximum value of the displacement of a particle from its mean position is called the amplitude.

### Intensity of Wave

energy transmitted per second per unit area placed normal to the direction of propagation of the wave.

It is denoted by  $I$ .

$$I = \frac{P}{A}$$

$$I \propto \frac{1}{r^2}$$

Consider a wave of amplitude  $A$  and frequency  $f$  passing through a surface of area  $A$ .

What are the factors on which the speed of sound depends upon? What was Newton's formula for the speed of sound? What was drawback in it, how it was corrected by Laplace?

### Speed of Sound in Air

The speed of sound in air is about 330 m/s at 15°C.

It is given by

Newton's formula for the speed of sound in air is given by

Factors of the medium ( $E$ )

Factors of the medium ( $\rho$ )

It is denoted by  $v$ .

The speed of the medium, then the speed  $v$  can be expressed as,

$$v = \sqrt{\frac{E}{\rho}}$$

► Speed of sound in air is about 330 m/s at 15°C.

Reason

► Since the speed of sound in air is about 330 m/s at 15°C, it is

quite quickly as that of

► The speed of sound in air is about 330 m/s at 15°C, it is

quite quickly as that of

because the air is more compressible

Newton's formula for the speed of sound in air

he found that the speed of sound in air is about 330 m/s at 15°C.

$$v = \sqrt{\frac{E}{\rho}}$$

$$v = \sqrt{\frac{E}{\rho}}$$

Calculation of modulus of elasticity

Newton's Assumption

► The formula for the speed of sound in air is given by

temperature of the medium is constant

and the air is not compressed

$P$  is constant

### Newton's Formula



Consider a sound wave of amplitude  $A$  and frequency  $f$  passing through a surface of area  $A$ .

It is denoted by  $I$ .

The speed of the medium, then the speed  $v$  can be expressed as,

Newton's formula for the speed of sound in air

he found that the speed of sound in air is about 330 m/s at 15°C.

Calculation of modulus of elasticity

Newton's Assumption

► The formula for the speed of sound in air is given by

temperature of the medium is constant

and the air is not compressed

$P$  is constant

Calculation of modulus of elasticity

Newton's Assumption

► The formula for the speed of sound in air is given by

temperature of the medium is constant

and the air is not compressed

$P$  is constant

- ▶  $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$
- ▶  $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$$v = \sqrt{\frac{kx^2}{m}} = \sqrt{\frac{(100 \text{ N/m})(0.1 \text{ m})^2}{0.5 \text{ kg}}} = 2 \text{ m/s}$$

- ▶  $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$
- ▶  $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

#### Check Your Information

| Value             | Unit |
|-------------------|------|
| $\frac{1}{2}mv^2$ | J    |
| $\frac{1}{2}kx^2$ | J    |
| $\frac{1}{2}mv^2$ | J    |
| $\frac{1}{2}kx^2$ | J    |

#### Drawback to Newton's Law

#### Gravitational Potential

- ▶  $U = mgh$
- ▶  $U = mgh$

$$U = mgh = (100 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m}) = 9800 \text{ J}$$

$$U = mgh = (100 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m}) = 9800 \text{ J}$$

$$PV = (P + \Delta P) \left( V + \Delta V \right)$$
$$P = P + \Delta P \left( 1 + \frac{\Delta V}{V} \right)$$

By Binomial expansion  $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$

$$P = P + \Delta P \left( 1 + \frac{\Delta V}{V} \right)$$
$$P = P + \Delta P \left( 1 + \gamma \frac{\Delta V}{V} \right)$$
$$P = P + \gamma P \frac{\Delta V}{V} + \Delta P = \gamma P \frac{\Delta V}{V} + \Delta P$$

Since  $\Delta P$  and  $\Delta V$  both are small  $\therefore$   $\Delta P = \gamma P \frac{\Delta V}{V}$

$$P = P + \gamma P \frac{\Delta V}{V} + \Delta P$$
$$P = P + \gamma P \frac{\Delta V}{V} + \Delta P$$
$$\gamma P \frac{\Delta V}{V} = \Delta P$$

$$\gamma = \frac{\Delta P}{\frac{\Delta V}{V} P}$$

Where  $\frac{\Delta P}{\frac{\Delta V}{V} P} = \frac{\text{volumetric stress}}{\text{volumetric strain}} = \gamma$  (modulus of elasticity)

$$\gamma = \frac{\Delta P}{\frac{\Delta V}{V} P}$$

Putting value of  $\gamma$  in the equation  $v = \sqrt{\frac{\gamma}{\rho}}$

$$v = \sqrt{\frac{\gamma}{\rho}}$$

Putting  $\gamma =$

$$v = \sqrt{\frac{\gamma}{\rho}}$$
$$v = \sqrt{\frac{1.4 \times 10^5}{1.29}}$$
$$v = 331.7 \text{ m/s} \approx 332 \text{ m/s}$$

Which is same as the experimental value  $332 \text{ m/s}$

2.8 What is effect of density, moisture, pressure, temperature and wind on speed of sound?

Effect of Various Factors on Speed of Sound in air

Sound waves are compressional mechanical waves propagating through a medium.

$$v = \sqrt{\frac{P}{\rho}}$$

The following factors affect the speed of sound in a gas

- Density:** The speed of sound in a gas varies inversely as the square root of its density.  
 $v \propto \frac{1}{\sqrt{\rho}}$
- Moisture:** The presence of moisture in the air reduces the resultant density.  
► Therefore the speed of sound increases with humidity.  
► Hence the velocity of sound in damp air is greater than dry air.
- Pressure:** For one mole of an ideal gas having volume  $V$  and pressure  $P$  at temperature  $T$ , we can write



$$PV = nRT \quad n = 1$$

$$\Rightarrow PV = RT$$

$$V = \frac{RT}{P} \quad \text{--- (2)}$$

Where  $R$  is a universal gas constant

If  $m$  is the mass of the gas then its density is

$$\rho = \frac{m}{V}$$

$$\rho = \frac{m}{V} = \frac{m}{\frac{RT}{P}} = \frac{mP}{RT}$$

$$\rho = \frac{mP}{RT}$$

$$\rho = \frac{mP}{RT}$$

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$$\rho = \frac{mP}{RT}$$

$$\rho = \frac{mP}{RT}$$

$$\rho = \frac{mP}{RT}$$

$$\frac{v}{v_0} = \frac{1}{2 \times 10^{-5}}$$

$$v = \frac{v_0}{2 \times 10^{-5}}$$

$$\text{Since } v_0 = 332 \text{ m/s}$$

$$v = \frac{332}{2 \times 10^{-5}}$$

$$v = v_0 + 0.61 \text{ } ^\circ\text{C}$$

$$v_0 = v - 0.61 \text{ } ^\circ\text{C}$$

Thus the velocity of sound increases by 0.61 m/s for each degree rise in temperature is 0.61 m/s

6. Wind is blowing with velocity  $w$  in the direction of wave propagation. The speed of sound in the direction of wave propagation is  $v$ . The speed of sound in the direction of wave propagation is  $v + w$ .
- The speed of sound in the direction of wave propagation is  $v + w$ .
- The speed of sound in the direction of wave propagation is  $v - w$ .

### Assignment 4.7

Compute the speed in sea water, if its density is  $1025 \text{ kg/m}^3$  and Elastic modulus is  $2.1 \times 10^{11} \text{ Nm}^{-2}$

Given Data  $\rho = 1025 \text{ kg/m}^3$   
 $E = 2.1 \times 10^{11} \text{ Nm}^{-2}$

Solution

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{2.1 \times 10^{11}}{1025}} \text{ m/s}$$

### MCQ's

- The speed of sound in a medium depends on
  - Temperature
  - Pressure
  - Density
  - All of these
- The speed of sound in a medium is
  - 20 m/s
  - 332 m/s
  - 343 m/s
  - 400 m/s
- Newton's formula for the velocity of sound in gas is related as
  - $v \propto \sqrt{\frac{p}{\rho}}$
  - $v \propto \sqrt{\frac{\rho}{p}}$
  - $v \propto \sqrt{\frac{p}{\rho}}$
  - $v \propto \rho p$
- The speed of sound is greater in solids due to their high
  - Density
  - Pressure
  - Temperature
  - Elasticity
- The speed of sound in air does not depend on
  - Density
  - Pressure
  - Temperature
  - All of these
- Increase in the speed of sound for  $1^\circ\text{C}$  rise in temperature is
  - 31 m/s
  - 33 m/s
  - 35 m/s
  - 37 m/s
- The speed of sound is greater in
  - Air
  - Steel
  - Aluminium
  - Vacuum
- Appropriate range of audible frequencies for a younger person
  - 20 - 20,000 Hz
  - 20 - 20,000 Hz
  - 20 - 20,000 Hz
  - 20 - 20,000 Hz
- Increase in speed of sound at  $1^\circ\text{C}$  is given as
  - $v = v_0 + 0.61$
  - $v = v_0 + 0.61$
  - $v = v_0 + 0.61$
  - $v = v_0 + 0.61$
- The velocity of sound in vacuum
  - 332 m/s
  - 333 m/s
  - 334 m/s
  - 335 m/s
- The pressure exerted by the column of mercury 76 cm high and  $0^\circ\text{C}$  scale
  - 1 atm
  - 1.013  $\times 10^5 \text{ Pa}$
  - 1.013  $\times 10^5 \text{ Pa}$
  - 1.013  $\times 10^5 \text{ Pa}$

1. Which of the following is not a characteristic of a wave?  
 A. It carries energy. B. It has a wavelength. C. It has a frequency. D. It has a period.
2. Sound waves travel faster in  
 A. air. B. water. C. glass. D. vacuum.
3. Sound waves cannot be  
 A. reflected. B. refracted. C. polarized. D. diffracted.
4. The expression for the speed of sound in air is  
 $v = \sqrt{\frac{\gamma P}{\rho}}$   
 where  $\gamma$  is the adiabatic index,  $P$  is the pressure, and  $\rho$  is the density of the air.
5. The speed of sound in air would be higher than at 10°C at a temperature of  
 A. 0°C. B. 20°C. C. 30°C. D. 40°C.
6. The approximate relation between speed of sound in air and absolute temperature is  
 $v \propto \sqrt{T}$   
 where  $T$  is the absolute temperature.
7. Speed of sound in air at 20°C is  
 A. 330 m/s. B. 340 m/s. C. 350 m/s. D. 360 m/s.
8. Speed of sound in copper is  
 A. 1100 m/s. B. 1200 m/s. C. 1300 m/s. D. 1400 m/s.
9. The ratio of the speed of sound in air to the speed of sound in water is  
 A. 1/4. B. 1/3. C. 1/2. D. 1/5.
10. The speed of sound in air is lower than in  
 A. water. B. glass. C. steel. D. iron.
11. The speed of sound in air is greater than in  
 A. water. B. glass. C. steel. D. iron.
12. The speed of sound in air is less than in  
 A. water. B. glass. C. steel. D. iron.

|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|

Q. 9. State and explain the principle of superposition.

Ans. Superposition Principle

If a particle is subjected to two simultaneous motions, then the resultant displacement of the particle is the vector sum of the individual displacements. This is called superposition principle.

Consider two waves, one moving along the x-axis and the other along the y-axis.

Let the displacement of the first wave be  $y_1 = a \sin(kx - \omega t)$  and the displacement of the second wave be  $y_2 = b \sin(ky - \omega t)$ .

Then the resultant displacement is  $y = y_1 + y_2$ .

When the two waves are in phase, the resultant displacement is the sum of the individual displacements.

When the two waves are out of phase, the resultant displacement is the difference of the individual displacements.

When the two waves are in phase, the resultant displacement is the sum of the individual displacements. When the two waves are out of phase, the resultant displacement is the difference of the individual displacements.

When the two waves are in phase, the resultant displacement is the sum of the individual displacements. When the two waves are out of phase, the resultant displacement is the difference of the individual displacements.

Q. 10. State and explain the principle of superposition.

Ans. Superposition Principle

If a particle is subjected to two simultaneous motions, then the resultant displacement of the particle is the vector sum of the individual displacements. This is called superposition principle.

Consider two waves, one moving along the x-axis and the other along the y-axis.

Let the displacement of the first wave be  $y_1 = a \sin(kx - \omega t)$  and the displacement of the second wave be  $y_2 = b \sin(ky - \omega t)$ .

Then the resultant displacement is  $y = y_1 + y_2$ .

When the two waves are in phase, the resultant displacement is the sum of the individual displacements.

When the two waves are out of phase, the resultant displacement is the difference of the individual displacements.

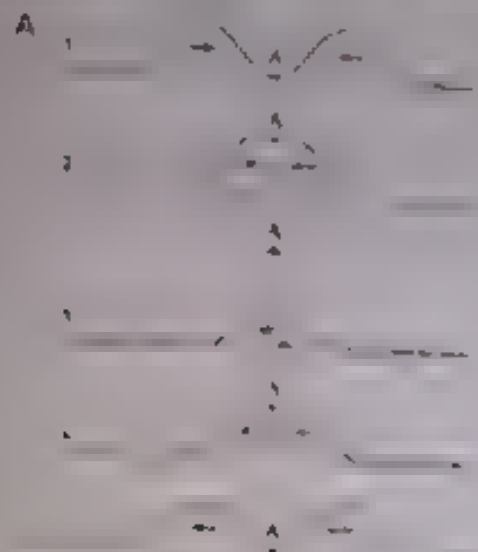
When the two waves are in phase, the resultant displacement is the sum of the individual displacements. When the two waves are out of phase, the resultant displacement is the difference of the individual displacements.

When the two waves are in phase, the resultant displacement is the sum of the individual displacements. When the two waves are out of phase, the resultant displacement is the difference of the individual displacements.

Cases of Superposition principle

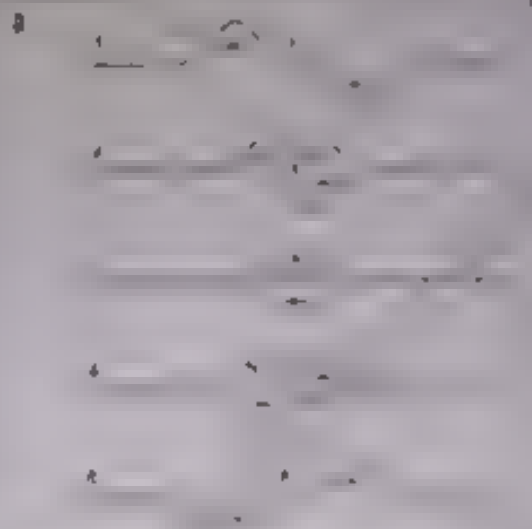
Interference

Part (a)



Constructive interference occurs when two waves of the same phase meet.

Part (b)

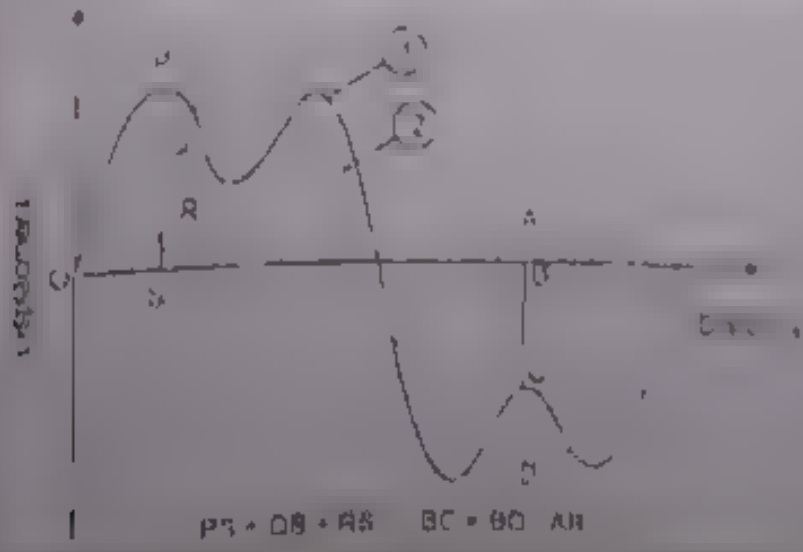


Destructive interference occurs when two waves of opposite phase meet.

Q10 What is interference of waves? Discuss types of interference.

Ans: Interference of Waves

- It is the phenomenon in which two or more waves meet at a point and their displacements are added or subtracted.
- It is a result of the superposition principle.
- It is a characteristic of all types of waves.
- It is a process in which two or more waves of the same frequency and amplitude meet at a point and their displacements are added or subtracted.
- It is a process in which two or more waves of the same frequency and amplitude meet at a point and their displacements are added or subtracted.



**Constructive interference** When two waves arrive at the same place at the same time in phase then they will combine to form a wave of greater amplitude.

- When two waves are in phase at a point, the path difference between them is an integer multiple of the wavelength.

► When two waves are in phase at a point, the path difference between them is an integer multiple of the wavelength.

- The wave from the first wave (Fig. 8.4) is shown

(i) PS, QS, RS

(ii) BT, CT, AT

- The wave from the second wave (Fig. 8.4) is shown

**Destructive interference** If two waves arrive at the same place at the same time but are out of phase by half a wavelength, then they will cancel each other out.

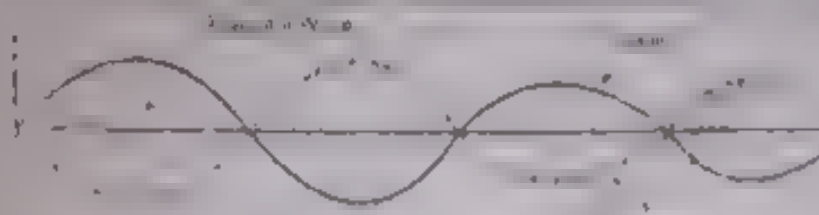


Fig. 8.4

- The path difference between the two waves is an odd multiple of half a wavelength.

- The path difference between the two waves is an odd multiple of half a wavelength.

### Conditions for interference

For two waves to interfere, the following conditions must be satisfied:

- The two waves must be of the same frequency.
- The two waves must be of the same amplitude.
- The two waves must be travelling in the same direction.
- The two waves must be coherent, i.e. they must have a constant phase difference.
- The two waves must be of the same type (e.g. both light waves or both sound waves).

For two waves to interfere, the path difference between the waves must be an integer multiple of the wavelength.

$$d = n \lambda$$

$$d = n \lambda$$

Where  $n$

**Q 11 Explain interference of the sound waves**

**Ans** Interference of sound waves

Definition

The phenomenon in which two or more waves overlap to form a new wave pattern is called interference.

- ▶ When two waves of the same frequency and amplitude are in phase, the resulting wave has a maximum amplitude.
- ▶ When two waves of the same frequency and amplitude are out of phase, the resulting wave has a minimum amplitude.

- ▶ When two waves of the same frequency and amplitude are out of phase by  $\pi$ , the resulting wave has a minimum amplitude.
- ▶ When two waves of the same frequency and amplitude are out of phase by  $\frac{\pi}{2}$ , the resulting wave has a minimum amplitude.

Activities

- ▶ Take two tuning forks of the same frequency and amplitude. Strike them and hold them near your ear. You will hear a loud sound.
- ▶ Now, move one of the tuning forks slightly. You will hear a soft sound.

- ▶ This is because of the interference of the sound waves.

- ▶ The sound is loud when the waves are in phase and soft when they are out of phase.

- ▶ As a result loud sound is heard.

- ▶ Now, move the tuning forks further apart. The sound will become soft.

- ▶ This is because the path difference between the two waves will be different from zero.

- ▶ When the path difference is zero, the sound is loud.

- ▶ When the path difference is  $\lambda$ , the sound is loud.

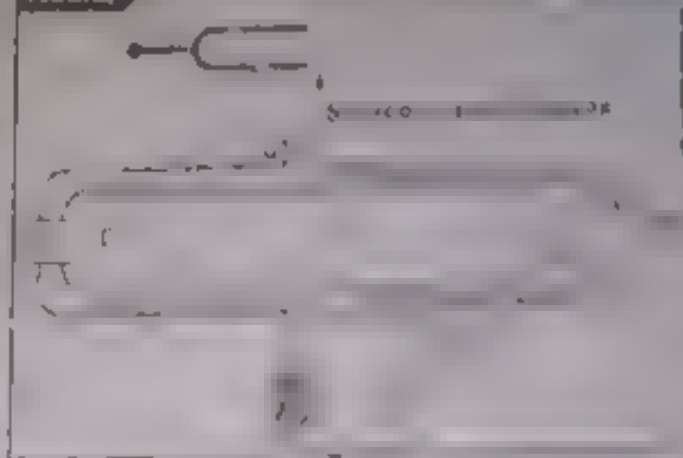
- ▶ When the path difference is  $2\lambda$ , the sound is loud.

- ▶ When the path difference is  $3\lambda$ , the sound is loud.

- ▶ When the path difference is  $4\lambda$ , the sound is loud.

- ▶ When the path difference is  $5\lambda$ , the sound is loud.

FIGURE



**Q 12 Define and explain beats?**

**Ans** Beats

"The periodic variations between maximum and minimum loudness of sound waves are called beats."

When two waves of slightly different frequencies are superimposed, the resulting wave has a frequency that is the average of the two frequencies.

The beat frequency is the difference between the two frequencies.

Beat frequency

The beat frequency is the number of beats per second. It is given by the formula:

$$f_b = |f_1 - f_2|$$

Explanation

Let us consider two waves of slightly different frequencies, say  $f_1$  and  $f_2$ . The resulting wave has a frequency that is the average of the two frequencies.

The beat frequency is the difference between the two frequencies.

For example, if  $f_1 = 440$  Hz and  $f_2 = 442$  Hz, the beat frequency is 2 Hz.

This means that the sound will have a maximum loudness every 0.5 seconds.

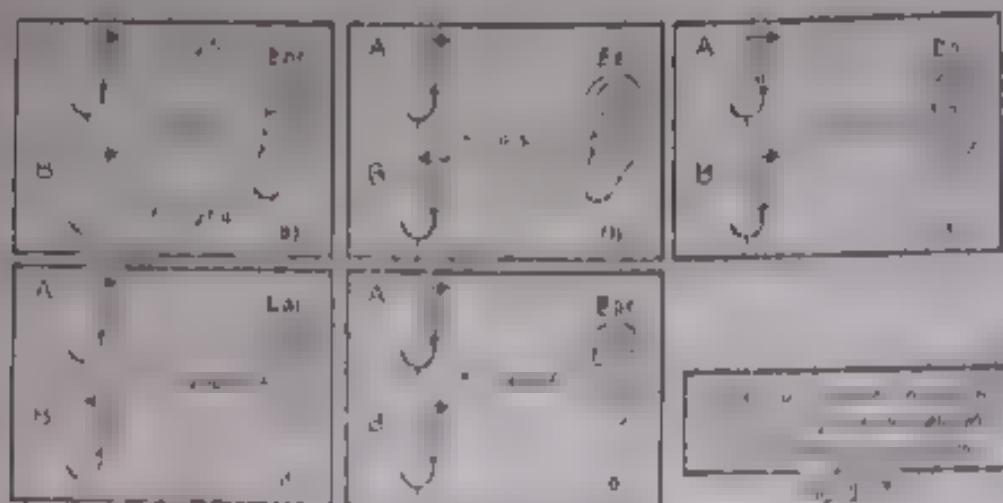
The beat frequency is the number of beats per second.

It is given by the formula:  $f_b = |f_1 - f_2|$

where  $f_1$  and  $f_2$  are the frequencies of the two waves.

For example, if  $f_1 = 440$  Hz and  $f_2 = 442$  Hz, the beat frequency is 2 Hz.

- ▶ The two compressions will arrive at the ear together and interfere constructively. We call this **loud**.
- ▶ As the tines of the fork B vibrates with a frequency lower than that of A, it will begin to lag behind.



After time  $t_1 = 1.4$  second

- ▶ Fork A will complete 128 vibrations (compression).
- ▶ Fork B will complete 120 vibrations (compression).
- ▶ The waves from both forks will reach the ear at the same time.

After time  $t_2 = 3.4$  second

- ▶ Fork A will complete 256 vibrations (compression).
- ▶ Fork B will complete 240 vibrations (compression).
- ▶ The waves from both forks will reach the ear at the same time.

After time  $t_3 = 3.4$  second

- ▶ Fork A will complete 256 vibrations (compression).
- ▶ Fork B will complete 240 vibrations (compression).
- ▶ The waves from both forks will reach the ear at the same time.

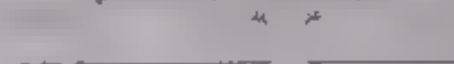
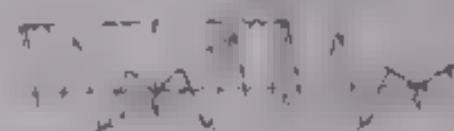
After  $t_4 = 1$  second

- ▶ Fork A will complete 256 vibrations (compression).
- ▶ Fork B will complete 240 vibrations (compression) fig B.17 (e).
- ▶ The waves from both forks will reach the ear at the same time.
- ▶ The waves from both forks will reach the ear at the same time.
- ▶ The waves from both forks will reach the ear at the same time.

$$N = f_1 - f_2$$

- ▶ We can understand the phenomenon of the beats by considering the displacement curves of the two waves produced by the two forks.
- ▶ The displacements of the particles of the medium due to two waves are plotted separately as a function of time as shown in fig B.18.
- ▶ If both the waves travel simultaneously along the same line then according to the superposition principle the resultant displacement of any particle will be the vector sum of the displacements due to each of the two waves.
- ▶ For a given wave which is produced as shown.
- ▶ It is seen that amplitude varies with time that gives rise to variation of loudness which we call beats.

- ▶ The time interval between the two successive loud sounds is  $\frac{1}{N}$ .
- ▶ During this time interval the number of cycles of  $f_1$  and  $f_2$  are  $N_1$  and  $N_2$  respectively. The wave  $f_1$  has made  $N_1$  cycles and the wave  $f_2$  has made  $N_2$  cycles.



$$N = f \cdot T \quad \text{where } N = \text{beat frequency}$$

Where  $f$  is a beat frequency and  $T$  is the time period.

### Uses of beats

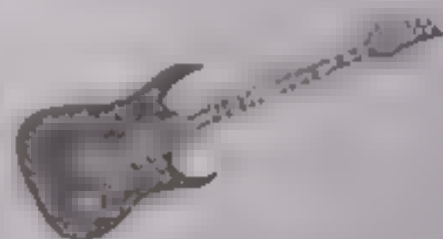
1. Beats are used to tune the musical instruments.
2. Beats are used to determine unknown frequencies.
3. Beats are used to produce variety in music.

### Example

The ear can hear beats. A string instrument such as piano or guitar is tuned by beating a note against a note of known frequency. The string can then be adjusted to the desired frequency by tightening or loosening it until no beat is heard.

### Explanation

During the process of tuning the piano or guitar, the tuner listens for beats. The beats are heard when the string is not in tune. The tuner then adjusts the string until the beats disappear, indicating that the string is in tune.



Tuning a musical instrument

- ▶ When the string is in tune, the beats disappear.
- ▶ When the string is not in tune, the beats are heard.
- ▶ When the string is not in tune, the tuner adjusts the string by tightening or loosening it.
- ▶ When the beats disappear, the string is in tune.

### NOTE

When the difference between the frequencies of two waves is small (say, 10 Hz), the beats are heard clearly.

### Example 8.2:

How many beats per second are heard when two tuning forks of 256 Hz and 259 Hz are sounded together?

Given Data  
 $f_1 = 256 \text{ Hz}$   
 $f_2 = 259 \text{ Hz}$

Let frequencies be  $N$

### Solution

$$N = 259 \text{ Hz} - 256 \text{ Hz}$$

$$N = 3 \text{ Hz}$$

### Q 13 Define and explain reflection of waves

#### Ans Define and explain reflection of waves

##### Reflection of waves

- ▶ When a wave pulse travels towards a boundary, it is reflected back.
- ▶ The pulse does not move up with the crest in same string upward, so the wall exerts a force on the string downward to such an amount that upward displacement pulse is reflected as downward displacement.
- ▶ A pulse on a string is reflected at a boundary between two media with different wave speeds.

and reflected pulses

- ▶ If the fixed end of the string is the boundary, the pulse is reflected inverted.
- ▶ If the free end of the string is the boundary, the pulse is reflected upright.

##### Reflection of light

#### Reflection of light

- ▶ Reflection of light occurs when electromagnetic waves such as light waves are reflected.
- ▶ The law of reflection states that the angle of incidence is equal to the angle of reflection.
- ▶ The law of reflection is used in many applications (radio audio detection and radar).

#### Reflection of Sound Waves

- ▶ The law of reflection for sound waves states that the angle of incidence is equal to the angle of reflection.
- ▶ The law of reflection is used in many applications (radio audio detection and radar).
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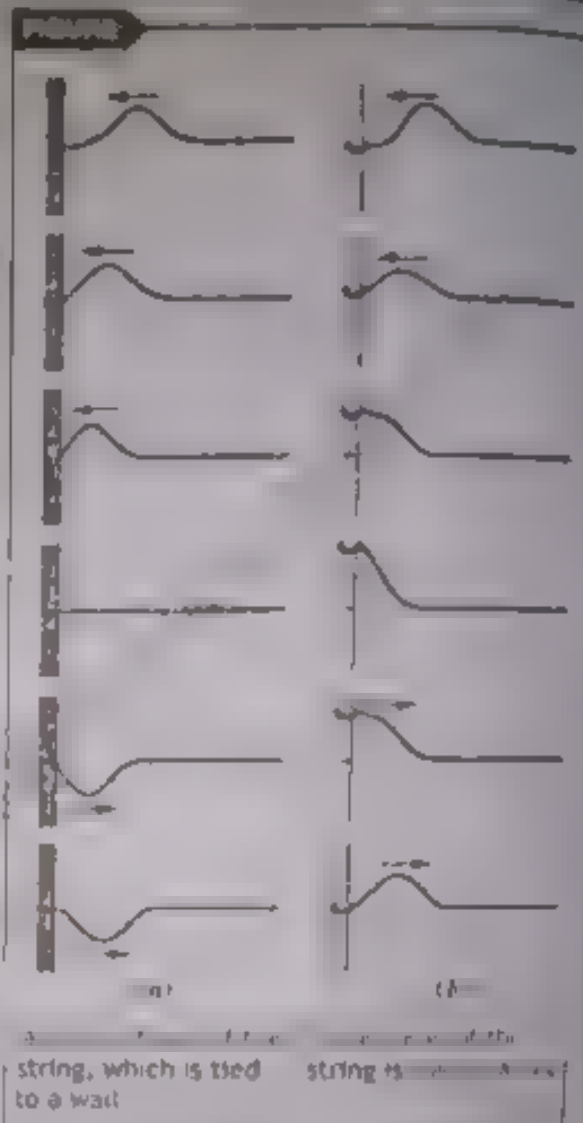


Fig 8.21 Guitar

## Q 14 What is Echo and reverberation?

## Ans: Echo

The reflection of sound waves from a reflecting surface is called echo.

Let the speed of sound is  $340 \text{ m/s}$ , then

$$\text{Speed of sound waves} = \frac{340 \times 2}{2} = 340 \text{ m/s}$$

- The reflection of sound waves in pipes, halls and auditoriums is called reverberation. It is a reflection of sound waves by reflecting proper direction and by avoiding sound loss.

## Reverberation

- When the reflecting surface is at a distance less than  $17 \text{ m}$ , the sound waves are reflected back so fast that they cannot be distinguished.
- The reflection of sound waves in pipes, halls and auditoriums is called reverberation.

## MCQ's

- Beats can be heard when difference of frequency is not more than  
A 4 Hz B 6 Hz C 8 Hz D 10 Hz
- in order to produce beats the two sound waves should have  
A same frequency B same amplitude C same frequency and amplitude D same frequency and phase
- Two tuning forks of frequencies 240 Hz and 243 Hz are sounded together. The number of beats per second will be  
A 3 B 4 C 5 D 6
- Beats are used to find unknown  
A frequency B wavelength C speed D intensity
- The periodic increase and decrease in loudness of sound are called  
A Beats B Interference C Beats D Polarization
- Which of the following changes due to interference of two sound waves of same frequency?  
A Frequency B Time period C Wave length D Amplitude
- Which of the following is the basic principle of beats?  
A Interference B Diffraction C Reflection D Refraction
- On loading the prong of a tuning fork with wax, the frequency of sound:  
A increases B decreases C increases D First increases then decreases
- Which of the following is the path difference for constructive interference?  
A  $2\lambda$  B  $\frac{\lambda}{2}$  C  $\frac{\lambda}{4}$  D  $\frac{3\lambda}{4}$
- The pitch of sound depends upon  
A intensity B speed C wavelength of sound D frequency of sound

## Answers Key

|     |     |     |     |     |     |     |     |     |      |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| 1 B | 2 D | 3 C | 4 A | 5 C | 6 D | 7 A | 8 B | 9 C | 10 C |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|

## Q 15 What are stationary waves? Explain

## Ans: Stationary waves (standing waves)

Two identical waves traveling with same speed along same line but in opposite direction superpose each other and give rise to the stationary wave.

- The sound produced by most of the string and wind type musical instruments is due to formation of stationary waves in these instruments.
- Consider a rubber string fixed at one end and held in hand by the other end. We wiggle the string from the end in hand. Then at a suitable frequency  $f$ , the wave is reflected from fixed end. The incoming wave and reflected wave combine to set up stationary waves.
- When wiggling frequency is doubled  $2f$ , then two stationary waves are set up.

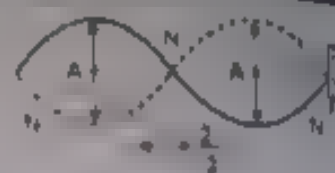


- ▶ When vibrating frequency is  $3f$  then three stationary waves are set up
- ▶ When vibrating frequency is  $4f$  then string vibrates in 4 loops.

**Nodes:**  $x = 0, \lambda/2, \lambda, 3\lambda/2, 2\lambda, 5\lambda/2, 3\lambda, 7\lambda/2, 4\lambda, \dots$  when stationary waves are set up in a

**Antinodes:**

$x = \lambda/4, 3\lambda/4, 5\lambda/4, 7\lambda/4, 9\lambda/4, 11\lambda/4, 13\lambda/4, 15\lambda/4, \dots$



### Properties of stationary waves

1. The string vibrates in stationary waves at particular frequencies, called natural frequencies.

- 2. The string vibrates in successive antinodes from the fixed end.
- 3. A point on the string does not move at all from its position.
- 4. All points on the string move in phase with the centre of the loop.

Distance between two consecutive antinodes is  $2 \times \lambda/2 = \lambda$  (1, 1)

Distance between two consecutive nodes is  $\lambda/2$ .

Distance between one node and next antinode is  $\lambda/4$

5. The string vibrates in stationary waves at particular frequencies, called natural frequencies.

6. The string vibrates in stationary waves at particular frequencies, called natural frequencies.

**Note**

- At their extreme positions P.E. is maximum and K.E. is zero.
- At their mean position K.E. is maximum and P.E. is zero.
- The string vibrates in stationary waves at particular frequencies, called natural frequencies.

in opposite direction.

**Q 16** Show that frequencies of stationary waves in a stretched string are quantized  
OR Prove that for stationary waves in a stretched string  $f_n = n f_1$

### Stationary Waves in Stretched String

#### Speed of waves on string

1. The speed of waves on a string is given by  $v = \sqrt{\frac{T}{\mu}}$  where  $T$  is the tension and  $\mu$  is the mass per unit length.

- ▶ When a string is fixed at both ends, the wave is reflected back and forth, creating a standing wave.
- ▶ The string vibrates in stationary waves at particular frequencies, called natural frequencies.
- ▶ The string vibrates in stationary waves at particular frequencies, called natural frequencies.
- ▶ The string vibrates in stationary waves at particular frequencies, called natural frequencies.

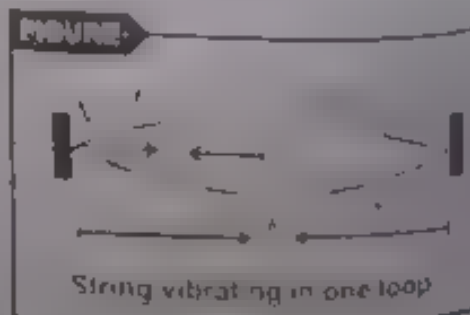
$$v = \sqrt{\frac{T}{\mu}}$$

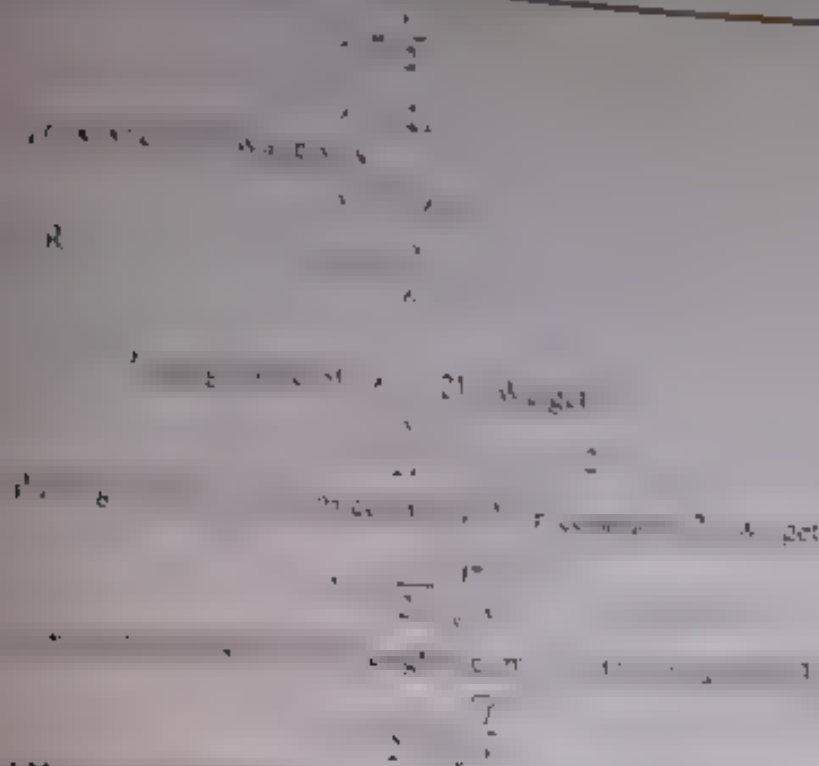
#### First mode of vibration

When a string is fixed at both ends, the first mode of vibration is the simplest. It is shown in the figure. Such a mode is called the first mode of vibration.

The distance between two consecutive nodes is  $\lambda/2$ .

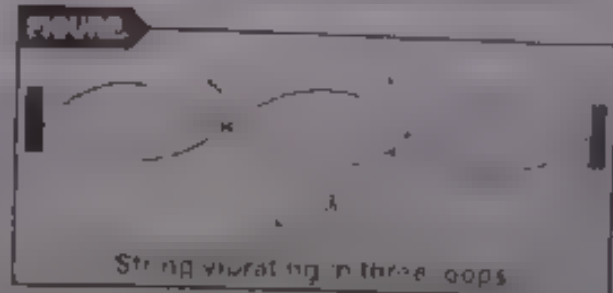
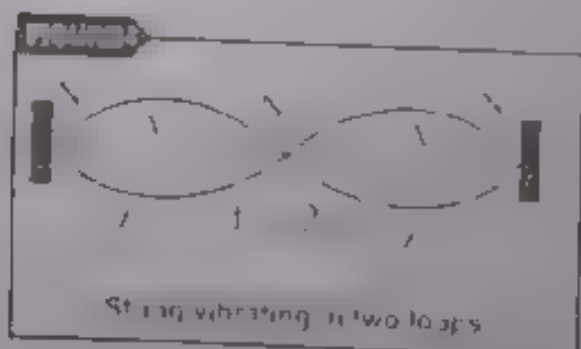
If the frequency of vibration is  $f$ , then the frequency of vibration is  $f_1$ .





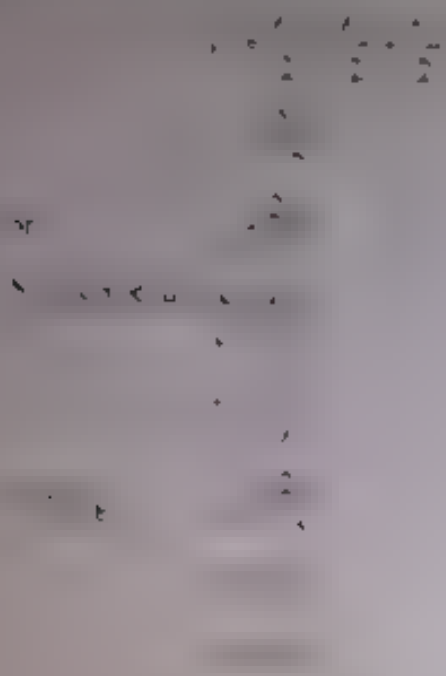
Second Mode of vibration

- ▶ When the length of the string is equal to two times the wavelength, then the string vibrates into two loops or two antinodes.
- ▶ In this mode, then



Third mode of vibration

- ▶ When the length of the string is equal to three times the wavelength, then the string vibrates into three loops or three antinodes.
- ▶ In this mode, then

**EXPLANATION**

A wave is a disturbance that travels through a medium. It can be a transverse wave, a longitudinal wave, or a surface wave. The wave travels through the medium, carrying energy and information. The wave is a disturbance that travels through a medium. It can be a transverse wave, a longitudinal wave, or a surface wave. The wave travels through the medium, carrying energy and information.

with mode of vibration

**Q 16** What is the relationship between the frequency of vibration and the frequency of the wave?

- ▶ The frequency of vibration is the same as the frequency of the wave.
- ▶ The frequency of vibration is the same as the frequency of the wave.
- ▶ The frequency of vibration is the same as the frequency of the wave.

**Q 17** How can we change the frequency of string on a musical instrument?

The frequency of a string can be changed by varying the tension, the length, or the mass per unit length of the string.

For example

- ▶ The frequency of a string can be changed by varying the tension, the length, or the mass per unit length of the string.

- ▶ (a) The speed of a wave is  $10 \text{ m/s}$ . If the frequency is  $100 \text{ Hz}$ , then the wavelength is  $0.1 \text{ m}$ .
- ▶ If the frequency is  $200 \text{ Hz}$ , then the wavelength is  $0.05 \text{ m}$ .

### Assignment 8.3c

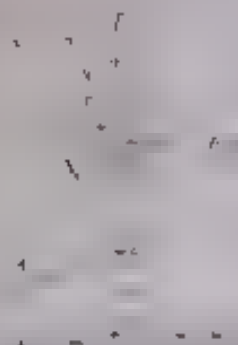
A  $40\text{-g}$  string  $2 \text{ m}$  in length vibrates in three loops. The tension in the string is  $270 \text{ N}$ . What is the wave length and frequency?

Given Data

Mass of string  $M = 40 \text{ g}$   
 Length of string  $L = 2 \text{ m}$   
 Tension  $T = 270 \text{ N}$

Wave length  
 Frequency

Solution



Wave length

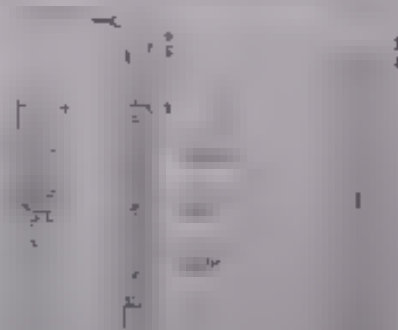


Frequency

### Q 18 Explain resonance in a air column

#### 8.13 Resonance of air Column and Organ pipes

- ▶ The phenomenon of resonance in a closed pipe is called resonance.
- ▶ When the frequency of the sound wave is equal to the natural frequency of the air column, the air column resonates.
- ▶ The resonance in a closed pipe is observed when the water level in a tube is adjusted so that the length of the air column is equal to the wavelength of the sound wave.
- ▶ Resonance occurs when the frequency of the periodic force is equal to the natural frequency of the system. In this case, the amplitude of the oscillation increases.



FIGURE

10.8.10



Resonating musical instruments

- ▶ The standing longitudinal waves in the air column in a pipe or tube are the sound waves. Sound is a longitudinal wave. Musical instruments such as flute, organ, trumpet, etc., are examples.

**Q 19** What are organ pipes? Derive expression for fundamental frequency and higher harmonics in closed pipe?

**Ans:** Organ Pipes

An organ pipe is the simplest example of an instrument which produces sound by means of a vibrating air column. Organ pipes are of two types.

- Closed organ pipe
- Open organ pipe

In both these types an air column is made to vibrate by blowing air into the whistle end, which is simplest in construction but its action is very simple.

Close organ pipe

- Let us consider an organ pipe of length  $l$  which is closed at one end.
- At the closed end we get node while at the open end we get an antinode.

Fundamental mode of vibration

- Fundamental mode of vibration has one node and one antinode.
- If  $\lambda$  is the wavelength of fundamental mode then length of the pipe is

$$l = \frac{\lambda}{4}$$

$$\text{Or } \lambda = 4l$$

Since  $v = f\lambda$  we have

$$\text{Or } f = \frac{v}{\lambda} \quad \text{putting value of } \lambda$$

$$f = \frac{v}{4l} \quad (1)$$

- The frequency  $f$  is called fundamental frequency.

Second Mode of vibration

- Second mode of vibration contains two nodes and two antinodes.
- If  $\lambda$  is the wavelength then length of the pipe is

$$l = \frac{\lambda}{4} + \frac{\lambda}{4}$$

$$l = \frac{2\lambda}{4}$$

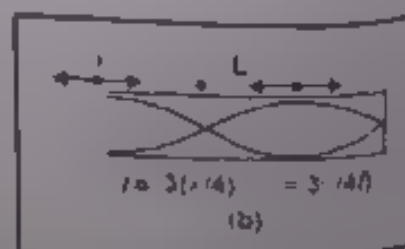
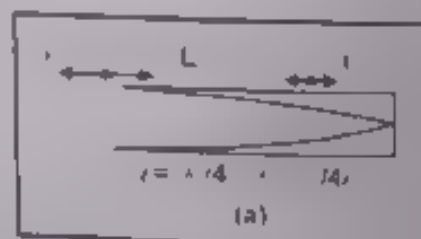
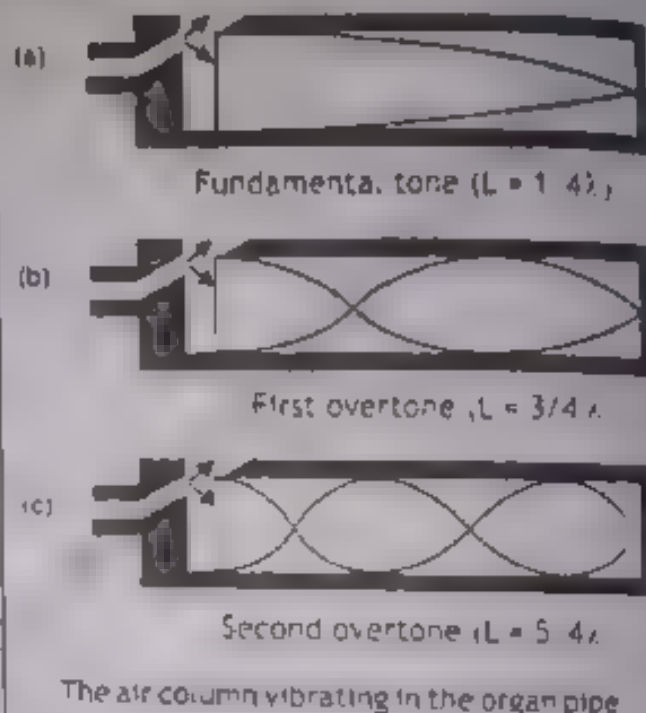
$$\lambda = \frac{4l}{2}$$

If  $v$  is a frequency of sound, then  $f_2 = \frac{v}{\lambda_2}$

putting value of  $\lambda$  we get

$$f_2 = \frac{3v}{4l}$$

**FIGURE**



or  $f_2 = 3 \left( \frac{v}{4L} \right)$

So  $f_2 = 3f_1$

[since  $\frac{v}{4L} = f_1$ ]

► This is called second harmonic.

### Third Mode of vibration

► Third mode of vibration contains three nodes and three antinodes.  
If  $\lambda_3$  is the wave length, the length of the pipe is

$$l = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$$

$$l = \left( \frac{1+2+2}{4} \right) \lambda_3$$

$$l = \frac{5\lambda_3}{4}$$

OR  $\lambda_3 = \frac{4l}{5}$

► If  $f_3$  is the frequency of sound, then speed becomes,  $v = f_3 \lambda_3$

OR  $f_3 = \frac{v}{\lambda_3}$

Putting value of  $\lambda_3 = \frac{4l}{5}$  we get

$$f_3 = \frac{v}{4L/5}$$

or  $f_3 = 5 \left( \frac{v}{4L} \right)$

So  $f_3 = 5f_1$  [since  $\frac{v}{4L} = f_1$ ]

Which is the frequency of third harmonic.

### Mode of vibration

► If a column vibrates in  $n$  loops then,

$$f_n = (2n-1) \frac{v}{4L}$$

Q.20 Derive expression for the fundamental frequency and higher harmonics in open pipe

### Open organ pipe (open at both ends)

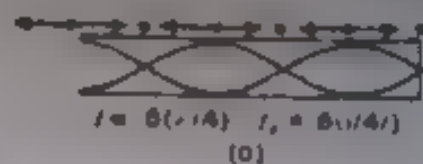
► Let us consider an organ pipe of length  $l$ , which is open at both ends.

► At the open ends air molecules have complete freedom of motion so it acts as antinode.

► Longitudinal waves set up inside the pipe have been represented by transverse curves which represent the displacement and amplitude of various parts of air at various points.

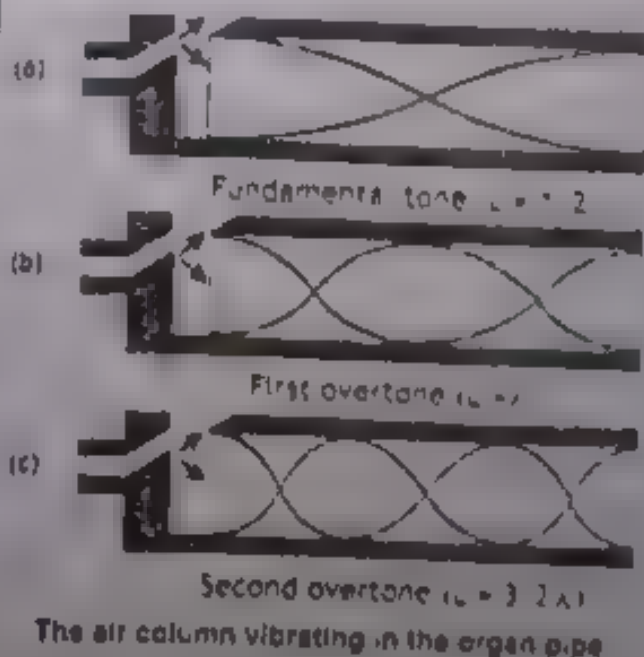
### Fundamental Mode of Vibration

► In this case there is only one node at the middle of the pipe.



Stationary longitudinal waves in a pipe open at both ends. Only odd harmonics are present.

FIGURE



- note the wave nodes at both the open ends
- the wave is in phase at

$$1 = \frac{\lambda}{2} + 1$$

$$1 = \frac{\lambda}{2}$$

►  $\lambda = 2L$  (where  $L$  is the length of the pipe)

►  $f = \frac{v}{\lambda} = \frac{v}{2L}$



### Standing waves in pipes

- the wave is in phase at
- the wave is in phase at

$$1 = \frac{\lambda}{2} + 1$$

$$1 = \frac{\lambda}{2}$$

$$1 = \frac{\lambda}{2}$$

$$1 = \frac{\lambda}{2}$$

$$1 = \frac{\lambda}{2}$$

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$$1 = \frac{\lambda}{2}$$

$$1 = \frac{\lambda}{2}$$

$$1 = \frac{\lambda}{2}$$



►  $f = \frac{v}{\lambda} = \frac{v}{2L}$

►  $f = \frac{v}{\lambda} = \frac{v}{2L}$

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►  $f = \frac{v}{\lambda} = \frac{v}{2L}$

### Standing waves in pipes

- the wave is in phase at
- the wave is in phase at

$$1 = \frac{\lambda}{2} + 1$$

$$1 = \frac{\lambda}{2}$$

$$1 = \frac{\lambda}{2}$$

$$1 = \frac{\lambda}{2}$$

$$1 = \frac{\lambda}{2}$$

$$1 = \frac{\lambda}{2}$$

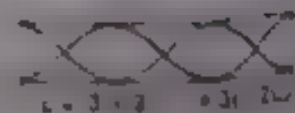


Fig A  
Standing longitudinal waves in a pipe open at both ends.

So the speed becomes,  $v = f \lambda$

$$f = \frac{v}{\lambda}$$

$$\text{Put } n\lambda = 2l \Rightarrow \lambda = \frac{2l}{n}$$

$$f = \frac{v}{\frac{2l}{n}}$$

$$f = \frac{nv}{2l}$$

$$\text{Since } \frac{\lambda}{2l} = \frac{1}{n}$$

► The frequency  $f$  is called third harmonic

### Mode of vibration

► If air column vibrates in  $n$  loops then

$$l_n = n \frac{\lambda}{2} = n \lambda$$

► The frequency  $f$  is known as harmonic frequency and the corresponding  $\lambda$  is called as harmonic wavelength

► So, the longitudinal stationary waves in a closed pipe are called odd harmonics as harmonics are

$$\text{And wave length is } \lambda_n = \frac{2l}{n}$$

where  $n = 1, 2, 3, 4, 5, \dots$

### Activity

If you place your ear near the neck of a soft bottle, as in fig. and blow softly across the opening, the sound wave reflected from the bottom of the bottle interferes with the wave you produce a standing wave. Since the bottle is closed at one end, there should be a displacement node at the bottom of the bottle.

Once you have heard one resonance, add varying amounts of water to raise the level within and listen for other resonances. The resulting sound is of easily higher than the original sound. Notice that the longer the air column within the bottle, the lower the pitch is.



### Assignment 2.5:

What length of closed pipe will produce a fundamental frequency of 256 Hz at 20°C?

Given Data:  $f = 256 \text{ Hz}$

temp =  $t = 20^\circ \text{C}$

Length of closed pipe = ?

### Solution:

Speed of sound at 20°C is

$$v = v_0 + 0.61t$$

$$v = 332 + 0.6 \times 20 = 332 + 12 = 344 \text{ m/s}$$

$$f = \frac{v}{\lambda}$$

$$l = \frac{v}{4f} = \frac{344}{4 \times 256} = 0.336 \text{ m}$$

$$l = 33.6 \text{ cm}$$

## MCQs

- 1 When two identical waves move in opposite directions superpose each other they give rise to
  - A a stationary wave
  - B two waves
  - C one wave
  - D no wave at all
- 2 Which of the following is the distance between two consecutive nodes or antinodes in stationary waves?
  - A  $\frac{\lambda}{2}$
  - B  $\frac{\lambda}{4}$
  - C  $\lambda$
  - D  $2\lambda$
- 3 On increasing the tension the frequency of the vibration of a string
  - A decreases
  - B increases
  - C remains the same
  - D becomes zero
- 4 Phase angle of  $180^\circ$  is equivalent to a path difference of
  - A  $\frac{\lambda}{2}$
  - B  $\frac{\lambda}{4}$
  - C  $\lambda$
  - D  $2\lambda$
- 5 A string vibrates in 4 loops then the wavelength of stationary wave will be.
  - A  $\frac{\lambda}{4}$
  - B  $\frac{\lambda}{2}$
  - C  $\lambda$
  - D  $2\lambda$
- 6 When a transverse wave is reflected from a fixed end the phase of the wave changes by
  - A  $180^\circ$
  - B  $90^\circ$
  - C  $270^\circ$
  - D  $360^\circ$
- 7 In stationary waves the velocity of particles is
  - A zero
  - B maximum
  - C constant
  - D variable
- 8 Which of the following is true wave of wavelength  $\lambda$  is reflected from a fixed end?
  - A  $\lambda$
  - B  $4\lambda$
  - C  $2\lambda$
  - D  $\lambda$
- 9 Which of the following is true for organ pipe closed at one end?
  - A  $\lambda$
  - B  $2\lambda$
  - C  $4\lambda$
  - D  $8\lambda$
- 10 Stationary waves are generated in a string fixed at both ends. The frequency is
  - A  $f$
  - B  $2f$
  - C  $4f$
  - D  $8f$
- 11 A stationary wave is established in a string which vibrates in 4 segments. The frequency of 12 Hz is its fundamental frequency.
  - A  $3\text{ Hz}$
  - B  $6\text{ Hz}$
  - C  $12\text{ Hz}$
  - D  $24\text{ Hz}$
- 12 A set of frequencies which is true for a string fixed at both ends is
  - A  $f, 2f, 3f, 4f, 5f, 6f, \dots$
  - B  $f, 2f, 4f, 8f, 16f, \dots$
  - C  $f, 2f, 3f, 4f, 5f, 6f, \dots$
  - D  $f, 2f, 4f, 8f, 16f, \dots$
- 13 The string of length  $L$  fixed at the both ends is vibrating in 4 segments. The wavelength of the wave is
  - A  $\frac{L}{4}$
  - B  $\frac{L}{2}$
  - C  $L$
  - D  $2L$
- 14 The frequency of a vibrating string is
  - A  $f$
  - B  $2f$
  - C  $4f$
  - D  $8f$
- 15 For same mass per unit length and length of string, the frequency of a vibrating string is decreased by 4 times if
  - A  $L$  is increased 4 times
  - B  $L$  is decreased 4 times
  - C  $T$  is increased 4 times
  - D  $T$  is decreased 4 times
- 16 A stretched string of length  $2\text{ m}$  vibrates in 3 segments. The distance between consecutive nodes is
  - A  $\frac{2}{3}\text{ m}$
  - B  $\frac{4}{3}\text{ m}$
  - C  $\frac{2}{3}\text{ m}$
  - D  $\frac{4}{3}\text{ m}$
- 17 The distance between node and antinode is equal to
  - A  $\frac{\lambda}{2}$
  - B  $\frac{\lambda}{4}$
  - C  $\lambda$
  - D  $2\lambda$
- 18 If the organ pipe is closed at one end the frequency of fundamental harmonic is
  - A  $f = \frac{v}{4L}$
  - B  $f = \frac{v}{2L}$
  - C  $f = \frac{v}{L}$
  - D  $f = \frac{v}{2L}$
- 19 The number of nodes between two consecutive antinodes is
  - A 2
  - B 1
  - C 3
  - D 4
- 20 A stretched string  $4\text{ m}$  long and it has 4 nodes of stationary waves. Then the wave length is
  - A  $4\text{ m}$
  - B  $2\text{ m}$
  - C  $1\text{ m}$
  - D  $0.5\text{ m}$
- 21 The distance between 1<sup>st</sup> node and 4<sup>th</sup> antinode is
  - A  $\frac{\lambda}{4}$
  - B  $\frac{\lambda}{2}$
  - C  $\frac{3\lambda}{4}$
  - D  $\lambda$

12. When a transverse wave travels through denser medium it is reflected from a denser medium. The phase change is  $\pi$ .

|      |      |      |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 A  | 2 A  | 3 B  | 4 B  | 5 A  | 6 C  | 7 D  | 8 B  | 9 B  | 10 B | 11 C | 12 B |
| 13 C | 14 B | 15 A | 16 A | 17 D | 18 B | 19 A | 20 C | 21 C | 22 D |      |      |

Q31 Define Doppler effect and discuss its different cases

Ans: Doppler Effect

The change in frequency or wavelength of a wave as observed by an observer moving relative to the source of the wave is called the Doppler effect. It is named after Christian Johann Doppler, an Austrian physicist and mathematician.

► The Doppler effect is observed in all types of waves, including sound waves, light waves, and water waves.

► The Doppler effect is observed in sound waves when the source and the observer are moving relative to each other.

► The Doppler effect is observed in light waves when the source and the observer are moving relative to each other.

► The Doppler effect is observed in water waves when the source and the observer are moving relative to each other.

► Consider a source "S" and listener "L" are at rest then the listener "L" receives "A" number of waves in one second.

► The frequency of waves received by the listener "L" is equal to the frequency of waves emitted by the source "S".

► The wavelength of waves received by the listener "L" is equal to the wavelength of waves emitted by the source "S".

Case 1: Source moving towards a stationary listener

► When a source of waves is moving towards a stationary listener, the frequency of waves received by the listener is higher than the frequency of waves emitted by the source.

At Source moving towards a stationary listener.

► The source "S" is moving with speed "u" towards the listener "L".

► The frequency of waves emitted by the source is "f".

► The wavelength of waves emitted by the source is " $\lambda$ ".

► The frequency of waves received by the listener is "f'".

► The wavelength of waves received by the listener is " $\lambda'$ ".

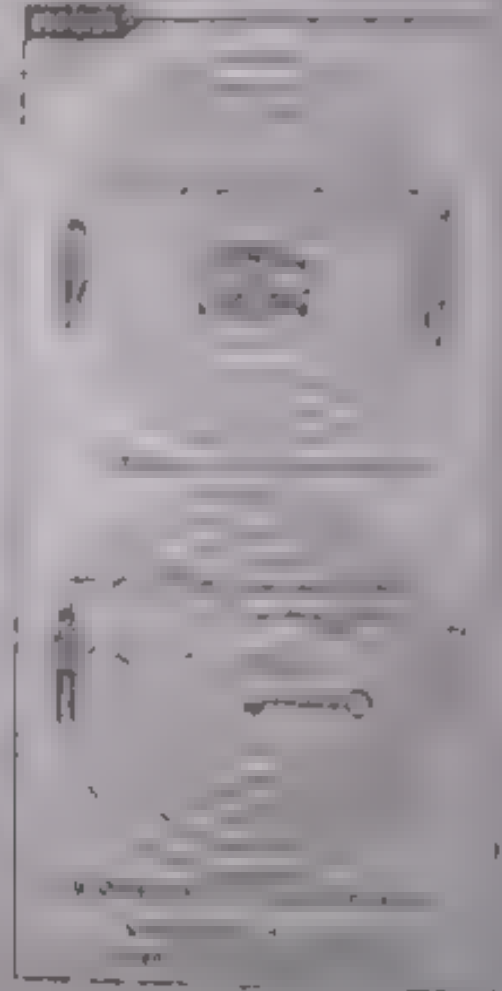
► The frequency of waves received by the listener is higher than the frequency of waves emitted by the source.

► The wavelength of waves received by the listener is smaller than the wavelength of waves emitted by the source.

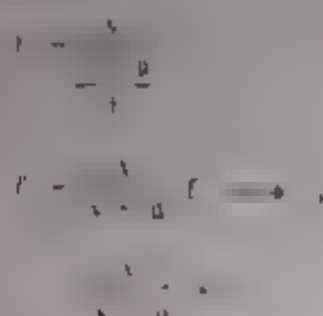
Since the frequency of waves received by the listener is higher than the frequency of waves emitted by the source, the wavelength of waves received by the listener is smaller than the wavelength of waves emitted by the source.

$$f' = \frac{v}{\lambda'}$$

The changed frequency f' is  $f' = \frac{v}{\lambda'}$



The Doppler effect



Therefore  $f' > f$

- If the observer is moving with speed  $v$  towards a stationary source, the observed frequency is given by

Source Moving Towards Stationary Listener

#### (1 B) The source of sound moves away from stationary listener

- Let the source  $S$  move with speed  $u$  away from a stationary listener  $L$ . The waves are emitted with wavelength  $\lambda$ .
- Since the source is moving,  $\lambda'$  becomes less than original wavelength  $\lambda$ .

$$\lambda' = \frac{v + u}{f}$$

The observer is stationary

Putting in the

$$\lambda' = \frac{v + u}{f}$$

So

- When the source is moving away from the stationary listener, the pitch of sound decreases

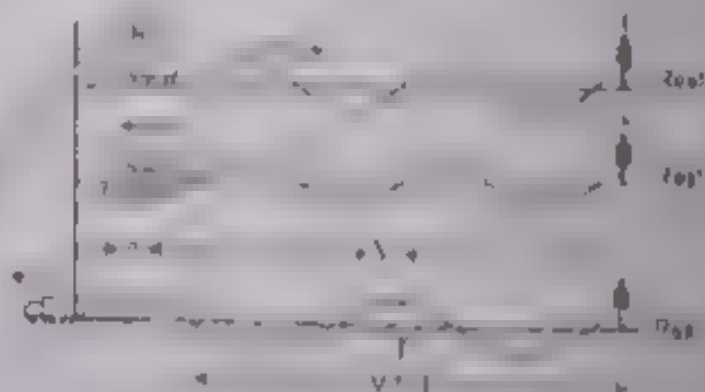
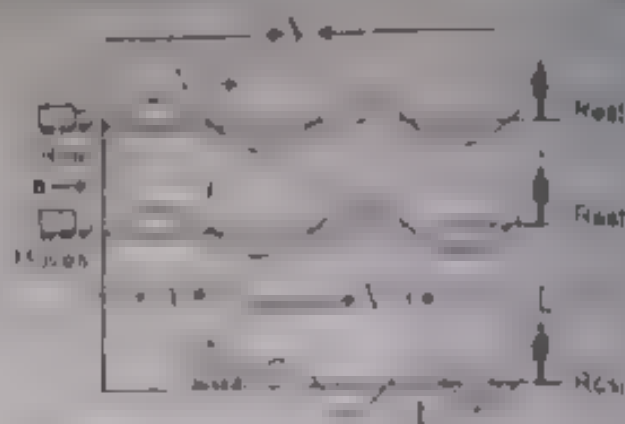
#### The source is at rest and listener is moving

When the listener is moving towards or away from the stationary sounding source, the pitch of sound changes

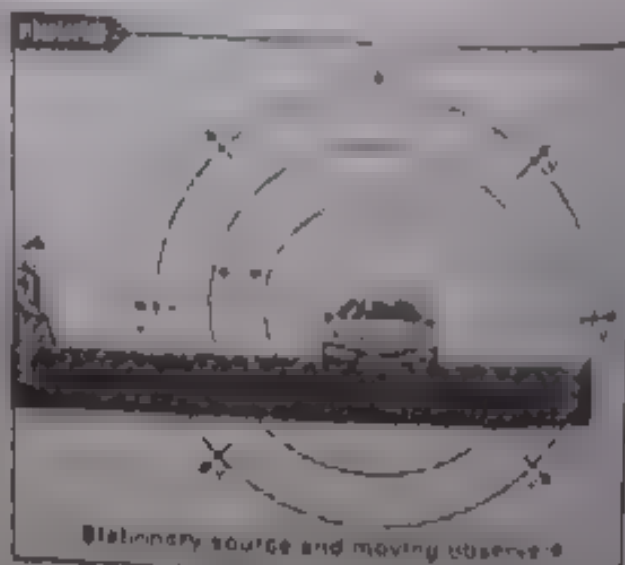
#### (1 A) The listener moves towards a stationary sounding source

- Let the listener  $L$  be moving with speed  $v$  towards a stationary sounding source  $S$  as shown in the fig 9.12
- Since speed of sound relative to the listener is  $(v + v)$  and wavelength remains the same

Therefore, the observed frequency  $f'$  is



Source Moving Away from Stationary Listener



$$f' = \frac{v+b}{v}$$

W. I.

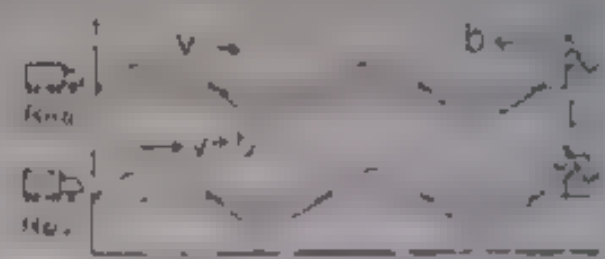
$$\frac{v}{f}$$

W. I. A.

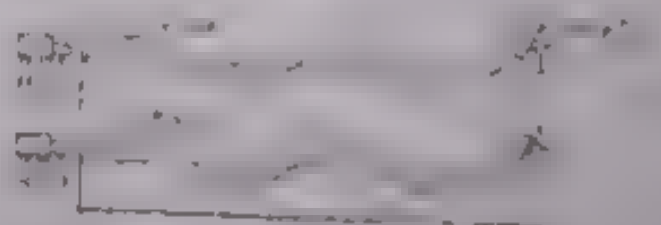
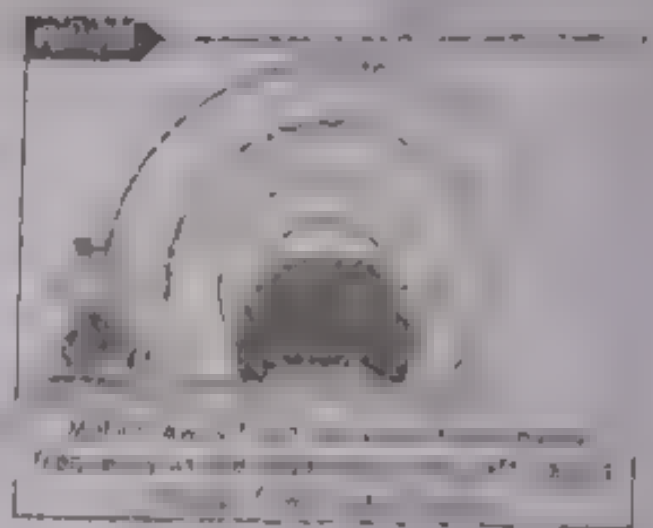
$$\frac{v+b}{v}$$

W. I. A.

$$\frac{v+b}{v}$$



Listener Moving Towards Stationary Source



Listener Moving Away from Stationary Source

$$\frac{v}{f}$$

When source and listener both move

When Source and Listener both move

It is a possibility that when source and listener both move the pitch of sound will change.

Source and listener both move towards each other.

If source and listener are approaching each other with velocities 'a' and 'b' respectively, the observed wave length  $\lambda'$  is given by

$$\lambda' = \frac{v-b}{f}$$

As the listener is moving towards the source (so  $b = (v+b)$ )

The apparent frequency  $f'$  is  $f' = \frac{v}{\lambda'}$

Putting value of  $v$  and  $\lambda'$  we get

$$f = \frac{v+b}{\frac{v-b}{f}}$$

$$f = \frac{v+b}{v-b} f$$

$$\frac{f}{f} = \frac{v+b}{v-b}$$

$$f = f$$

► When waves are sent by the source, the frequency of the waves is  $f$ .

When source and listener move away from each other

When the source and listener are moving away from each other, the frequency of the waves received by the listener is  $f'$ .

$$\lambda' = \frac{v+b}{f}$$

Since the speed of the waves to the listener is  $v' = (v - b)$

received frequency is

$$f'$$

When the source and listener are moving away from each other, the frequency of the waves received by the listener is  $f'$ .

$$\frac{f}{f'} = \frac{v-b}{v+b}$$

$$\frac{f}{f'} = \frac{v-b}{v+b}$$

$$f' = f \frac{v+b}{v-b}$$

When source and listener are moving away from each other, the frequency of the waves received by the listener is  $f'$ .

**Q 22** Discuss various applications of Doppler effect

**1.775 Applications of Doppler Effect**

1. The Doppler effect is the apparent change in frequency of a wave as it moves relative to the observer.

2. The Doppler effect is used in many applications.

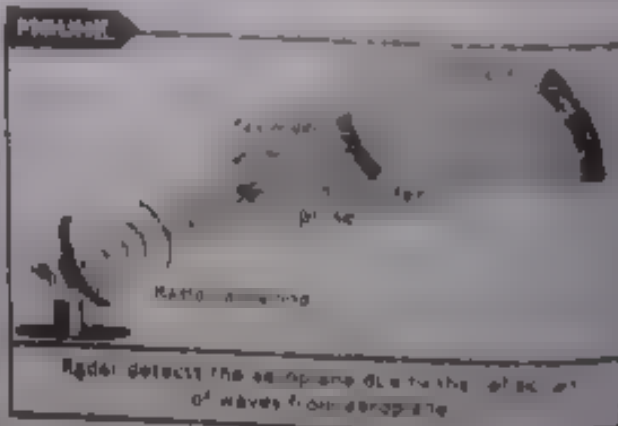
► The frequency of light from a star can be determined by observing the shift in the spectral lines. If the star is moving towards or away from the observer, the frequency of the light will be shifted. This is the Doppler effect.

3. The Doppler effect is used in radar to detect the speed of moving objects. The frequency of the reflected waves is shifted due to the Doppler effect.

► The frequency of reflected waves **decreases** if the object is moving away from the source.

► The frequency of reflected waves **increases** if the object is moving towards the source. The speed and direction of the plane can be determined by the Doppler effect.

4. When sound waves are reflected from a moving submarine, their frequency is changed.



- ▶ The change in frequency is called the Doppler shift. The speed of the object and the direction of its motion are the factors that determine the Doppler shift in frequency.
- ▶ Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

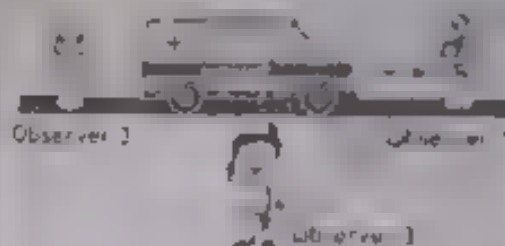


### QUIZ

1. Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

2. A stationary observer observes the Doppler shift of a sound source moving at a constant velocity.

3. The sound is observed as shown below. Which observer hears the highest frequency? Which observer hears the lowest frequency? What can be said about the frequency observed by observer 3?



Ans: 1. An ambulance with a siren moving towards you would be heard as a higher frequency when it was going towards you and as a lower frequency when it has passed by. This would help me know whether I needed to pull over and clear the road for ambulance.

### Assignment 2.6:

a. What frequency is received by a woman watching an oncoming ambulance moving at 110 km/h and emitting a steady 800 Hz sound from its siren? The speed of sound on this day is 345 m/s. (b) What frequency does she receive after the ambulance has passed?

Given Data

$$f = 800 \text{ Hz}$$

$$v = 345 \text{ m/s}$$

Required

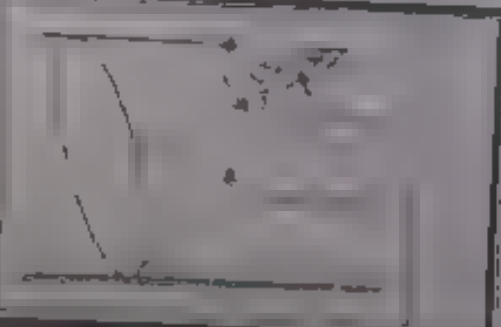
Solution

When the ambulance is approaching the stationary observer, the frequency heard is,

$$f' = \left( \frac{v}{v - v_s} \right) f = \left( \frac{345}{345 - 30.6} \right) \times 800 = 878 \text{ Hz}$$

### For Your Information

A shock wave is formed when the speed of sound is exceeded. The speed of sound is the speed at which the wave travels. When the speed of the object is greater than the speed of sound, the wave fronts pile up on one another. These wave fronts pile up together and form a shock wave passing a given point at the same time. All the energy of the sound waves is concentrated into a very small area called the shock wave. This very concentrated energy of sound builds up into a shock wave which causes an extremely loud sound called shock boom.



**Q 23** What are ultrasonic waves? Discuss the uses, generation and detection of ultrasonic waves.

### ANSWER

Ultrasonic waves are those waves whose frequency is higher than the frequency of the sound waves. The frequency of the sound waves is less than  $20,000 \text{ Hz}$  and that of the ultrasonic waves is more than  $20,000 \text{ Hz}$ .

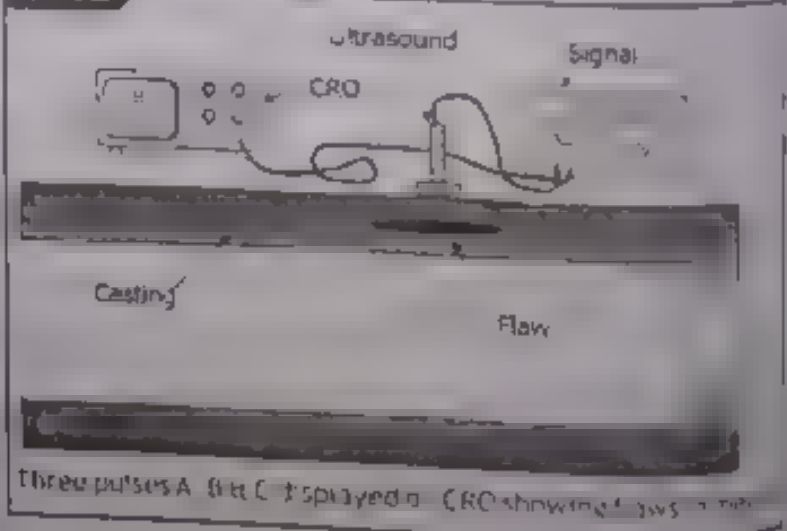
- ▶ Ultrasonic waves are used in many ways. They are used in the medical field for the diagnosis of diseases.
- ▶ They are used in the industrial field for the detection of defects in the materials.
- ▶ They are used in the naval field for the detection of submarines.
- ▶ They are used in the agricultural field for the detection of the growth of the crops.
- ▶ They are used in the geological field for the detection of the structure of the earth's crust.
- ▶ They are used in the meteorological field for the detection of the weather.
- ▶ They are used in the astronomical field for the detection of the distance of the stars.
- ▶ They are used in the biological field for the detection of the life of the organisms.
- ▶ They are used in the chemical field for the detection of the concentration of the solutions.
- ▶ They are used in the physical field for the detection of the properties of the materials.

### Uses of ultrasonic waves

Ultrasonic waves have many uses. Some of the important uses of ultrasonic waves are the following:

- ▶ **Medical uses:** Ultrasonic waves are used in the medical field for the diagnosis of diseases. They are used in the ultrasound imaging technique to produce the images of the internal organs of the body. The images are used to detect the presence of the diseases.
- ▶ **Industrial uses:** Ultrasonic waves are used in the industrial field for the detection of defects in the materials. They are used in the ultrasonic testing technique to detect the defects in the materials.
- ▶ **Naval uses:** Ultrasonic waves are used in the naval field for the detection of submarines. They are used in the sonar technique to detect the presence of the submarines.
- ▶ **Agricultural uses:** Ultrasonic waves are used in the agricultural field for the detection of the growth of the crops. They are used in the ultrasonic sensing technique to detect the growth of the crops.
- ▶ **Geological uses:** Ultrasonic waves are used in the geological field for the detection of the structure of the earth's crust. They are used in the ultrasonic seismic technique to detect the structure of the earth's crust.
- ▶ **Meteorological uses:** Ultrasonic waves are used in the meteorological field for the detection of the weather. They are used in the ultrasonic weather sensing technique to detect the weather.
- ▶ **Astronomical uses:** Ultrasonic waves are used in the astronomical field for the detection of the distance of the stars. They are used in the ultrasonic distance sensing technique to detect the distance of the stars.
- ▶ **Biological uses:** Ultrasonic waves are used in the biological field for the detection of the life of the organisms. They are used in the ultrasonic life sensing technique to detect the life of the organisms.
- ▶ **Chemical uses:** Ultrasonic waves are used in the chemical field for the detection of the concentration of the solutions. They are used in the ultrasonic concentration sensing technique to detect the concentration of the solutions.
- ▶ **Physical uses:** Ultrasonic waves are used in the physical field for the detection of the properties of the materials. They are used in the ultrasonic property sensing technique to detect the properties of the materials.

FIGURE 8.10



Three pulses A, B, and C displayed on CRO showing flaws in the casting.

- ▶ Sound waves and electromagnetic waves are in use for the treatment of a number of diseases like rheumatism and arthritis.
- ▶ Ultrasound waves are used in a process, such as cavitation, which is pain relieving.
- ▶ When sound waves are focused on a small space or point, very high intensities can be produced. These intensities are used and a large number of holes are formed. This process is called lithotripsy.
- ▶ The use of ultrasound produces violently destructive forces near the solid surface in the liquid.

### Do You Know?

A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance. A use of the time delay caused by a bat to sense distances.



### Generation of ultrasonic sound waves

- ▶ Sound waves can be generated by any object which is capable of vibrating at a frequency higher than 20 kHz.
- ▶ In most applications, ultrasonic waves are generated by applying an electric current to a specialized kind of material known as piezoelectric crystal. This crystal converts electrical energy into mechanical energy. This energy is used to vibrate at high frequency, generating ultrasonic waves.
- ▶ In another method, a high voltage is applied to a special crystal which causes the crystal to vibrate at a frequency above 20 kHz, generating ultrasonic waves.

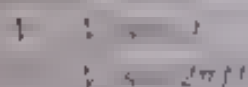
### Detection of Ultrasonic Waves

- ▶ Ultrasonic waves can be detected but we need to consider a few methods.

#### (a) Piezoelectric Detection method

- ▶ A crystal is used to detect compressions and rarefactions so when they are allowed to vibrate, an electric current or voltage is produced across the crystal faces.

This current is then amplified in an amplifier and the ultrasonic waves are detected.



#### (b) Kundt's tube method

- ▶ A horizontal glass tube supported horizontally with air column in it.
- ▶ Fine powder like lycopodium is sprinkled in the tube.
- ▶ When sound waves are passed through this Kundt's tube, the lycopodium powder in the tube collects at the nodes and leaves gaps at the antinodes.
- ▶ The length of the tube is very small.

### MCQ's

- Which of the following is used in radar?  
 (A) sound waves (B) visible waves (C) radio waves (D) microwaves
- Radar is the application of  
 (A) chemical effect (B) electric effect (C) Doppler's effect (D) magnetic effect
- Stars moving towards the earth show  
 (A) blue shift (B) red shift (C) UV shift (D) long wavelength
- Car A has a siren sounding a note of 540 Hz. A listener in car B hears a note of 564 Hz. Both move in the same direction. One concludes that  
 (A) B leads A and moves faster (B) B is behind A and moves faster (C) B is behind A and moves slower (D) B is ahead of A and moves slower

4. The detection of submarines can be detected by  
 A. Doppler effect B. Fermi phase effect C. Diffraction D. Compton effect
5. Star moving away from earth shows  
 A. Red shift B. Blue shift C. No shift D. All of these

**ANSWERS KEY**

|     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1 + | 2 C | 3 A | 4 C | 5 A | 6 B |
|-----|-----|-----|-----|-----|-----|

**FORMULAE**

|                                                        |                                               |  |
|--------------------------------------------------------|-----------------------------------------------|--|
| Relation between wave length and frequency             | $\lambda = \frac{v}{f}$                       |  |
| Relation between frequency and time period             | $f = \frac{1}{T}$                             |  |
| Relation between phase difference and path difference  | $\Delta \phi = \frac{2\pi}{\lambda} \Delta x$ |  |
| Speed of sound                                         | $v = \frac{\lambda}{T}$                       |  |
| Boyer's law for sound process                          | $v \propto \sqrt{\frac{E}{\rho}}$             |  |
| Newton's formula for speed                             | $v = \sqrt{\frac{p}{\rho}}$                   |  |
| Boyer's law for sound process                          | $v \propto \sqrt{\frac{E}{\rho}}$             |  |
| Speed of sound in air                                  | $v = 332 \text{ m/s}$                         |  |
| Relation between speed of sound and temperature        | $v \propto \sqrt{T}$                          |  |
| Condition for constructive interference in sound waves | $\Delta x = n\lambda$                         |  |
| Condition for destructive interference in sound waves  | $\Delta x = (2n+1)\frac{\lambda}{2}$          |  |
| Beat frequency                                         | $f_b =  f_1 - f_2 $                           |  |
| Velocity of transverse waves on string                 | $v = \sqrt{\frac{T}{\mu}}$                    |  |
| Fundamental frequency of stationary waves on string    | $f_1 = \frac{v}{2L}$                          |  |
| Frequency of stationary waves on string other order    | $f_n = \frac{nv}{2L}$                         |  |

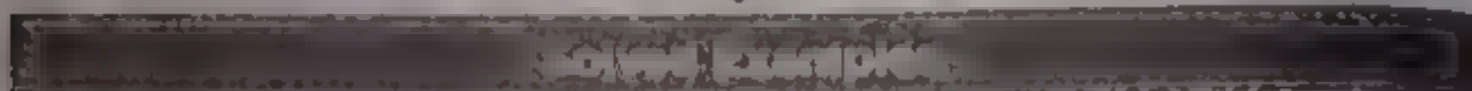
|                                                                                         |  |                                              |
|-----------------------------------------------------------------------------------------|--|----------------------------------------------|
| Wave length of stationary waves on string (nth mode)                                    |  |                                              |
| Frequency of longitudinal stationary waves in organ pipe open at both ends (nth mode)   |  | $f_n = n f$<br>where $n = 1, 2, 3, 4, \dots$ |
| Wave length of longitudinal stationary waves in organ pipe open at both ends (nth mode) |  |                                              |
| Apparent frequency when observer(A) moves towards stationary source                     |  |                                              |
| Apparent frequency when observer(B) moves away from stationary source                   |  |                                              |
| Doppler shift                                                                           |  |                                              |
| Apparent frequency when source moves towards stationary observer(C)                     |  |                                              |
| Apparent frequency when source(S) moves away from stationary observer(D)                |  |                                              |



### Key Points

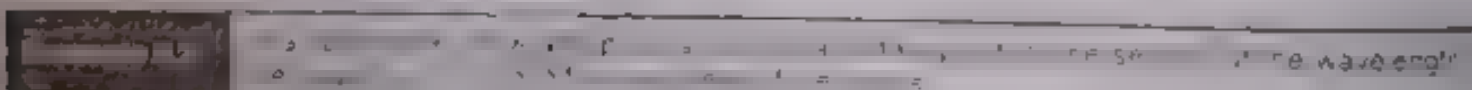
- Wave is a disturbance that travels with energy & matter. It is called wave.
- Mechanical Waves** These are waves which require a medium for their propagation. These are called mechanical waves.
- Electromagnetic Waves** These are waves which do not require a medium for their propagation. These are called electromagnetic waves.
- Transverse Waves** A wave in which the particles of the medium vibrate perpendicular to the direction of propagation of the wave is called transverse wave.
- Longitudinal Wave** A wave in which the particles of the medium vibrate parallel to the direction of propagation of the wave is called longitudinal wave.
- Sound** The stimulus due to a vibrating body, capable of producing sensation of hearing is called sound.
- Interference of Waves** The effect produced due to the superposition of two waves is called interference.
- Beats** The periodic variation in the intensity of sound which is heard when two waves of slightly different frequencies are played simultaneously, is known as beats.
- Stationary Waves** The superposition of two plane waves having the same amplitude & frequency, travelling in opposite directions along a line, produces a wave known as stationary wave.
- Resonance** The vibration of a body or the air column under the influence of periodic force having the same frequency is called resonance.

- ♦ Organ Pipes are closed at one end and open at the other. The closed end is a node and the open end is an antinode.
- ♦ Doppler's Effect is the change in frequency of a wave due to the relative motion of the source and the observer.
- ♦ Ultrasonic Waves are sound waves having frequency greater than  $2 \times 10^4$  Hz and used in many applications.



A wave generator gives out 2 pulses in 1 s. Find the period and frequency of the pulses.

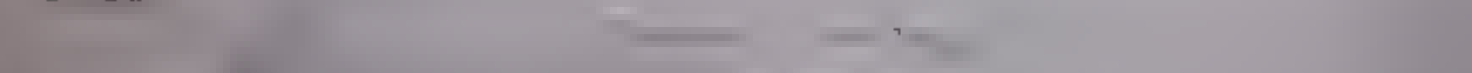
**Solution**



**Solution**



**Formula**



Find the speed of sound in a steel railway track if the density of steel is  $7800 \text{ kg m}^{-3}$  and Young's modulus is  $2.0 \times 10^{11} \text{ Nm}^{-2}$ .

**Solution**



**Solution**



Find the speed of sound in a Neon gas at  $0^\circ\text{C}$  ( $m = 20 \text{ g/mol}$  and  $\gamma$  for monoatomic gas = 1.67)

Given

$$\gamma = 1.67$$

$$m = 20 \text{ g/mol}$$

$$R = 8.314 \text{ J/K mol}$$

Required

$$v = ?$$

Solution

$$v = \sqrt{\frac{\gamma R T}{m}}$$

$$= \sqrt{\frac{1.67 \times 8.314 \times 273}{20 \times 10^{-3}}} = 432 \text{ ms}^{-1}$$

Two pianos sound the same note. If the vibration from one is 221.60 Hz and that of the other is 221.40 Hz. What is the beat frequency between the two notes?

Solution

$$\text{Beat frequency} = f_1 - f_2$$

$$f = 221.60 \text{ Hz} - 221.40 \text{ Hz}$$

$$f = 0.20 \text{ Hz}$$

The speed of a wave on a particular string is 24 m/s. If the string is 80 m long to what driving frequency will it resonate

Solution

3 possible resonance wavelength are given by

$$\lambda = \frac{2L}{n}$$

$$\lambda = \frac{2 \times 80}{1} = 160 \text{ m}$$

$$\lambda = \frac{2L}{2} = 80 \text{ m}$$

$$\lambda = \frac{2L}{3} = 53.33 \text{ m}$$

$$\lambda = \frac{2L}{4} = 40 \text{ m}$$

A string 40 m long has a mass of 30 g. One end of the string is fastened to a staff and the other end hangs over a pulley with a 20 kg mass attached. What is the speed of a transverse wave in this string?

Solution

$$L = 40 \text{ m}$$

$$M = 30 \text{ g} = 0.03 \text{ kg}$$

$$\text{Mass per unit length } \mu = \frac{M}{L} = \frac{0.03}{40} = 7.5 \times 10^{-4} \text{ kg/m}$$

$$\text{Mass hanging with the string } = 20 \text{ kg}$$

$$\text{Force } F = mg = 20 \times 9.8 = 196 \text{ N}$$

$$\text{Speed of transverse wave } v = \sqrt{\frac{F}{\mu}}$$

A pipe open at one end and closed at the other end is 82 cm long. What are the three lowest frequencies to which it will resonate? Take the speed of sound as  $340 \text{ m s}^{-1}$ .

Solution

A car is moving at  $20 \text{ m s}^{-1}$  on a straight road with its  $500 \text{ Hz}$  horn sounding. You are standing at the roadside. What frequency do you hear as the car is (a) approaching, (b) receding from you at  $20 \text{ m s}^{-1}$ ? Take the speed of sound as  $340 \text{ m s}^{-1}$ .

Solution

### Text Book Exercises

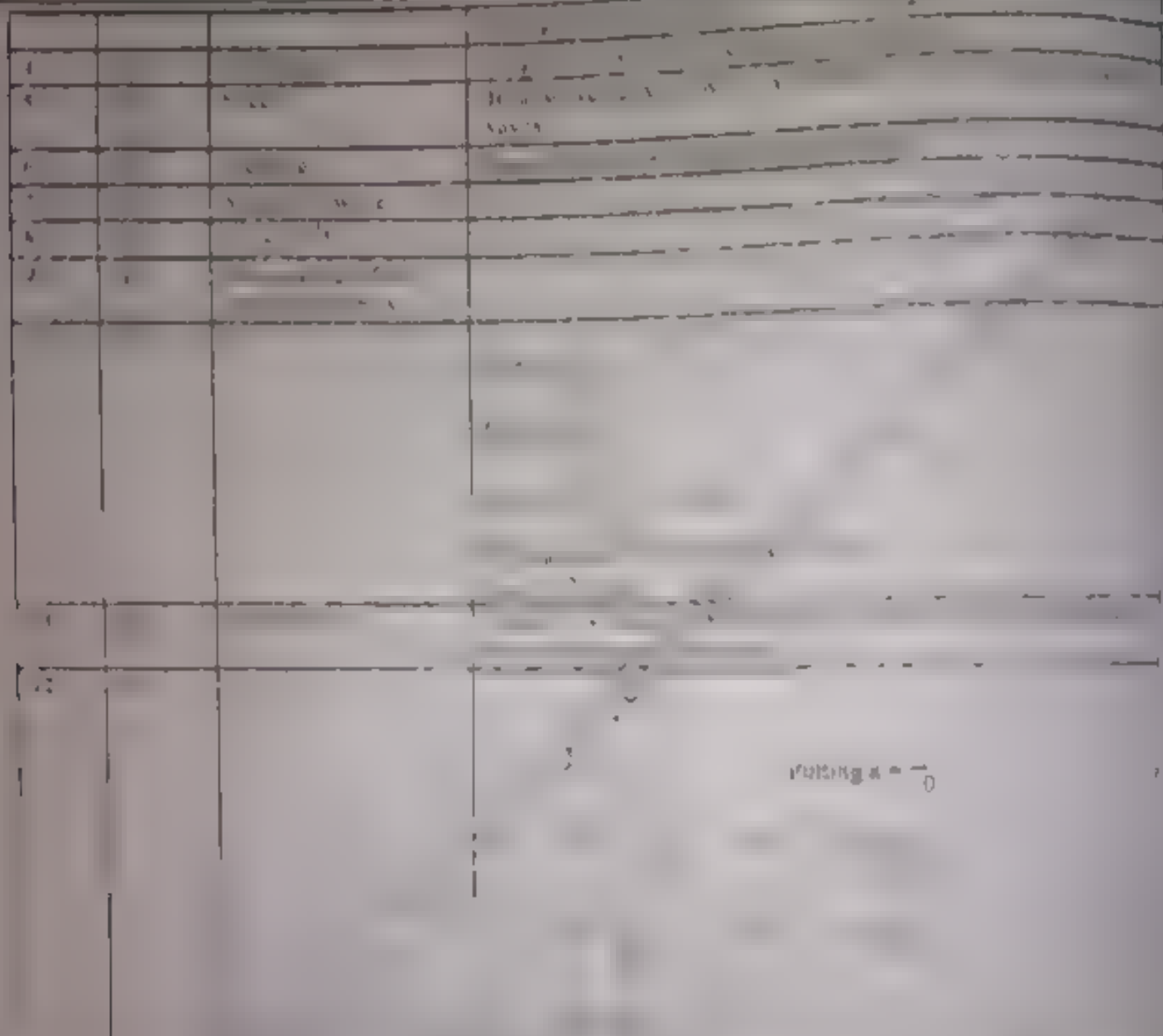
Q 1 Select the correct answer of the following questions.

1. When a wave goes from one medium to another medium, which one of the following characteristics of the wave remains constant?

(a) wavelength (b) frequency (c) wave speed (d) wave period

1. When a transverse wave travelling through a rare medium is reflected from a denser medium then it undergoes a phase change of  $\pi$ .
2. If the tension in the string is doubled and its mass per unit length is reduced to half. The speed of transverse wave on it is  $2\sqrt{3}$  times.
3. Which one of the following properties is not exhibited by the longitudinal waves?  
(a) Reflection (b) Refraction (c) Interference (d) Polarization
4. A sound source and a listener are both at rest relative to each other. If the listener moves towards the source, then which one of the following does not change?  
(a) Frequency (b) Speed (c) Wavelength (d) Temperature
5. Which one of the following factors have no effect on the speed of sound in a gas?  
(a) Pressure (b) Density (c) Temperature (d) Humidity
6. There is no net transfer of energy by progressive wave motion.  
(a) Longitudinal wave (b) Transverse wave (c) Stationary wave (d) Electromagnetic wave
7. Which one of the following could be the frequency of a progressive wave?  
(a) 0 Hz (b)  $10^{-15}$  Hz (c)  $10^{15}$  Hz (d)  $10^{16}$  Hz
8. When a stationary wave is formed, then its frequency is  
(a) Same as the frequency of the individual waves (b) Twice that of the individual waves (c) Half that of the individual waves (d) Double that of the individual waves
9. The fundamental frequency of a closed organ pipe is  $f$ . If both the ends of pipe are opened then its fundamental frequency will be  
(a)  $f$  (b)  $0.5f$  (c)  $2f$  (d)  $4f$
10. If the amplitude of wave is doubled, then its intensity is  
(a) Doubled (b) Four times (c) One fourth (d) One sixteenth
11. A sound source is moving towards a listener with a velocity  $v$  of sound. The ratio of apparent to real frequency is  
(a)  $\frac{v}{v-v}$  (b)  $\frac{v}{v+v}$  (c)  $\frac{v+v}{v}$  (d)  $\frac{v-v}{v}$

| Q. No. | Ans. | Explanatory                                                                                                                                                                                                                       |
|--------|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1      | (a)  | When a transverse wave travelling through a rare medium is reflected from a denser medium then it undergoes a phase change of $\pi$ .                                                                                             |
| 2      | (b)  | If the tension in the string is doubled and its mass per unit length is reduced to half. The speed of transverse wave on it is $2\sqrt{3}$ times.                                                                                 |
| 3      | (d)  | Which one of the following properties is not exhibited by the longitudinal waves?<br>(a) Reflection (b) Refraction (c) Interference (d) Polarization                                                                              |
| 4      | (a)  | A sound source and a listener are both at rest relative to each other. If the listener moves towards the source, then which one of the following does not change?<br>(a) Frequency (b) Speed (c) Wavelength (d) Temperature       |
| 5      | (c)  | Which one of the following factors have no effect on the speed of sound in a gas?<br>(a) Pressure (b) Density (c) Temperature (d) Humidity                                                                                        |
| 6      | (c)  | There is no net transfer of energy by progressive wave motion.<br>(a) Longitudinal wave (b) Transverse wave (c) Stationary wave (d) Electromagnetic wave                                                                          |
| 7      | (c)  | Which one of the following could be the frequency of a progressive wave?<br>(a) 0 Hz (b) $10^{-15}$ Hz (c) $10^{15}$ Hz (d) $10^{16}$ Hz                                                                                          |
| 8      | (a)  | When a stationary wave is formed, then its frequency is<br>(a) Same as the frequency of the individual waves (b) Twice that of the individual waves (c) Half that of the individual waves (d) Double that of the individual waves |
| 9      | (c)  | The fundamental frequency of a closed organ pipe is $f$ . If both the ends of pipe are opened then its fundamental frequency will be<br>(a) $f$ (b) $0.5f$ (c) $2f$ (d) $4f$                                                      |
| 10     | (c)  | If the amplitude of wave is doubled, then its intensity is<br>(a) Doubled (b) Four times (c) One fourth (d) One sixteenth                                                                                                         |
| 11     | (c)  | A sound source is moving towards a listener with a velocity $v$ of sound. The ratio of apparent to real frequency is<br>(a) $\frac{v}{v-v}$ (b) $\frac{v}{v+v}$ (c) $\frac{v+v}{v}$ (d) $\frac{v-v}{v}$                           |



$$y = 0.1 \sin(2\pi x / \lambda + \phi)$$



of a short wave is the following relation

Find the ratio of the wave frequency to the wave velocity

1. Difference between a progressive wave and a stationary wave

| S. No. | Question                                                                  | Answer                                                                                                                                                                                                                                                     |
|--------|---------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1      | What is the difference between a progressive wave and a stationary wave?  | A progressive wave is a wave that travels through a medium, carrying energy from one point to another. A stationary wave is a wave that does not travel, but rather oscillates in place.                                                                   |
| 2      | What is the difference between a transverse wave and a longitudinal wave? | A transverse wave is a wave in which the particles of the medium oscillate perpendicular to the direction of wave propagation. A longitudinal wave is a wave in which the particles of the medium oscillate parallel to the direction of wave propagation. |
| 3      | What is the difference between a wave and a pulse?                        | A wave is a disturbance that travels through a medium, carrying energy from one point to another. A pulse is a single disturbance that travels through a medium, carrying energy from one point to another.                                                |
| 4      | What is the difference between a wave and a particle?                     | A wave is a disturbance that travels through a medium, carrying energy from one point to another. A particle is a small object that moves back and forth in a medium, but does not travel with the wave.                                                   |
| 5      | What is the difference between a wave and a sound wave?                   | A wave is a disturbance that travels through a medium, carrying energy from one point to another. A sound wave is a type of wave that travels through a medium, carrying energy from one point to another.                                                 |

22. Clearly explain the difference between  $\sin^2 \theta$  and  $\sin \theta$  in wave optics.

23. How are beats formed?

24. Tuning of musical instruments

- Beat
- Frequency of the untuned instrument by beats by increasing the string.
- When the frequency of the musical instrument is equal to frequency

25. Two wave pulses travel towards each other. What happens in the case of constructive interference?

- The amplitude of the wave is double of the individual waves.
- The frequency of the wave is same as the individual waves.
- The wavelength of the wave is same as the individual waves.
- The speed of the wave is same as the individual waves.

26. What are the conditions for interference of waves?

27. The condition is for interference of waves

- The waves must be coherent.
- The waves must be monochromatic.
- The waves must be of the same frequency.
- The waves must be of the same amplitude.

28. Constructive interference

- If two waves are in phase at a point, the resultant wave is of maximum amplitude.
- When waves are in phase, the path difference is an integral multiple of the wavelength.
- For constructive interference, the path difference is an integral multiple of the wavelength.

$$2\pi x = 2\pi \lambda$$

$$x = \lambda$$

$$x = 0.5 \lambda$$

### (c) Destructive interference

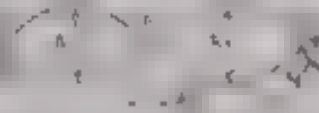
- ▶ For destructive interference, the path difference must be an odd multiple of  $\lambda/2$ .
- ▶ For destructive interference, the path difference must be an odd multiple of  $\lambda/2$ .

$$\lambda$$

26. (a) The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

(b) The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.
- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.
- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.
- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.



27. (a) The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

(b) The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.
- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.
- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.
- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

28. (a) The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

(b) The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

$$\lambda$$

- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

$$x = \lambda$$

- ▶ The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

29. (a) The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

(b) The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

(c) The wave is a transverse wave. The wave is a transverse wave. The wave is a transverse wave.

$$x = \lambda$$

$$x = 0.5 \lambda$$

$$x = 0.5 \lambda$$

$$x = 0.5 \lambda$$

$$x = 0.5 \lambda$$

$$x = 0.5 \lambda$$

$$x = 0.5 \lambda$$

$$x = 0.5 \lambda$$

$$x = 0.5 \lambda$$

Therefore  $v \propto \sqrt{T}$

- The speed of sound is directly proportional to the square root of temperature.
- When temperature of medium increases the speed of sound increases.
- When temperature of medium decreases the speed of sound decreases.
- Therefore speed of sound changes with change in temperature of gas medium.
- The speed of sound increases  $0.61 \text{ m/s}$ ,  $0.61 \text{ cm/s}$ , with  $1^\circ\text{C}$  rise in temperature.  
 $= v_0 + 0.61$

Q.10 Is it possible for two astronauts to talk directly to one another even if they remove their helmets?

**Ans:** No

- It is not possible for two astronauts to talk directly to one another if they remove their helmets.
- It is because the sound waves are mechanical waves which require a medium for propagation.
- As there is no medium between the two astronauts, they cannot hear each other.

Q.11 Estimate the frequencies at which a test tube 15 cm long resonates when you blow across its lips?

**Ans:** Length of tube =  $L = 15 \text{ cm} = 0.15 \text{ m}$

Speed of sound at  $0^\circ\text{C} = v = 332 \text{ m/s}$

Fundamental frequency for closed tube is

$$f_1 = \frac{v}{4L}$$

$$f_1 = \frac{332}{4 \times 0.15}$$

$$f_1 = 553 \text{ Hz}$$

$$f_3 = 3 f_1 = 3 \times 553 = 1660 \text{ Hz}$$

$$f_5 = 5 f_1 = 5 \times 553 = 2765 \text{ Hz}$$



## Comprehensive Education

Q3 Give a short response to the following questions.

1. What is meant by wave motion? Define the terms wave length and frequency and derive the relationship between them.

**Ans:** See Q. 1 and Q. 4 from book

2. Describe longitudinal and transverse wave with examples and clearly explain the difference between them.

**Ans:** See Q. 4 and Q. 5 from book

3. Explain the following terms

(a) Crest

(b) Trough

(c) Compression

(d) Rarefaction

(e) Node

(f) Anti-node

**Ans:** See Q. 4 and Q. 5 from book

4. What do you mean by stationary waves? Show that as the string vibrates in more and more loops, its frequency increases and wavelength decreases

**Ans:** See Q. 6 from book

- 5 Explain Newton's formula for the speed of sound. Show that how it was corrected by a French scientist Laplace?

**Ex. 8.1** See Example 8.1

- 6 Explain the speed of sound in a gas and give all the factors which affect the speed of sound in the air.

**Ex. 8.2** See Example 8.2

- 7 How the speeds of sound in the air varies with temperature and hence show that for each one degree cent grade rise in temperature the speed of sound increases by  $0.61 \text{ m s}^{-1}$ .

**Ex. 8.3** See Example 8.3

- 8 What are beats? Explain how they are produced and show that the number of beats per second is equal to the difference in frequencies of the two sources.

**Ex. 8.4** See Example 8.4

- 9 What is Doppler's Effect? Derive expression for the frequencies heard  
 a) When the sounding source approaches a stationary listener.  
 b) When listener moves towards a stationary sounding source.

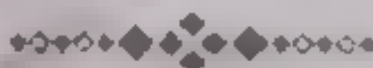
**Ex. 8.5** See Example 8.5

- 10 What are Organ Pipes? Show that an open organ pipe has four harmonics than a closed organ pipe?

**Ex. 8.6** See Example 8.6

- 11 Explain the vibrations in a closed organ pipe and show that the frequency of third harmonic is  $\frac{5}{4}f_1$ .

**Ex. 8.7** See Example 8.7



## Numerical Problems

- 1 What are the wave lengths of a television station which transmits via an antenna  $600 \text{ MHz}$  and sound  $5 \text{ GHz}$  respectively? Take speed of electromagnetic wave as  $3 \times 10^8 \text{ m s}^{-1}$ .

**Given Data**  $f_1 = 600 \text{ MHz} = 600 \times 10^6 \text{ Hz}$   
 $f_2 = 5 \text{ GHz} = 5 \times 10^9 \text{ Hz}$   
 Speed of wave  $c = 3 \times 10^8 \text{ m s}^{-1}$   
 Wave length  $\lambda = ?$

**Solution**

**Formula**

$$c = f\lambda$$

$$\lambda = \frac{c}{f}$$

$$\lambda_1 = \frac{c}{f_1}$$

$$\lambda = \frac{3 \times 10^8}{600 \times 10^6}$$

$$\lambda = 0.5 \text{ m} \quad \text{or} \quad \lambda_1 = 0.5 \text{ m}$$

$$\lambda_2 = \frac{c}{f_2}$$

$$\lambda = \frac{c}{f}$$

$$\lambda_2 = \frac{c}{f_2}$$

$$\lambda = \frac{v}{f}$$

$$\left[ \frac{\text{m}}{\text{s}} \right] \text{ or } \left[ \frac{\text{m}}{\text{s}} \right]$$

2. A person on the sea shore observes that 48 waves reach the shore in one minute. If the wavelength of the waves is 10 metre then find the velocity of the waves.

Given Data

$n = 48$  waves in 1 min

$\lambda = 10$  m

$t = 60$  s

$f = ?$

Wavelength  $\lambda = 10$  m

Velocity of wave  $v = ?$

Solution

Formula

$$v = \lambda f$$

$$v = \lambda \times \frac{n}{t}$$

$$v = \frac{\lambda n}{t}$$

3. In a ripple tank 60 waves pass through a certain point in 14 second. The speed of the wave is 3.5 m/s then find the wave length of the waves.

Given Data

$n = 60$  waves in 14 s

$t = 14$  s

Speed  $v = 3.5$  m/s

Wavelength  $\lambda = ?$

Solution

$$v = \lambda f$$

$$v = \lambda \times \frac{n}{t}$$

$$\lambda = \frac{v t}{n}$$

Wave length

$$\lambda = \frac{v t}{n}$$

$$\lambda = \frac{3.5 \times 14}{60}$$

$$\lambda = \frac{49}{60}$$

$$\lambda = 0.8167$$

$$\lambda = 0.82$$

$$\lambda = 0.82 \text{ m}$$

4. A string of a guitar is 3 m long vibrates with 4 nodes. 2 of them at the two ends. Find the wavelength & speed of the wave in the string if it vibrates at 500 Hz.

Given Data

Length  $L = 3$  m

No. of loops  $n = 3$

Frequency  $f = 500$  Hz

(a) Wavelength  $\lambda = ?$

(b) Speed  $v = ?$

Solution

Formula

$$\lambda = \frac{2L}{n}$$

$$\lambda = \frac{2 \times 3}{3}$$

$$\lambda = 2 \text{ m}$$

$$\lambda = 2 \text{ m}$$

(b)

$$v = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{340}{866} = 0.392 \text{ m}$$

$$\lambda = 0.392 \text{ m}$$

5. A tension of 400 N causes a 300 g wire of length 1.6 m to vibrate with a frequency of 40 Hz. What is wavelength of transverse waves? Find the speed of sound in Helium gas at 27°C ( $\gamma = 1.66$  and  $R = 8334 \text{ J/k mol}$ .)

Data

$$T = 400 \text{ N}$$

$$L = 1.6 \text{ m}$$

$$M = 300 \text{ g} = 0.3 \text{ kg}$$

$$f = 40 \text{ Hz}$$

$$\lambda = ?$$

Solution

Wavelength =  $\lambda = ?$   
speed of wave in stretched string is

$$v = \sqrt{\frac{T}{\mu}} \quad \text{Putting values}$$

$$v = \sqrt{\frac{400}{0.3/1.6}}$$

$$v = \sqrt{\frac{400 \times 1.6}{0.3}}$$

$$v = \sqrt{\frac{6400}{0.3}}$$

$$v = \sqrt{21333.33}$$

$$v = 146 \text{ m/s}$$

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{146}{40}$$

$$\lambda = 3.65 \text{ m}$$

6. Compare the theoretical speed of sound in hydrogen ( $M_H = 2.0 \text{ g/mol}$ ,  $\gamma_H = 1.4$ ) with helium ( $M_{He} = 4.0 \text{ g/mol}$ ,  $\gamma_{He} = 1.66$  &  $R = 8334 \text{ J/k}^\circ\text{mol}$ ) at 0°C

Given Data

$$M_H = 2.0 \text{ g/mol}$$

$$\gamma_H = 1.4$$

$$M_{He} = 4.0 \text{ g/mol}$$

$$\gamma_{He} = 1.66$$

$$R = 8334 \text{ J/k}^\circ\text{mol}$$

$$T_{\text{temp}} = T = 0^\circ\text{C} = 273 \text{ K}$$

Solution.

$$v = \sqrt{\frac{\gamma R T}{M}}$$

$$\frac{v_H}{v_{He}} = \sqrt{\frac{\gamma_H R T}{M_H}} \div \sqrt{\frac{\gamma_{He} R T}{M_{He}}}$$

$$\frac{v_H}{v_{He}} = \sqrt{\frac{\gamma_H M_{He}}{\gamma_{He} M_H}} = \sqrt{\frac{1.4 \times 4}{1.66 \times 2}} = \sqrt{1.68}$$

$$\frac{v_H}{v_{He}} = \sqrt{1.68}$$

$$v_{He} = 0.77$$

$$v_H = 0.77 v_{He}$$

$$v_H = 0.77 v_{He}$$

The speed of sound in air at  $0^{\circ}\text{C}$  is  $332 \text{ m s}^{-1}$ . What will be the speed of sound at  $22^{\circ}\text{C}$ ?

**Given Data**  
 Speed of sound in air at  $0^{\circ}\text{C} = v_0 = 332 \text{ m s}^{-1}$   
 $t_0 = 0 + 273 = 273 \text{ K}$   
 $T = 22^{\circ}\text{C} = 22 + 273 = 295 \text{ K}$   
 Speed of sound at  $22^{\circ}\text{C} = v = ?$

**Solution**

**Formula**

$$v = v_0 \sqrt{\frac{T}{t_0}}$$

$$= 332 \sqrt{\frac{295}{273}}$$

$$v = 332 (1.0395)$$

$$v = 345.11 \text{ m s}^{-1}$$

### ALTERNATE METHOD

$$v = v_0 + 0.61 t \quad (\text{where } t = 22^{\circ}\text{C})$$

$$v = 332 + 0.61 (22)$$

$$v = 332 + 13.2$$

$$v = 345.2 \text{ m s}^{-1}$$

Two tuning forks P and Q give 4 beats per second. On loading Q lightly with wax we get 3 beats per second. What is the frequency of Q before and after loading if the frequency of P is  $512 \text{ Hz}$ ?

**Given Data**  
 Frequency of tuning fork P =  $f_P = 512 \text{ Hz}$   
 Frequency of tuning fork Q =  $f_Q = ?$   
 No. of beats before loading tuning fork Q = 4 beats  
 No. of beats after loading tuning fork Q = 3 beats  
 To find:  $f_Q$

$$f = f_P \pm 4$$

$$f = 512 \pm 4 = 516 \text{ Hz}$$

**Formula**

$$f = 512 \pm 4$$

$$512 - 4 = 508 \text{ Hz}$$

$$512 + 4 = 516 \text{ Hz or } 508 \text{ Hz}$$

As the frequency of tuning fork Q is  $516 \text{ Hz}$  when tuning fork Q is loaded with wax its frequency decreases.  
 As the frequency of tuning fork Q is  $508 \text{ Hz}$  or it can produce 3 beats per second when P is 512 Hz.  
 As the frequency of P is 512 Hz  
 $512 - f_Q = 3$   
 $f_Q = 512 - 3 = 509 \text{ Hz}$   
 Therefore, frequency of tuning fork Q before loading =  $516 \text{ Hz}$   
 After loading =  $509 \text{ Hz}$

On a sunny day, the speed of sound in the air is  $340 \text{ m s}^{-1}$ , 2 tuning forks A & B are sounded simultaneously. The wave length of the sounds emitted are  $1.5 \text{ m}$  and  $1.68 \text{ m}$  respectively. How many beats will produce per second?

**Given Data**

$$\text{Speed of sound} = v = 340 \text{ m s}^{-1}$$

$$\text{Wavelength of wave produced by tuning fork A} = \lambda_A = 1.5 \text{ m}$$

$$\text{Wavelength of wave produced by tuning fork B} = \lambda_B = 1.68 \text{ m}$$

$$\text{No. of beats} = \text{beat frequency} = f_b = ?$$

## Solution

Frequency of sound waves produced by tuning fork A is

$$f_A = \frac{v}{\lambda_A} \quad \text{or} \quad (f_A \lambda_A)$$

$$= \frac{330}{5}$$

$$= 66 \text{ Hz}$$

Frequency of sound waves produced by tuning fork B

$$f_B = \frac{v}{\lambda_B} \quad \text{or} \quad (f_B \lambda_B)$$

$$= \frac{330}{4}$$

$$= 82.5 \text{ Hz}$$

$$\therefore \text{Difference in frequency} = f_B - f_A$$

$$= 82.5 - 66$$

$$= 16.5 \text{ Hz}$$

$\therefore$  The difference in frequency is 16.5 Hz.

10. A sound source vibrates at 200 Hz and is receding from a stationary observer at 18 m/s. If the speed of sound is 331 m/s, then what frequency does the observer hear?

## Given Data

$$\text{Frequency of sound source } f_s = 200 \text{ Hz}$$

$$\text{Speed of sound } v = 331 \text{ m/s}$$

$$\text{Speed of source } v_s = 18 \text{ m/s}$$

$$\text{Observer is stationary}$$

## Solution

As the sound source is receding from the stationary observer, the apparent frequency is given by

$$f_a = f_s \left( \frac{v}{v + v_s} \right)$$

$$= 200 \left( \frac{331}{331 + 18} \right)$$

$$= 187.65 \text{ Hz}$$

$$\therefore f_a = 187.65 \text{ Hz}$$

11. Suppose a train that has a 150 Hz horn is moving at 35 m/s in still air on the day when the speed of sound is 340 m/s (a) what frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes and moves away from him? (b) what frequency is observed by the train's engineer traveling on the train?

## Given Data

$$\text{Frequency of sound source } f_s = 150 \text{ Hz}$$

$$\text{Speed of sound } v = 340 \text{ m/s}$$

$$\text{Speed of train } v_s = 35 \text{ m/s}$$

## To Find

(a) (i) The apparent frequency when the train approaches  $f_a$

(ii) The apparent frequency when the train recedes past the station  $f_r$

(b) Frequency observed by the train's engineer traveling with the train  $f_e$

## Solution

(a) (i) When the sound source approaches the stationary listener, the apparent frequency is given by

$$f_a = f_s \left( \frac{v}{v - v_s} \right)$$

$$= 150 \left( \frac{340}{340 - 35} \right)$$

$$= 167 \text{ Hz}$$

(ii) When the train is moving away from stationary observer, the apparent frequency heard is

$$f = \frac{v}{\lambda}$$
$$= \left( \frac{340}{1.5 + 1.5} \right) \times 150$$
$$f = 340 \text{ Hz}$$

h. There is a change in frequency observed by the train as it recedes from the platform. As the speed of sound is speed of train therefore he observes the frequency to be lower.

12. The first overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe 3.6 m in length. What is the length of the open organ pipe?

Given Data  
Length of closed pipe =  $L = 3.6 \text{ m}$   
First overtone of open pipe =  $f_2 = 2f_1$

$$\lambda_{\text{open}} = \frac{v}{2L}$$
$$f_2 = 2\left(\frac{v}{2L}\right) \dots\dots (1)$$

Frequency of first overtone of close pipe =  $f_2 = 3f_1$

Put the value of  $f_1 = \frac{v}{4L}$

According to given condition  
First overtone of open pipe = First overtone of close pipe

Put the values from Equation (1) and (2)

$$2(f_1)_{\text{open}} = 3(f_1)_{\text{close}}$$
$$2\left(\frac{v}{2L}\right) = 3\left(\frac{v}{4L}\right)$$

$$L = 1.44 \text{ m}$$

4.2 (For book answer) The first harmonic of an open organ pipe has the same frequency as the first harmonic of a closed pipe 3.6 m in length. What is the length of the open organ pipe?

Given Data  
Length of closed pipe =  $L = 3.6 \text{ m}$

$$f_1 = \frac{v}{4L}$$

Frequency of first harmonic of closed pipe

$$f_1 = \frac{v}{4L}$$

Solution  
According to given condition frequency of first harmonic of closed pipe = frequency of first harmonic of open pipe

$$f_1 = \frac{v}{4L}$$

Put the values of  $f_1$  and  $L$

$$\frac{v}{4L} = \frac{v}{4L} \quad (\text{same})$$

Length of open pipe

$$L = 3.6 \text{ m}$$
$$L = 3.6 \text{ m}$$
$$L = 3.6 \text{ m}$$

- [illegible]

74. 4 000

1 Why does the flash of light is seen earlier than hearing the thunder?

3. How much greater the speed of sound in hydrogen to that of oxygen?

- 3] Is the speed of transverse wave on a string the same as the speed at which a particle on the string moves?

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and the role of the accounting department in ensuring the integrity of the financial data.

$$y = 7.4x$$

4. At which temperature will the velocity of sound in air become double than the velocity in air at  $0^\circ\text{C}$ ?

(1)

Part 18  
There is a ... and  $T = 273 K$

$$\frac{2V}{V_1} = \frac{T}{273}$$

$$2 = \frac{T}{273}$$

Working ...

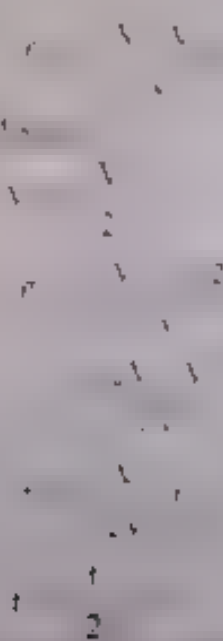
$$T = 546 K$$

$$T = 273 + 273$$

$$T = 546$$

A boy moves away with a half the speed of sound emitted by a speaker of a message. What frequency of sound is heard by him?

When ...  
him ...



where  $\lambda$  ...

$$\lambda = \frac{v}{f}$$

$$\lambda' = \frac{v - v/2}{f}$$

$$\lambda' = \frac{v/2}{f}$$

$$\lambda' = \frac{\lambda}{2}$$

The time ...

As a result of distant explosion, an observer senses a ground tremor and then hears the explosion. Explain the time difference?

Explanation

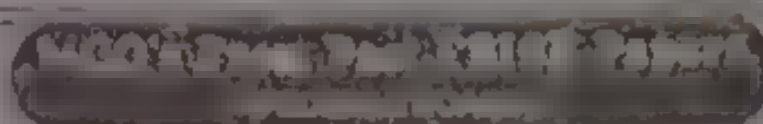
The speed of sound is given by

Since the speed of waves in solids is greater than the speed of waves in air, the observer senses the ground tremor first and then hears the explosion. This is why the observer senses the ground tremor first and then hears the explosion.

Why is a diver under water unable to hear the sound produced in air?

The water surface reflects the most of the sound waves back into the air. And only a small fraction of the sound waves is refracted through water. Due to this reason the diver under water cannot hear the sound produced in air.

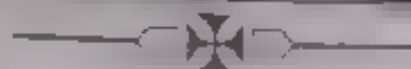




1. If stretching force  $T$  of wire increases then its frequency
  - (a) Increases
  - (b) Decreases
  - (c) Remains the same
  - (d) All of these
2. A stretched string of length 2m vibrates in four loops. Its wave length
  - (a) 1m
  - (b) 0.5m
  - (c) 2m
  - (d) 4m
3. In a stretched string, if the tension is increased four times then speed of wave increases
  - (a) 2 times
  - (b) 4 times
  - (c) 16 times
  - (d) 8 times
4. The speed of a wave in a string is 16m/s. If the tension is increased four times then the speed of wave
  - (a) 4m/s
  - (b) 8m/s
  - (c) constant
  - (d) 16m/s
5. When a wave travels from a medium to another medium, its
  - (a) frequency changes
  - (b) frequency
  - (c) speed
  - (d) wavelength
6. The speed of sound in air at  $0^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
7. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
8. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
9. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
10. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
11. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
12. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
13. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
14. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
15. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
16. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
17. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
18. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$
19. The speed of sound in air at  $10^\circ\text{C}$  is 332 m/s. At what temperature will the speed of sound be 340 m/s?
  - (a)  $546^\circ\text{C}$
  - (b)  $546^\circ\text{C}$
  - (c)  $5^\circ\text{C}$
  - (d)  $5^\circ\text{C}$

20. For  $1^{\circ}\text{C}$  rise in temperature, the speed of sound increases by  
 (a)  $0.6 \text{ ms}^{-1}$  (b)  $0.16 \text{ ms}^{-1}$  (c)  $6.0 \text{ ms}^{-1}$  (d)  $1.6 \text{ ms}^{-1}$  [P.B. SE (3M), 2018]
21. Which of the following factors has no effect on the speed of sound in a gas?  
 (a) Pressure (b) Temperature (c) Density (d) Humidity [P.B. SE (3M), 2019]
22. There is no net transfer of energy by particles of medium in  
 (a) longitudinal wave (b) Transverse wave (c) Progressive wave (d) Stationary wave [P.B. SE (3M), 2018]

| Q. No. | Ans. | Q. No. | Ans. | Q. No. | Ans. | Q. No. | Ans. |
|--------|------|--------|------|--------|------|--------|------|
| 1      | h    | 11     | L    | 21     | d    | 31     | J    |
| 2      | s    | 12     | h    | 22     | i    | 32     | J    |
| 3      | J    | 13     | t    | 23     |      | 33     |      |
| 4      | e    | 14     |      | 24     |      | 34     |      |



## SELF-ASSESSMENT PAPER

Total Mark 40

(1 x 5 = 5)

Question No 1 Choose the correct answer from the given options

## SECTION - A

- 1 The waves in which particles of the medium have displacement along the direction of propagation of waves are called.  
 A transverse waves B longitudinal waves  
 C electromagnetic waves D stationary wave
- 2 A taut wire is clamped at two points 1 m apart. It is plucked near one end. Which are the three longest wavelengths present on the vibrating wire?  
 A 1.0 m, 0.50 m and 0.25 m B 1.0 m, 0.67 m and 0.50 m  
 C 2.0 m, 1.0 m and 0.40 m D 2.0 m, 1.0 m and 0.57 m
- 3 When the amplitude of a wave becomes double its energy becomes  
 A one half B double C four times D six times
- 4 Speed of sound in vacuum is  
 A 330 m/s B 330 m/s<sup>2</sup> C 330 m/s<sup>2</sup> D zero
- 5 If a train of waves moving along a rope has a velocity of 100 m/s and a wavelength of 20 m, then the time period is  
 A 0.5 second B 2.0 second C 2.0 second D 0.5 second
- 6 A sound source is moving towards stationary listener with  $\frac{1}{10}$  of the speed of sound. The ratio of apparent to real frequency is:  
 A  $\frac{10}{9}$  B  $\frac{9}{10}$  C  $\frac{11}{10}$  D  $\frac{10}{11}$

Question No 2 Give short answers of following

3 x 7 = 21

## SECTION - B

- (i) Why sound produced due to sudden expansion going on in the sun cannot be heard on the earth?
- (ii) When tension produced in a wire is increased by 16 times then what will be the effect on speed of wave in the stretched string?
- (iii) Why does sound travel faster in solids than in gases?
- (iv) Clearly explain the difference between longitudinal and transverse waves.
- (v) Estimate the frequency of a wave whose crest takes 5 m to pass a point when you blow across its lips?
- (vi) A 40 g string 2 m in length vibrates in three loops. The tension in the string is 2.0 N. What is the wave length and frequency?
- (v) Give a brief definition of stationary waves.

Question No 3 Extensive Questions

(13)

## SECTION - C

- (a) Define Doppler effect and discuss its different cases. (8)
- (b) Suppose a train that has a 500 Hz horn is moving at 35 m/s in station on the day when the speed of sound is 340 m/s.  
 (i) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes and moves away from him?  
 (ii) What frequency is observed by the train's engineer traveling on the train? (5)

The End

# PHYSICAL OPTICS

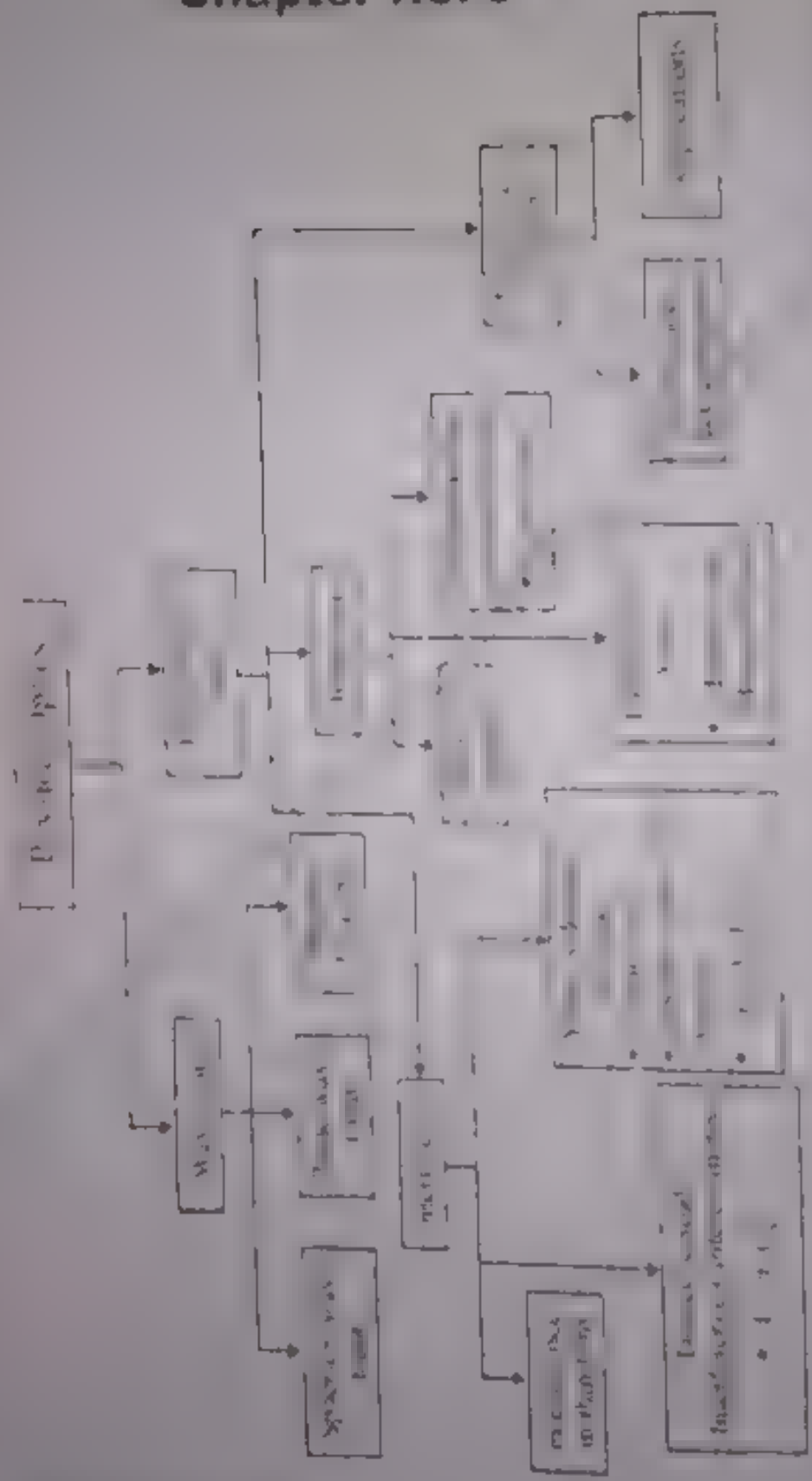
158

## Learning Objectives

- Describe light waves as a perturbation in the electromagnetic field.
- Describe the concept of wave front
- State Huygen's principle and its application to the propagation of light
- State the necessary condition for interference
- Describe Young's double slit experiment and its results provides to support the wave theory of light
- Explain interference in thin films
- Describe the production of Newton's rings
- Explain diffraction and identify the conditions for diffraction that have been observed
- Describe Fraunhofer diffraction
- Describe a diffraction grating and its use
- Describe the use of a diffraction grating in the determination of wavelengths using  $d \sin \theta = n\lambda$
- Describe the phenomenon of diffraction of light by a single slit
- Explain polarization as a phenomenon associated with transverse waves
- Identify and express that polarization is a vector property
- Explain the effect of rotation of plane of polarization
- Explain how plane polarized light is produced by reflection

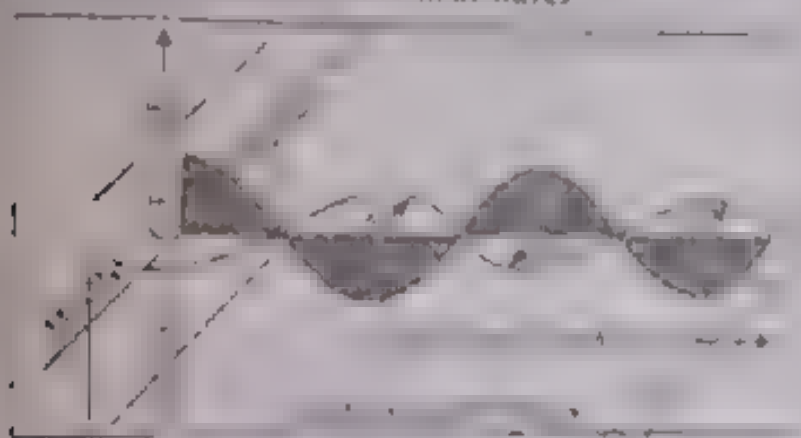
# Chapter No. 9

## CONCEPT MAP



The transfer of energy through a medium is called wave.

- ▶ Light is a transverse wave.
- ▶ Light is an electromagnetic wave.
- ▶ The wave nature of light is illustrated through experiment through a slit in.
- ▶ Maximal destructive interference occurs at angles where the path difference is an odd multiple of half wavelength.
- ▶ Light is a form of energy which travels in form of waves.



spaced 1 unit apart. The arrows represent rays.

When segments of large spherical wavefronts are separated a great distance.

#### EXPLANATION

The wave fronts far away from the source are very small portion of the sphere which the plane wave front.

## 21 What is a wave front?

### Ans: Wave Fronts

Surfaces of constant phase of waves have same phase at a given time is known as wave front.

#### Explanation

- ▶ Suppose a point source  $S$  propagates outward in all directions with speed  $v$ . At time  $t$ , the waves reach the surface of an imaginary sphere of radius  $vt$  as  $S$  and radius as " $vt$ ".
- ▶ As the distance of a wave point from the source is same to all the points on the surface of the sphere, the phase of vibration at such a surface is known as wave front.

#### Note

- ▶ The wave front from a point source is spherical.
- ▶ Thus wave propagates in wave fronts.
- ▶ The distance between two consecutive wave fronts is called wave length.

### Ray of Light

- ▶ The line normal to the wave front is called ray of light.

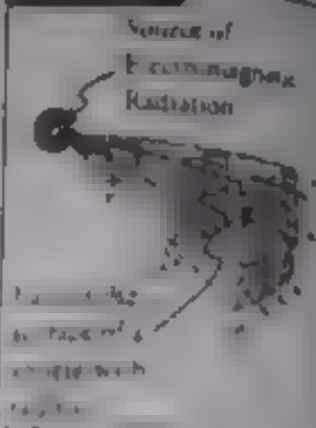
#### Spherical wave front

The wave front in which the light waves propagate in all directions from a point source is called spherical wave front.



- ▶ Light travels in a straight line as medium is homogeneous and isotropic.
2. **Plane wave front**
- ▶ At very large distance from a point source, the wave front becomes nearly plane like a wave from a distant source. It is called plane wave front.
- ▶ Plane wave front is perpendicular to the direction of propagation.
- ▶ Plane wave front is perpendicular to the direction of propagation.

## Problem 9.3



Q 2 State and explain the Huygen's principle?

**Huygen's Principle**

- ▶ Huygen's principle states that every point on a wave front acts as a source of secondary wavelets. The secondary wavelets spread out in all directions with the same speed as the primary wave. The new wave front is the surface tangential to all the secondary wavelets.
- ▶ Huygen's principle is used to explain the reflection, refraction, diffraction and interference of light.
- ▶ Huygen's principle is used to explain the formation of the rainbow.

Explanation

- ▶ Huygen's principle states that every point on a wave front acts as a source of secondary wavelets. The secondary wavelets spread out in all directions with the same speed as the primary wave. The new wave front is the surface tangential to all the secondary wavelets.
- ▶ Huygen's principle is used to explain the reflection, refraction, diffraction and interference of light.
- ▶ Huygen's principle is used to explain the formation of the rainbow.

**MCQs**

1. The angle of diffraction is the angle between the ray of light and the wave front?
 

A.  $90^\circ$  B.  $45^\circ$  C.  $120^\circ$  D.  $180^\circ$
2. A case of point source the shape of wave front is
 

A. Spherical B. Cylindrical C. Elliptical D. None of these
3. According to following is the phase difference between two points on wave front?
 

A.  $0$  B.  $\pi$  C.  $\frac{\pi}{2}$  D.  $\frac{\pi}{4}$
4. The wave nature of light was proposed by
 

A. Huygens B. Galileo C. Huygens D. Newton
5. Light from a point source the wave front is the form of
 

A. Spherical wave front B. Plane wave front C. Elliptical wave front D. Huygen's wave front
6. The distance between the two consecutive wave fronts is called
 

A. Amplitude B. Frequency C. Wavelength D. Displacement
7. When the sunlight passes through atmosphere, the total energy is reduced due to
 

A. Reflection B. Scattering by dust particles C. Absorption by dust particles D. All of these
8. The wave nature of light is due to
 

A. Reflection B. Refraction C. Diffraction D. Scattering

According to Huygens's principle, each point on a wave front acts as a source of secondary wavelets.

|     |     |     |     |     |     |     |     |     |      |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| 1 C | 2 F | 3 A | 4 G | 5 B | 6 C | 7 D | 8 J | 9 A | 10 A |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|

### Coherent Sources

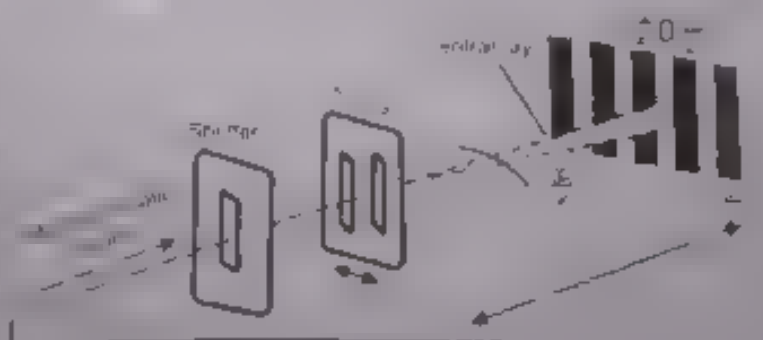
1. The first part of the paper is devoted to a review of the literature on the topic of the role of the state in the development of the economy. It is found that the state has played a significant role in the development of the economy in many countries, particularly in the case of developing countries. The state has been able to mobilize resources, provide infrastructure, and create a favorable environment for investment and growth.

which gives light within a narrow band of wave-length

## 15.4 Interference of Light Waves

### Constructive interference

### Constructive interference



F 0 9 5

constructive interference

- Constructive interference  
 Destructive interference  
 Path difference  $d = 0, \lambda, 2\lambda, 3\lambda, \dots$   
 Phase difference  $\phi = 0, 2\pi, 4\pi, 6\pi, \dots$   
 Phase differences are  $0, \pi, 2\pi, 3\pi, \dots$

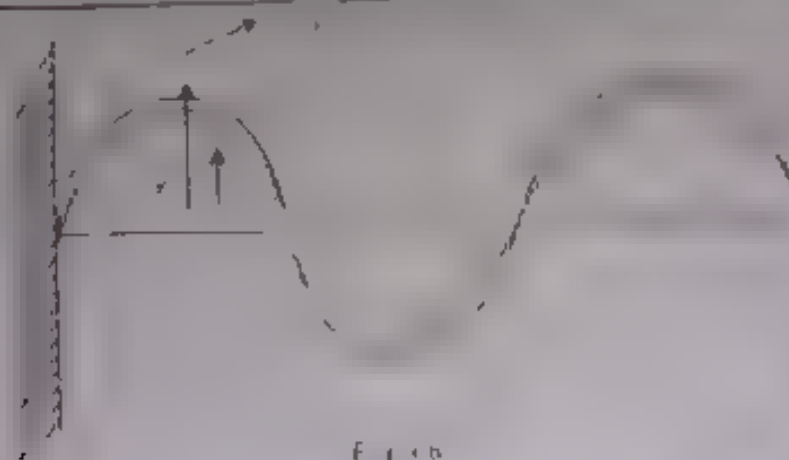


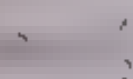
Fig. 3.6

**(ii) Destructive interference**

If the crest of a wave falls on the trough of the other wave, then the waves cancel each other out. Such an interference is known as destructive interference.

Consider the following:

1. Crest of wave 1 falls on trough of wave 2.
2. Crest of wave 1 falls on crest of wave 2.



Please refer to Fig. 3.7 (a) and (b).

**Conditions for detectable interference pattern**

The following conditions must be satisfied in order to observe the interference phenomenon.

1. The interfering beams must have the same frequency.
2. The interfering beams must have a constant phase difference.
3. The sources should have the same amplitude.
4. The intensity of the two sources must be comparable.

**Monochromatic Sources**

- The sources which emit the light of a single wavelength are called monochromatic sources.

**Coherent Sources**

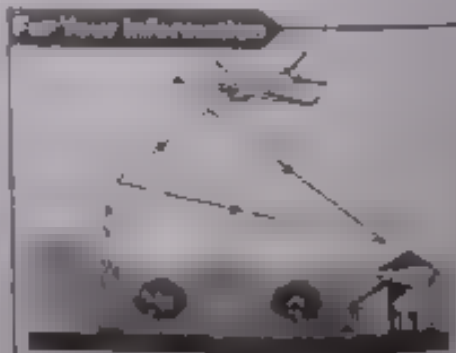
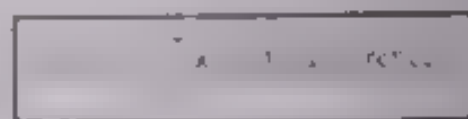
- Coherent sources are the sources of light which emit waves having a constant phase difference and constant wavelength.

**How to obtain coherent sources**

- A common method to obtain the coherent light sources is to use a monochromatic source of light (laser) which emits light of a single wavelength.
- The light emerging from the two slits in a double-slit experiment can be regarded as two coherent light sources because the two slits are illuminated by the same wave front.
- The points on a spherical wave front which are equidistant from the wavelets are also coherent sources of light.



Fig. 3.7

**FOR YOUR INFORMATION:**

Do you know how a radio receiver receives signals from a distant station?

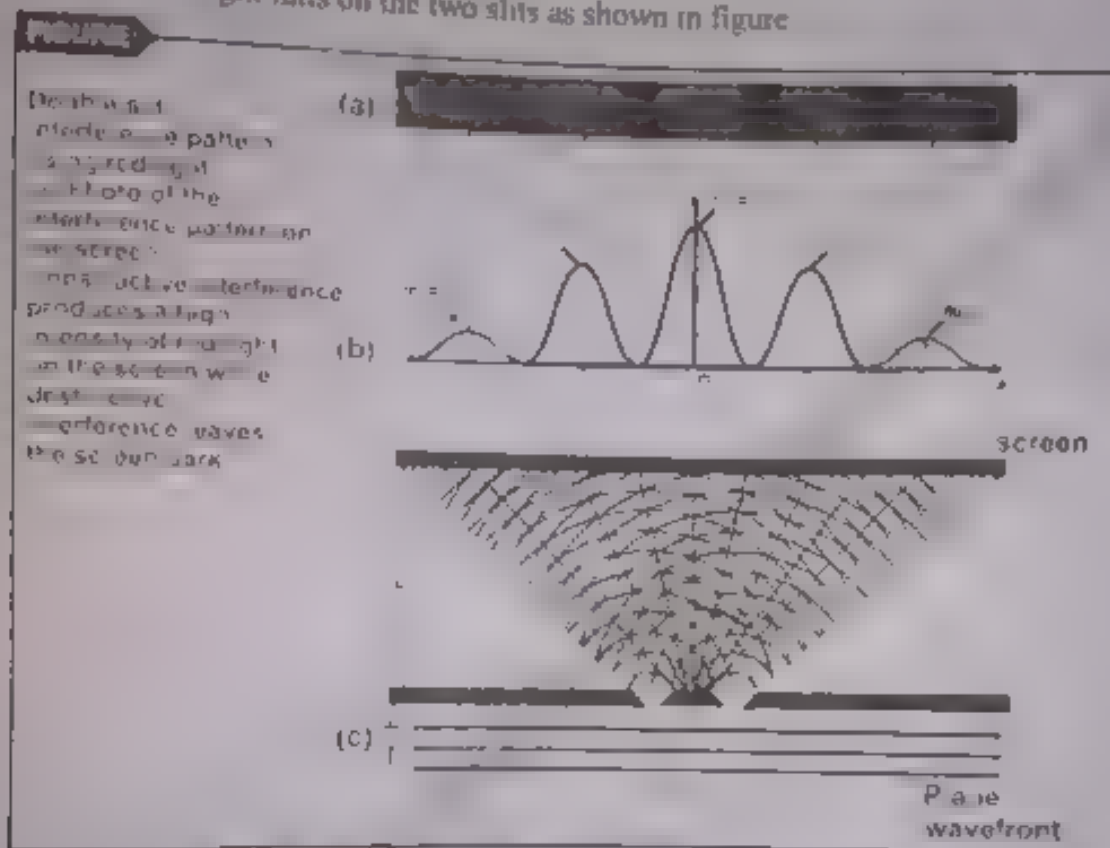
The radio waves from the distant station reach the receiver. The receiver is tuned to the frequency of the radio waves. The receiver then picks up the signals and converts them into sound waves.

Similarly, the light waves from a distant star reach the Earth. The light waves are then picked up by the telescope and converted into an image of the star.

Q 5 Describe the Young's double slit experiment for demonstration of interference of light. Derive an expression for fringe spacing

### Young's Double Slit Experiment

- ▶ The Young's double slit experiment is based on division of wave front.
- ▶ Monochromatic light falls on the two slits as shown in figure



- ▶ The portion of wave front which passes through the slits behaves like the source of secondary wavelets.
- ▶ These wavelets interfere with each other. If these wavelets result into the series of maxima and minima, they are termed as the second order maxima if the distance parallel to the central axis is  $2\lambda$ .

### Conditions for Maxima and Minima

Consider two slits  $S_1$  and  $S_2$  are passing light through them. Let  $B$  and  $C$  at point  $P$  on the screen.

Distance  $S_1P = AP$

Distance  $S_2P = BP$

$BP - AP$

- ▶ For maxima,  $BP - AP = n\lambda$  where  $n = 0, 1, 2, 3, \dots$
- ▶ For minima,  $BP - AP = (n + \frac{1}{2})\lambda$  where  $n = 0, 1, 2, 3, \dots$

- ▶ It can be proved that  $BP - AP = n\lambda$  by assuming that  $AQ$  is a normal to  $BP$ .
- ▶ The path difference between the wavelets leaving  $S_1$  and arriving at  $P$  is  $BQ$ .

### Constructive interference

$$AP = BP \Rightarrow \sqrt{a^2 + y^2} = \sqrt{a^2 + (y-d)^2}$$

$$BP = a + d$$

$$\text{Where } a = \frac{d}{2}$$

$$\text{Where } d = \text{distance between slits}$$

$$\text{From eqn (3) \& (4)}$$

$$\sqrt{a^2 + y^2} = a + d$$

$$\sqrt{\left(\frac{d}{2}\right)^2 + y^2} = \frac{d}{2} + d$$

$$S = \frac{d}{2}$$

► At central maxima  $y = 0$

$$m = 0, \pm 1, \pm 2, \dots$$

► For  $m^{\text{th}}$  maxima  $y = m \lambda \frac{L}{d}$

### Destructive interference

► If dark fringes  $y = \frac{(m + \frac{1}{2}) \lambda L}{d}$

$$\text{Where } m = 0, \pm 1, \pm 2, \dots$$

$$S = d \sin \theta = d \frac{y}{L}$$

$$y = \frac{S L}{d}$$

$$\text{Then } y = \frac{d \sin \theta L}{d} = L \sin \theta$$

$$\text{Where } m = 0, \pm 1, \pm 2, \dots$$

### Position of Fringe on screen

$$\text{Let } y = \text{distance of } m^{\text{th}} \text{ fringes from } O$$

$$S = \frac{AP}{L} \quad \text{and} \quad S = \frac{BP}{L}$$

$$y = \frac{AP}{L} \quad \text{and} \quad y = \frac{BP}{L}$$

$$\text{But } y = \frac{AP}{L} \quad \text{and} \quad y = \frac{BP}{L}$$

$$S = \frac{d}{L}$$

$$y = \frac{d}{L}$$

$$y = \frac{d}{L}$$

### For $m^{\text{th}}$ Bright Fringe

$$\text{Putting value of } S \text{ from equation (1) in eqn (2)}$$

$$y = \frac{d}{L} \Rightarrow \frac{d}{L} = \frac{d \sin \theta}{L} \Rightarrow \sin \theta = \frac{y}{L}$$

$$\text{Putting } y = m \lambda \frac{L}{d} \text{ in equation (3)}$$

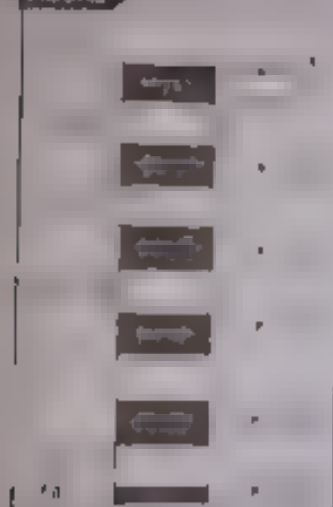
► For 1<sup>st</sup> Dark Fringe

Distance of 1<sup>st</sup> dark fringe from center of screen is  $x_1$   
 Distance of 2<sup>nd</sup> dark fringe from center of screen is  $x_2$   
 Distance of 3<sup>rd</sup> dark fringe from center of screen is  $x_3$

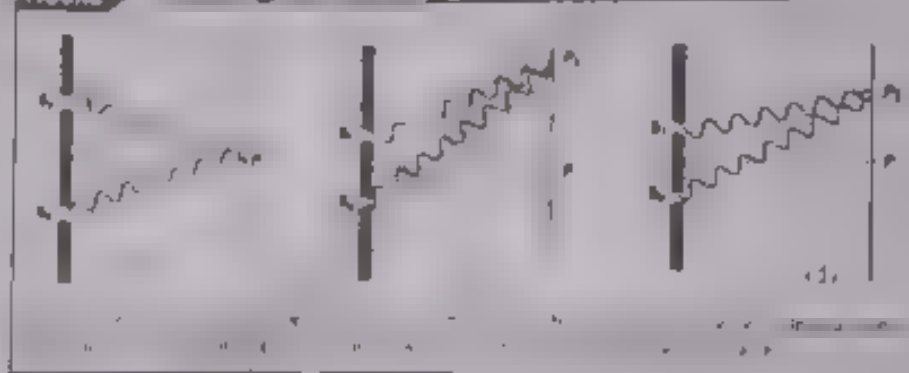
Distance of 4<sup>th</sup> dark fringe from center of screen is  $x_4$   
 Distance of 5<sup>th</sup> dark fringe from center of screen is  $x_5$

Distance of 6<sup>th</sup> dark fringe from center of screen is  $x_6$

FIGURE



FIGURE



Distance of 7<sup>th</sup> dark fringe from center of screen is  $x_7$

► Distance of 8<sup>th</sup> dark fringe from center of screen is  $x_8$

Distance of 9<sup>th</sup> dark fringe from center of screen is  $x_9$

► Distance of 10<sup>th</sup> dark fringe from center of screen is  $x_{10}$

► Distance of 11<sup>th</sup> dark fringe from center of screen is  $x_{11}$

Distance of 12<sup>th</sup> dark fringe from center of screen is  $x_{12}$

Distance of 13<sup>th</sup> dark fringe from center of screen is  $x_{13}$

Distance of 14<sup>th</sup> dark fringe from center of screen is  $x_{14}$

Distance of 15<sup>th</sup> dark fringe from center of screen is  $x_{15}$

Distance of 16<sup>th</sup> dark fringe from center of screen is  $x_{16}$

Distance of 17<sup>th</sup> dark fringe from center of screen is  $x_{17}$

Distance of 18<sup>th</sup> dark fringe from center of screen is  $x_{18}$

Distance of 19<sup>th</sup> dark fringe from center of screen is  $x_{19}$

Distance of 20<sup>th</sup> dark fringe from center of screen is  $x_{20}$

► For Two Dark Fringes,

Distance between two adjacent dark bands on the screen (1<sup>st</sup> and 2<sup>nd</sup> fringes) are  $x_2 - x_1$

Distance of 1<sup>st</sup> dark fringe from center of screen is  $x_1$

Distance of 2<sup>nd</sup> dark fringe from center of screen is  $x_2$

$$W = \frac{v}{f} = \frac{v}{\frac{c}{\lambda}} \quad \text{putting value of } f$$

$$W = \frac{v}{c} \lambda$$

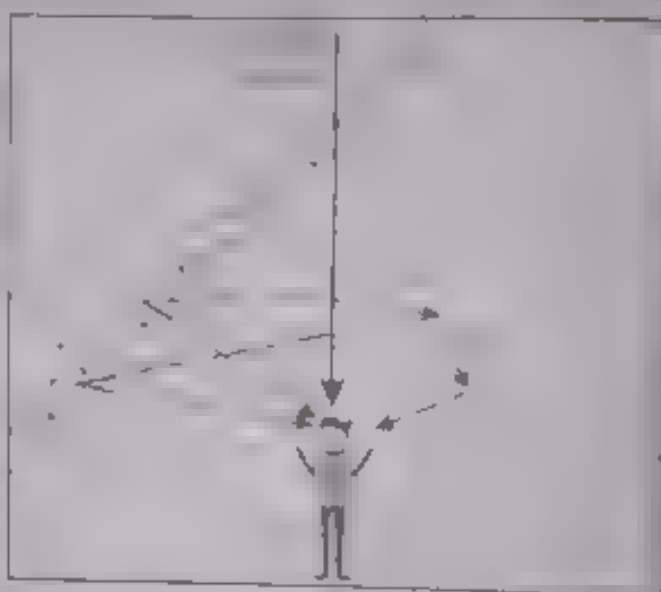
$$W = \lambda$$

Thus the wavelength and thickness of each slit are equal.

- The fringe spacing varies linearly with the thickness of the slit.
- The fringe spacing varies linearly with wavelength.
- The fringe spacing does not vary with the distance between the slits.

### For Your Information

Short wave lengths are scattered from the direct beam from the sun at midday more effectively than longer wavelengths. These scattered waves produce the blue sky while we see the sky as blue not looking directly at the sun. We are seeing light that has been scattered multiple times which concentrates the shorter blue light with a shorter wavelength. The shorter reaches our eyes. Since the scattering of sunlight occurs more by shorter wavelength than longer wavelength, the shorter wave lengths have a direct forward path.



### QUIZ

**Q** Young's double slit experiment breaks a single light beam into two sources. Would the same pattern be obtained for two independent sources of light such as the headlights of a distance car?

**A** No.

#### Explanation

Because the light of two independent sources have different wave fronts, it cannot produce interference.

### Assignment 9.1:

The 3<sup>rd</sup> bright fringe in a double slit experiment makes a 2.4° angle with respect to the central line. The wavelength of the monochromatic light used is 480 nm. Find the distance  $d$  between the two sources.

#### Given Data

Wave length of light  $\lambda = 480 \text{ nm}$

Order of fringe  $n = 3$

Angle  $\theta = 0.24^\circ$

Distance between the slits  $d = ?$

**Solution**

**MCQ's**

7. In which colour of light the fringe spacing will be maximum  
 A. Red B. Yellow C. Green D. Blue
8. In Young's double slit experiment, the path difference for 3rd order maximum is  
 A.  $3\lambda$  B.  $2\lambda$  C.  $\lambda$  D.  $\frac{\lambda}{2}$
9. Brilliant and beautiful colours of soap bubble are due to  
 A. Reflection B. Refraction C. Interference D. Diffraction
10. The distance between the two adjacent bright fringes is  
 A.  $\frac{\lambda D}{d}$  B.  $\frac{\lambda d}{D}$  C.  $\frac{D}{\lambda d}$  D.  $\frac{d}{\lambda D}$
11. The appearance of colours in thin film is due to  
 A. Reflection B. Refraction C. Interference D. Diffraction
12. What happens to the interference pattern produced by two slits if you decrease the slit spacing  
 A. Fringe spacing increases B. Fringe spacing decreases C. Fringe spacing remains same D. Fringe spacing becomes zero
13. A maxima is produced at points where the path difference between two waves is  
 A.  $0$  B.  $\frac{\lambda}{2}$  C.  $\lambda$  D.  $2\lambda$
14. Formula for fringe spacing is  
 A.  $\frac{\lambda D}{d}$  B.  $\frac{\lambda d}{D}$  C.  $\frac{D}{\lambda d}$  D.  $\frac{d}{\lambda D}$
15. The distance between two adjacent bright or dark fringes is  
 A.  $\frac{\lambda D}{d}$  B.  $\frac{\lambda d}{D}$  C.  $\frac{D}{\lambda d}$  D.  $\frac{d}{\lambda D}$
16. Fringe spacing is inversely proportional to  
 A. Slit spacing B. Distance to the screen C. Wavelength D. All of these

**Answer Key**

|      |      |      |      |      |      |      |      |      |       |
|------|------|------|------|------|------|------|------|------|-------|
| 1. A | 2. B | 3. D | 4. D | 5. C | 6. B | 7. A | 8. B | 9. A | 10. B |
|------|------|------|------|------|------|------|------|------|-------|

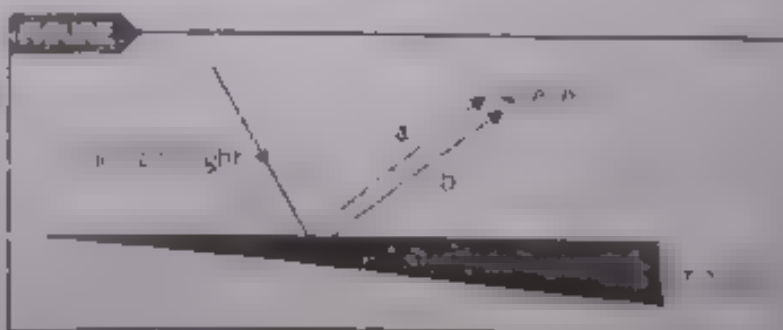
**Q6** Explain the phenomena of interference of light in a thin film?

**Interference in a Thin Film**

A transparent medium whose thickness is very small (comparable with the wavelength of light) is called thin film.

**Examples**

1. Thin oil film on water
2. Thin soap film
3. Thin layer of paint



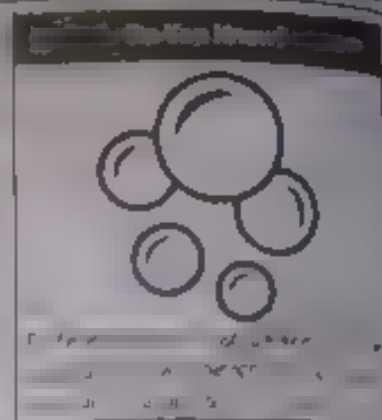
**Explanation**

A beam of light strikes the thin film at an angle. Part of the light is reflected from its upper surface. It is partially coherent. The other part of the light enters the film and is reflected from its lower surface. This film is very thin, so the path difference between the two rays is very small. The two rays are coherent and the result of the interference will be visible. The thickness of the film is very small.

The path difference between two rays depends on

1. Wavelength of the light
2. Nature of the film
3. Angle of incidence

- ▶ If the two reflected waves recombine each other, they will be bright.
  - ▶ However, if the thickness of the film and the angle of incidence are such that the two reflected waves cannot each other, they will be dark.
- (a) When a transverse wave travels from a medium of low refractive index to a medium of high refractive index, it undergoes a phase change of  $180^\circ$  and after reflection.
- (b) When a transverse wave travels from a medium of high refractive index to a medium of low refractive index, it undergoes no phase change after reflection.



### Interference of White Light

If white light is incident on a film of irregular thickness at all possible angles, we will observe interference pattern of each spectral colour separately.

But if the thickness of film and the angle of incidence are such that the path difference is an integral multiple of wavelength, all the colours will be in phase to reinforce each other.

- Q 7 Describe the principle, construction and working of Michelson's interferometer. How can you find the wave length of light used?

### ANS: Michelson's Interferometer

Michelson's interferometer is an optical instrument. It is used to study interference of light and find wave length.

**Principle** The principle of Michelson's interferometer is based on the division of light into two parts by reflection and transmission of light from a beam splitter into two medium.

### For Your Information

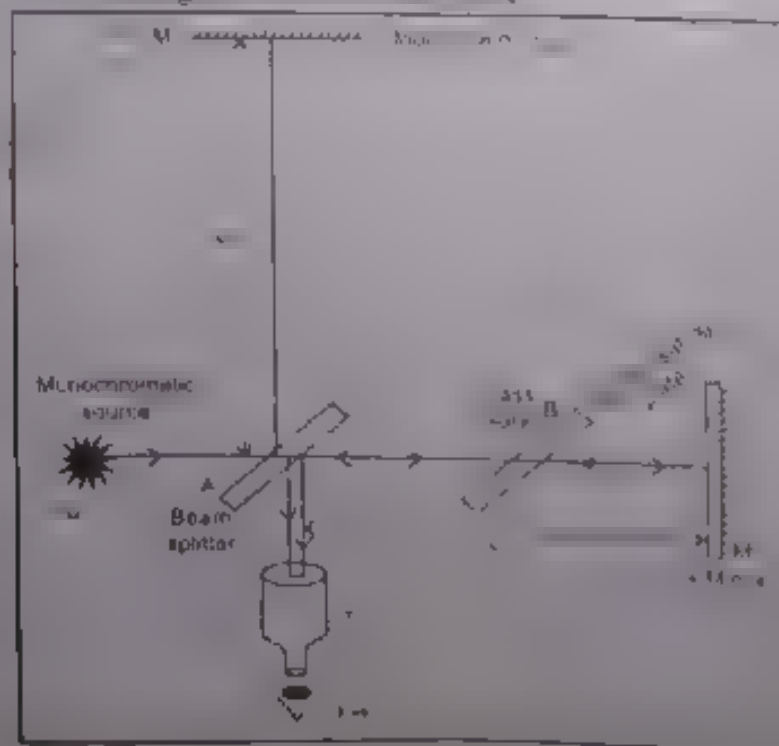


A photograph of Michelson's interferometer

### Construction

The essential parts of Michelson's interferometer are:

- ▶ Two plane mirrors M & M'
- ▶ Two glass plates A & A' forming the beam splitter.



- ▶ The plate "A" is rigidly attached to the back of the mirror.
- ▶ The mirror "M" is movable.
- ▶ The mirror "M" is fixed.

### Theory and working

A beam of light from a monochromatic source is split into two parts.

- ▶ The first part is reflected from plate "A" and is recombined with the mirror "M" to form a fringe.
- ▶ The second part of the light forms the other half of the recombined beam. There is a phase change of  $\pi$  in the reflected ray from plate "A".
- ▶ The plate "B" is moved in the vertical direction of the beam and the path is

### Conditions for constructive interference

- ▶ The path difference between the two rays must be an integral multiple of the wavelength.

$$d = m\lambda$$

$$m = 0, 1, 2, 3, \dots$$

When

### Conditions for destructive interference

- ▶ The path difference between the two rays must be an odd multiple of half the wavelength.

$$d = (2m+1)\frac{\lambda}{2}$$

$$m = 0, 1, 2, 3, \dots$$

When

the path difference  $d$  between the two rays is an integral multiple of the wavelength.

- ▶ The plate "B" is moved in the vertical direction of the beam and the path is
- ▶ Some of the light is reflected from the back of the mirror "M" and is recombined with the mirror "M" to form a fringe.

### Alternate Bright and Dark Fringes

- ▶ As the mirror "M" is moved further down, the path difference  $d$  between the two rays increases and the fringes will shift.
- ▶ When the mirror "M" is further down, the path difference  $d$  between the two rays will become  $\lambda$  and the next bright fringe will appear.
- ▶ As the mirror "M" is moved further down, the path difference  $d$  between the two rays will become  $\frac{3\lambda}{2}$  and the next dark fringe will appear.

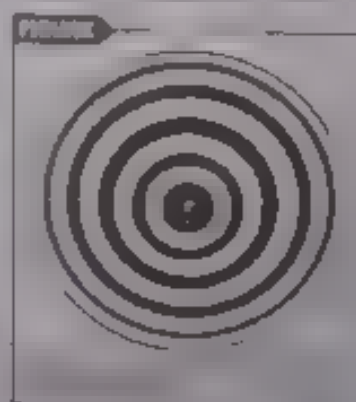
### Wavelength of light

- ▶ A bright fringe is formed when the path difference  $d$  between the two rays is an integral multiple of the wavelength  $\lambda$ .
- ▶ A dark fringe is formed when the path difference  $d$  between the two rays is an odd multiple of half the wavelength  $\frac{\lambda}{2}$ .

$$d = \frac{m\lambda}{2}$$

$$\lambda = \frac{2d}{m}$$

Knowing the value of  $d$  and  $P$  we can find  $\lambda$ .



340

Q808

1. A double-slit interference experiment is set up with a distance of 0.5 mm between the slits. A distance of 1.0 m is observed between the central maximum and the first minimum. The wavelength of the light used is 600 nm. Calculate the distance between the slits.
- (1) 0.5 mm
- (2) 1.0 mm
- (3) 1.5 mm
- (4) 2.0 mm

**MCQs**

1. When the monochromatic light of wavelength  $\lambda$  is incident on a double-slit interference experiment, a distance of 0.5 mm between the slits is observed. The distance between the central maximum and the first minimum is 1.0 mm. The wavelength of the light used is 600 nm. Calculate the distance between the slits.
- (1) 0.5 mm
- (2) 1.0 mm
- (3) 1.5 mm
- (4) 2.0 mm
2. In a double-slit interference experiment, the distance between the slits is 0.5 mm. The distance between the central maximum and the first minimum is 1.0 mm. The wavelength of the light used is 600 nm. Calculate the distance between the slits.
- (1) 0.5 mm
- (2) 1.0 mm
- (3) 1.5 mm
- (4) 2.0 mm
3. Constructive interference of two waves occurs when the path difference is an integral multiple of the wavelength. The path difference is 1.0 mm. The wavelength of the light used is 600 nm. Calculate the order of the maximum.
- (1) 1
- (2) 2
- (3) 3
- (4) 4

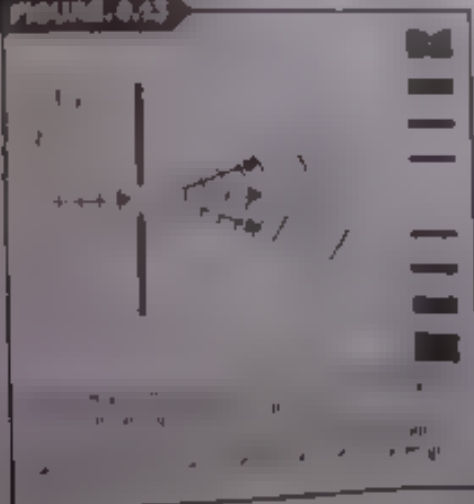
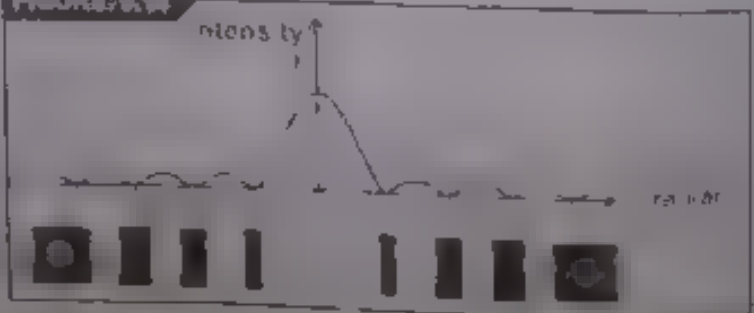
**Answers Key**

Q8 What is meant by diffraction of light?

**1.573** Diffraction of Light

1. The property of bending of light around obstacles and spreading of light waves into the geometrical shadows of an obstacle is called diffraction.
- (1) The property of bending of light around obstacles and spreading of light waves into the geometrical shadows of an obstacle is called diffraction.
- (2) The property of bending of light around obstacles and spreading of light waves into the geometrical shadows of an obstacle is called diffraction.
- (3) The property of bending of light around obstacles and spreading of light waves into the geometrical shadows of an obstacle is called diffraction.
- (4) The property of bending of light around obstacles and spreading of light waves into the geometrical shadows of an obstacle is called diffraction.

- Def
- The property of bending of light around obstacles and spreading of light waves into the geometrical shadows of an obstacle is called diffraction.
- (1) The property of bending of light around obstacles and spreading of light waves into the geometrical shadows of an obstacle is called diffraction.
- (2) The property of bending of light around obstacles and spreading of light waves into the geometrical shadows of an obstacle is called diffraction.
- (3) The property of bending of light around obstacles and spreading of light waves into the geometrical shadows of an obstacle is called diffraction.
- (4) The property of bending of light around obstacles and spreading of light waves into the geometrical shadows of an obstacle is called diffraction.

**FIGURE 9.23****FIGURE 9.24**

- (a) The condition of minimum path difference is  
 $\sin \theta = \frac{\lambda}{2a}$   
 It should be noted that the minimum path difference is  $\frac{\lambda}{2}$ .

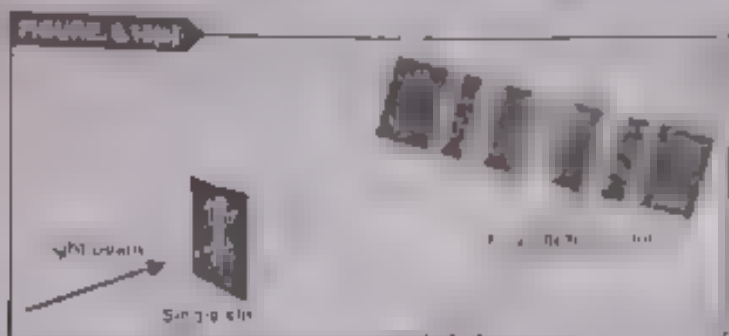
- (b) Similarly, the condition for maximum path difference is  $\sin \theta = \frac{\lambda}{a}$  which will give  $\theta = \sin^{-1} \frac{\lambda}{a}$ .

### Q9 Discuss Fraunhofer diffraction of light through a narrow slit?

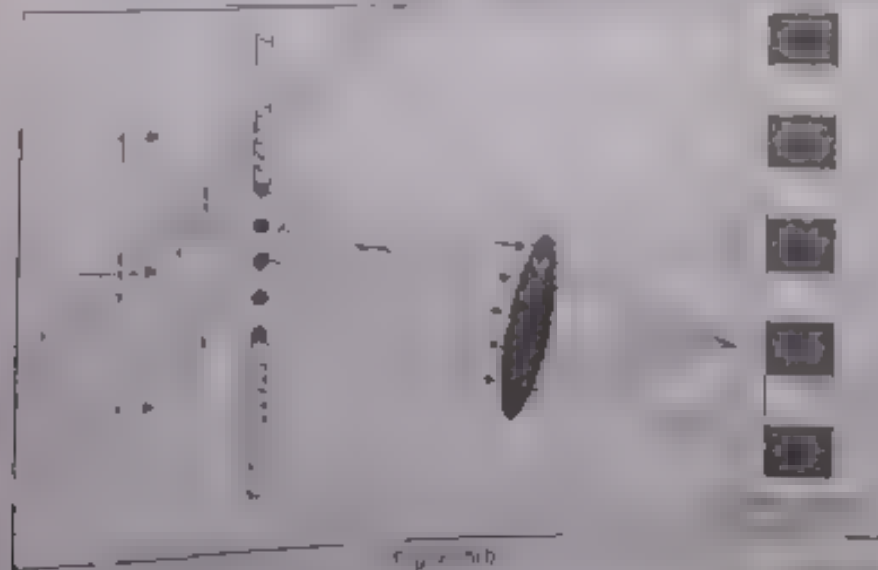
#### Ans Fraunhofer diffraction of light

"The Fraunhofer diffraction is observed when the source of light is at infinity from the slit and the screen is also at infinity from the slit."

- Consider the parallel plane wave fronts shown in Fig. 1.11(a).



- Here the wavefronts are plane waves and the source and screen are at infinity.  
 ► Moreover, the wavefronts are parallel to each other and there is no sort of interference effect.  
 ► Consider plane wave fronts which are incident on a slit as shown in Fig. 1.11(b).



According to Huygen's principle every point on the wavefront acts as a source for the secondary wavelet.

- At the point O in the screen, wavelets from A & B arrive. These points are equidistant from O. Path difference between the waves coming from the points A and B is  $\Delta x = \sin \theta \cdot \lambda$ . Hence constructive interference will take place and the intensity is observed.



Figure 1.11 (b) Intensity distribution of light in Fraunhofer diffraction



## Grating element

- ▶ The distance between two adjacent slits is called the grating element.
- ▶ It is denoted by  $d$ .
- ▶ If  $N$  is the number of slits per unit length, then  $d = \frac{1}{N}$ .
- ▶ The width of each slit is denoted by  $a$ .
- ▶ The distance between the centers of two adjacent slits is denoted by  $d$ .

## Working and Theory

- ▶ A monochromatic light of wavelength  $\lambda$  is incident on the grating. The light falling on the grating sends out waves from each slit.
- ▶ The rays from the slits are diffracted.
- ▶ These rays are collected by a telescope.
- ▶ The path difference between the rays should be equal to  $\lambda$ .

$$AB = d$$

In the triangle  $ABP$  we know that

$$\sin \theta = \frac{AB}{BP}$$

$$\lambda = d \sin \theta$$

$$\lambda = d \sin \theta$$

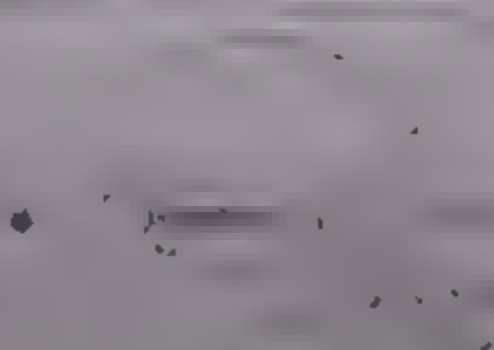
$$\lambda = d \sin \theta$$

- ▶ The angle  $\theta$  is called the angle of diffraction.
- ▶ The angle  $\theta$  is measured from the normal to the grating.
- ▶ The angle  $\theta$  is measured from the normal to the grating.

- ▶ This is the principle of the grating.
- ▶ The grating is used to measure the wavelength of light.
- ▶ The grating is used to measure the wavelength of light.

Grating Spectrometer is used to calculate the wavelength of light. The diagram in which a monochromatic light falls on the grating and all the light passes through the grating. The diffracted light passes the grating at an angle. A telescope is used to view the diffracted light. In this way angles for various orders are measured on each side of the central maximum. These angles are measured to calculate the wavelength.

## Grating Spectrometer



**Q 11** Describe the diffraction of x rays through crystals? Also describe the Bragge equation and its different uses?

### 11.1 Diffraction of X Rays by Crystals

- In 1895, Wilhelm Conrad Roentgen discovered x-rays while working on cathode rays.
- In 1912, Max von Laue discovered that crystals act as diffraction gratings for x-rays.



Fig. 9.11

The diffraction of x-rays by crystals is a phenomenon that has been studied extensively. It is a result of the periodic arrangement of atoms in a crystal lattice. The x-rays are scattered by the atoms, and the scattered waves interfere with each other to produce a diffraction pattern. This pattern can be used to determine the structure of the crystal.



### Experiment

- The experiment is performed using a x-ray source, a crystal, and a detector. The x-rays are incident on the crystal, and the diffracted rays are detected. The intensity of the diffracted rays is measured as a function of the angle of incidence.
- The Bragg equation is used to determine the structure of the crystal. The equation is  $n\lambda = 2d \sin \theta$ , where  $n$  is the order of diffraction,  $\lambda$  is the wavelength of the x-rays,  $d$  is the distance between the atomic planes, and  $\theta$  is the angle of incidence.
- The experiment is used to determine the structure of crystals. It is a powerful tool for studying the atomic structure of materials.
- The experiment is also used to study the properties of x-rays. It can be used to determine the wavelength of x-rays and the intensity of the diffracted rays.



Bragg's Law

- The condition for constructive interference is that the path difference between the rays must be an integral multiple of the wavelength.
- The path difference is given by  $2d \sin \theta$ , where  $d$  is the distance between the planes and  $\theta$  is the angle of incidence.
- The condition for constructive interference is  $2d \sin \theta = n\lambda$ , where  $n$  is an integer.

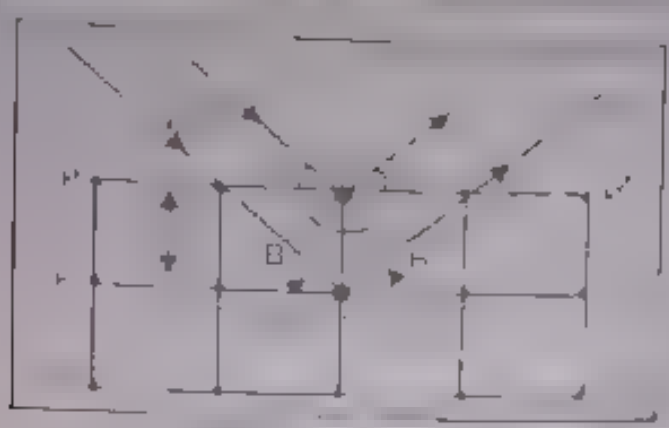


Fig 9.20 b shows Bragg's Law of Diffraction

- The condition for constructive interference is that the path difference between the rays must be an integral multiple of the wavelength.
- The path difference is given by  $2d \sin \theta$ , where  $d$  is the distance between the planes and  $\theta$  is the angle of incidence.

$$2d \sin \theta = n\lambda$$

For  $n=1$

Path difference =  $\lambda$

For  $n=2$  Path difference =  $2\lambda$

Similarly for  $n=3, 4, 5, \dots$

$$2d \sin \theta = n\lambda$$

For  $n=1$   $2d \sin \theta = \lambda$

For  $n=2$   $2d \sin \theta = 2\lambda$

Path difference =  $2\lambda$  for  $n=2$

Path difference =  $3\lambda$  for  $n=3$

Path difference =  $4\lambda$  for  $n=4$

Similarly for  $n=5, 6, 7, \dots$

Thus the condition for constructive interference is

Ex:

If the wavelength of the incident X-rays is  $0.1 \text{ nm}$  and the distance between the planes is  $0.2 \text{ nm}$ , find the angle of incidence for which the first order diffraction is observed.

Sol: Given  $\lambda = 0.1 \text{ nm}$

For first order diffraction  $n=1$   $2d \sin \theta = n\lambda$

Substituting the values  $2 \times 0.2 \times \sin \theta = 1 \times 0.1$

$0.4 \sin \theta = 0.1$   $\sin \theta = \frac{0.1}{0.4} = \frac{1}{4}$   $\theta = \sin^{-1} \left( \frac{1}{4} \right)$

**Problem 9.22**

A beam of X-rays of wave length  $0.3 \text{ nm}$  is incident on a crystal and gives a first order maximum when the angle of incidence is  $9^\circ$  degrees. Find the atomic spacing.

Sol: Given

Sol: Given

**Model**

Let the atomic spacing be  $d$ . The path difference between the rays is  $d \sin \theta$ . For a first order maximum, the path difference must be equal to the wavelength  $\lambda$ .  

$$d \sin \theta = \lambda$$

$$d \sin 9^\circ = 0.3 \text{ nm}$$

$$d = \frac{0.3 \text{ nm}}{\sin 9^\circ}$$

$$d = \frac{0.3 \times 10^{-9} \text{ m}}{0.1564}$$

$$d = 1.92 \times 10^{-9} \text{ m}$$

$$d = 1.92 \text{ nm}$$

**Problem 9.23**

Q.23. Explain the phenomenon of polarization. How plane polarized light is produced and how it is detected?

Sol: The phenomenon of polarization is the process by which the electric field vector of a light wave oscillates in a single plane perpendicular to the direction of propagation of the wave.

Ans:

- ▶ Plane polarized light is the light in which the electric field vector oscillates in a single plane perpendicular to the direction of propagation of the wave.
- ▶ Plane polarized light can be produced by reflection, refraction, double refraction, and scattering.
- ▶ Plane polarized light can be detected by using a polarizer. A polarizer is a device that allows only light with a specific polarization to pass through it.
- ▶ The intensity of plane polarized light is maximum when the plane of polarization is parallel to the plane of vibration of the particles of the wave and zero when it is perpendicular to it.

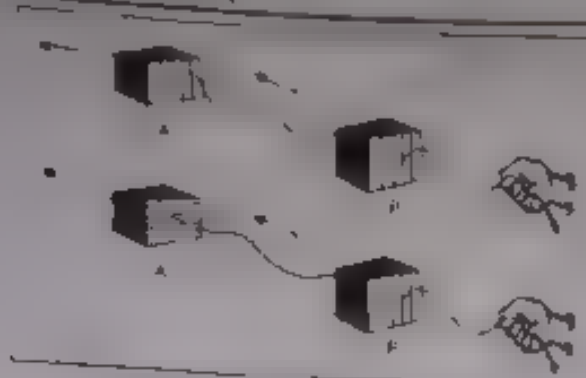


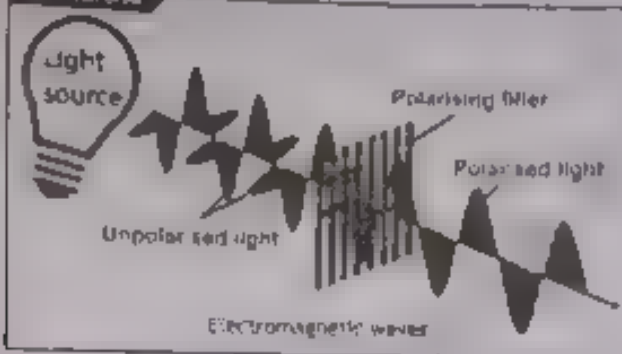
Fig 8.21 Polarisation of light (by reflection)

### Ordinary light

Question arises that why ordinary light is not polarized?

To answer this question, we know that light wave is emitted when an electron orbiting around a nucleus jumps from higher energy level to a lower energy state.

FIGURE 8.22



#### Do You Know?

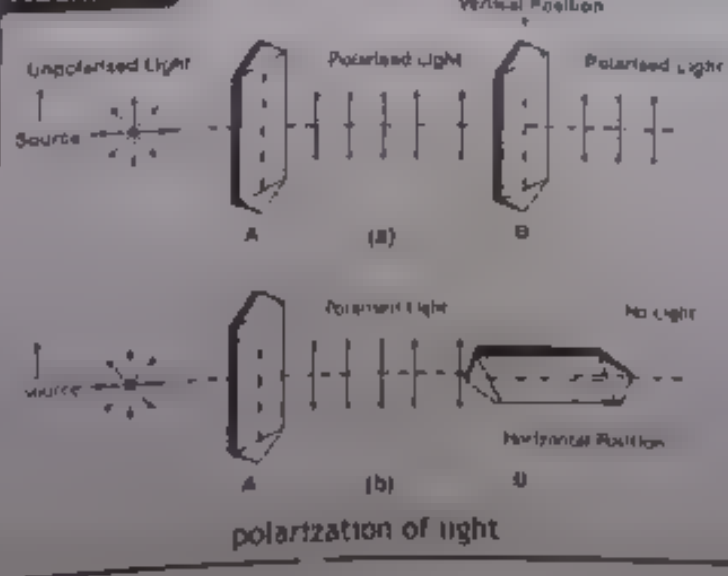
It is very effective to use a polarized sun glasses than using ordinary sun glasses.

- ▶ If the transition is due to an electron orbiting in a vertical plane, the light wave emitted is polarized in the vertical plane.
- ▶ But if the transition is due to an electron orbiting in a horizontal orbit then the light wave emitted is polarized in the horizontal plane.
- ▶ The light waves emitted from a source such as candle flame, a filament of bulb or the sun is non-polarized. This is because the light waves are from different atoms whose electrons experiencing the transition in different planes in all direction.
- ▶ Therefore, light waves are electromagnetic waves which consist of periodic vibrations of electric field vectors  $E$  accompanied by the magnetic field vector  $B$  at right angled to each other as shown in Fig 8.22.

### Polarized light

The beam of light in which vibrations are confined to a single plane of vibration is called polarized light.

FIGURE 8.23







## MCQ's

1. Which phenomena shows that light is transverse in nature?  
A. Reflection (d) Polarization (e) Interference (f) Refraction
2. The process of confining the beam of light to vibrate in one plane is called  
A. interference (B) Diffraction (c) reflection (d) refraction
3. If the unpolarized light is made incident on sheet of polythene, the transmitted beam of light will be  
A. unpolarized (B) Plane polarized (c) partially polarized (d) unpolarized
4. Optical rotation, a property of optically active substances is used to determine the  
A. velocity (B) viscosity (c) refractive index (d) optical density
5. Which characteristic of light is evidence from polarization of light?  
A. wave nature (B) Particle nature (c) wave speed (d) Light waves are transverse
6. Optically active crystals rotates the  
A. vibrating plane (B) Polarization plane (c) frequency (d) intensity of wave
7. Light is polarized by using  
A. glass (B) C. mica (c) mica (d) mica

## Answers Key

|      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|
| 1. B | 2. C | 3. B | 4. C | 5. A | 6. B | 7. B |
|------|------|------|------|------|------|------|

## FORMULAE

|                                                        |                                                       |                                                                     |
|--------------------------------------------------------|-------------------------------------------------------|---------------------------------------------------------------------|
| Path difference for constructive interference in light | $\Delta x = n\lambda$ where $n = 0, 1, 2, 3$          |                                                                     |
| Path difference for destructive interference in light  | $\Delta x = (2m+1)\frac{\lambda}{2}$<br>where $m = 0$ | $d \sin \theta = (2m+1)\frac{\lambda}{2}$<br>where $m = 0, 1, 2, 3$ |
| Position of $m$ th bright fringe                       | $y = m\lambda$                                        |                                                                     |
| Position of $m$ th dark fringe                         | $y = (m+\frac{1}{2})\lambda$                          |                                                                     |
| Fringe width / spacing                                 | $\Delta y = \frac{\lambda D}{d}$                      |                                                                     |
| Grating element                                        | $d = \frac{a+b}{\sin \theta}$                         | $d = \frac{a+b}{\sin \theta}$ when $\lambda = 1m$                   |
| Displacement of mirror in Michelson interferometer     | $\Delta x = \frac{\lambda}{2}$                        | $\Delta x = \frac{\lambda}{2}$                                      |
| Bragg's Law                                            | $2d \sin \theta = n\lambda$<br>Where $n = 1, 2, 3$    |                                                                     |



## Key Points

- ❖ **Nature of light** According to modern concept the nature of light is dual. Some time it behaves as a wave such as in reflection, refraction, diffraction, interference and polarization. But some time it behaves as the locus of all the points in a medium which have the same phase is called a wave front.
- ❖ **Wave front** Diffraction, interference and polarization. But some time it behaves as the locus of all the points in a medium which have the same phase is called a wave front.
- ❖ **Ray** The arrows to indicate the direction of wave fronts are known as the rays. The rays are always perpendicular to the wave fronts.

- ◆ **Phase coherent sources** The two sources of light which maintains a constant phase relation between their waves during emission are called phase coherent sources.
- ◆ **Interference of light waves** The effect produced by the superposition of two coherent waves is known as interference. The interference may be constructive or destructive.
- ◆ The principle of Young's double slits experiment and diffraction grating is based on the division of a wave front. While the principle of Michelson's interference thin film and Newton's ring is based on the division of amplitude.
- ◆ Michelson's interferometer is used for the precise measurement of wavelength.
- ◆ **Diffraction** The bending of light waves around an obstacle and spreading into its geometrical shadow is called diffraction. For a diffraction grating with grating element  $d$  and  $m$ th order bright fringe, we have  $d \sin \theta = m\lambda$ .
- ◆ The diffraction of X-rays can be obtained by means of crystals. For diffraction of X-rays the Bragg's law is  $2d \sin \theta = n\lambda$ .
- ◆ **Polarization of light** The process by which the electric and magnetic vibrations of light waves are restricted to a single plane of vibration is called polarization of light.



### Solved Example

**Example 2.1:** Yellow light from a sodium vapour lamp of wavelength  $5893 \times 10^{-10}$  m is directed upon two narrow slits 0.10 cm apart of the Young's experiment. Find the position of the first bright and dark fringes on a screen 100 cm away.

**Solution**

- Wavelength of light =  $\lambda = 5893 \times 10^{-10}$  m  
 Separation of slits =  $d = 0.10$  cm = 0.001 m  
 Distance of slits from the screen =  $L = 100$  cm = 1 m  
 Order of the fringe =  $m = 1$   
 Position of the 1st bright fringe =  $y_{\text{bright}} = ?$   
 Position of the 1st dark fringe =  $y_{\text{dark}} = ?$

(a) As for the bright fringe is

$$y_{\text{bright}} = L \frac{\lambda}{d} = \frac{1 \times 5893 \times 10^{-10}}{0.001} = 5.893 \times 10^{-4} \text{ m}$$

(b) And

$$y_{\text{dark}} = L \frac{\lambda}{2d} = \frac{1 \times 5893 \times 10^{-10}}{2 \times 0.001} = 2.953 \times 10^{-4} \text{ m}$$

**Example 2.2:** Light of wavelength 546 nm produces Young's interference pattern. The second order dark fringe is along the direction that makes an angle of  $18^\circ$  min related to the direction to the central maximum. What is the distance between the slits?

**Solution**

- Wavelength of light =  $\lambda = 546 \text{ nm} = 546 \times 10^{-9} \text{ m}$   
 Order of dark fringe =  $m = 2$   
 Angle =  $\theta = 18^\circ \text{ min} = \frac{18}{60} \times 10^\circ$

(As  $1^\circ = 60$  minutes)

For  $m$ th dark fringe we know that  $d \sin \theta = (m + \frac{1}{2}) \lambda$

$$d = \frac{\lambda}{2 \sin \theta}$$

$$d = \frac{1}{2} \times \frac{536 \times 10^{-9}}{\sin 33^\circ}$$

$$d = \frac{1.6 \times 536 \times 10^{-9}}{0.5446}$$

$$d = 1.47 \times 10^{-6} \text{ m}$$

**Example 9.3**

A red laser light of wavelength 670 nm is used in a Michelson interferometer. While keeping the mirror M<sub>1</sub> fixed, mirror M<sub>2</sub> is moved. The fringes are found to move past a fixed cross hair in the view. Find the distance the mirror M<sub>2</sub> is moved for a single fringe to move past the reference line.

**Given Data** $\lambda = 670 \text{ nm}$ **Required**the distance moved by M<sub>2</sub>**Solution**

For a single fringe to move past the reference line, the path difference between the two beams must be  $\lambda$ .

$$2\Delta x = \lambda$$

$$\Delta x = \frac{\lambda}{2}$$

**Example 9.4**

The deviation of the second order diffracted image formed by an optical grating having 5000 lines per centimeter is  $32^\circ$ . Calculate the wavelength of light used.

**Solution**

Number of lines per cm = 5000

or

Grating spacing =  $d$ 

$$d = \frac{1}{\text{number of lines per cm}} = \frac{1}{5000} = 2 \times 10^{-4} \text{ cm}$$

Wavelength =  $\lambda$ or  $\lambda =$ 

$$\lambda = \frac{d \sin \theta}{n}$$

$$= \frac{2 \times 10^{-4} \times \sin 32^\circ}{2}$$

$$\lambda = 6.4 \times 10^{-5} \text{ cm} = 640 \text{ nm}$$

**Example 9.5**

X-rays of wave length 3nm are incident on a crystal for which the lattice spacing is 5nm. Calculate the angle at which the first Bragg diffraction is observed.

**Solution**Wave length =  $\lambda = 3 \text{ nm}$ Lattice spacing =  $d = 5 \text{ nm}$ Angle =  $\theta$  (to be found)or  $\theta =$ **Step 1**

According to

$$2d \sin \theta = n\lambda$$

$$\sin \theta = \frac{n\lambda}{2d}$$

or  $\theta = \sin^{-1} \left( \frac{n\lambda}{2d} \right)$ or  $\theta =$ 

# Text Book Exercises

Q.1 Select the correct answer of the following questions:

- 1) The principle of Young's double slit experiment is based on the division of
  - (a) Amplitude
  - (b) Frequency
  - (c) Velocity
  - (d) Wave front
- 2) Which one of the following properties proves the transverse wave nature of light
  - (a) Interference
  - (b) Reflection
  - (c) Polarization
  - (d) Diffraction
- 3) Coloured fringes observed in soap bubbles are the examples of
  - (a) Diffraction
  - (b) Interference
  - (c) Polarization
  - (d) Reflection
- 4) During a sunny day we see the objects in a class room even when all the electric lights are off, due to
  - (a) Reflection of light
  - (b) Refraction of light
  - (c) Diffraction of light
  - (d) Interference of light
- 5) The principle of Michelson interferometer is based on the division of
  - (a) Wave front
  - (b) Amplitude
  - (c) Frequency
  - (d) Speed of light
- 6) In the Young's double slit experiment the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is
  - (a) Halved
  - (b) Unchanged
  - (c) Doubled
  - (d) Quadrupled
- 7) Signal from a remote control to the device operated by it travels with the speed of
  - (a) Sound
  - (b) Light
  - (c) Radio
  - (d) Supersonic
- 8) Light of wavelength  $\lambda$  is incident normally on a diffraction grating for which the slit spacing is equal to  $3\lambda$ . What is the sine of the angle between the second order maximum and the normal?
  - (a)  $\frac{1}{6}$
  - (b)  $\frac{1}{3}$
  - (c)  $\frac{2}{3}$
  - (d) 1
- 9) Which of the following gives three regions of the electromagnetic spectrum in order of increasing wavelength, visible radiation.
  - (a) Gamma rays, infrared waves, visible radiation
  - (b) Radio waves, ultraviolet, X-rays
  - (c) Ultraviolet, infrared, microwaves
  - (d) Visible radiation, gamma rays, radio waves
- 10) Two monochromatic radiations X and Y are incident normally on a diffraction grating. The second order intensity maximum for X coincides with the third order intensity maximum for Y. What is the ratio wavelength of X, wavelength of Y?
  - (a)  $\frac{1}{2}$
  - (b)  $\frac{2}{3}$
  - (c)  $\frac{3}{2}$
  - (d)  $\frac{2}{1}$
- 11) The tip of a needle does not give a sharp image. It is due to
  - (a) Reflection
  - (b) Refraction
  - (c) Diffraction
  - (d) Dispersion

| No. | Option | ANSWER              | EXPLANATION                                                                                                                                                                                                                                                          |
|-----|--------|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1   | d      | Wave front          | The correct option should be "wave front" Young is based on the principle of division of wave front                                                                                                                                                                  |
| 2   | c      | Polarization        |                                                                                                                                                                                                                                                                      |
| 3   | b      | Interference        |                                                                                                                                                                                                                                                                      |
| 4   | a      | Reflection of light |                                                                                                                                                                                                                                                                      |
| 5   | b      | Amplitude           |                                                                                                                                                                                                                                                                      |
| 6   | d      | Quadrupled          | $\lambda = \frac{\lambda L}{d}$ $d \rightarrow \text{halved and } L \rightarrow \text{doubled then}$ $\lambda = \frac{\lambda \cdot 2L}{d/2}$ $\lambda' = 4 \left( \frac{\lambda}{d} \right) \quad (2)$ Putting value from equation (1) in (2) $\lambda = 4 \lambda$ |

|    |   |                                      |                                                                                                                                                                                                                                                                                                                            |
|----|---|--------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| -  | b | pm                                   |                                                                                                                                                                                                                                                                                                                            |
| 8  | c | $\frac{2}{3}$                        | $d \sin \theta = m\lambda$ putting $d = 3\lambda$ and $m = 2$<br>$3\lambda \sin \theta = 2\lambda$<br>$\sin \theta = \frac{2}{3}$                                                                                                                                                                                          |
| 9  | c | 1. travel of infra red rays involves |                                                                                                                                                                                                                                                                                                                            |
| 10 | c | $\frac{3}{2}$                        | $d \sin \theta = m\lambda$<br>For x light wave length $\lambda$ , $m = 2$<br>So $d \sin \theta = 2\lambda$ (1)<br>For y light wavelength $\lambda_2$ , $m = 3$<br>So $d \sin \theta = 3\lambda_2$ (2)<br>Comparing equations (1) and (2) we have<br>$2\lambda = 3\lambda_2$<br>$\frac{\lambda_1}{\lambda_2} = \frac{3}{2}$ |
| 11 | c | diffraction                          |                                                                                                                                                                                                                                                                                                                            |

Short Answers of the Exercise

Q 2 Write short answers of the following questions

Q 1 A soap bubble looks black when it bursts why?

Ans When soap bubble is about to burst then its thickness is very small (almost zero)

- ▶ As one part of light rays is reflected from denser medium from upper side of soap bubble and have a path difference of  $\lambda/2$
- ▶ While a part of light rays is reflected from rarer medium from lower side of soap bubble and have no path difference
- ▶ Therefore the two reflected rays have path difference  $\lambda/2$  due to which destructive interference produces and soap bubble looks dark

Q 2 What is the difference between interference and diffraction?

Ans

Interference

- (i) Interference is due to superposition of two waves coming from two different wave fronts
- (ii) Interference bright fringes are of uniform high intensity
- (iii) The dark fringes are perfectly dark
- (iv) Interference fringes are equally spaced

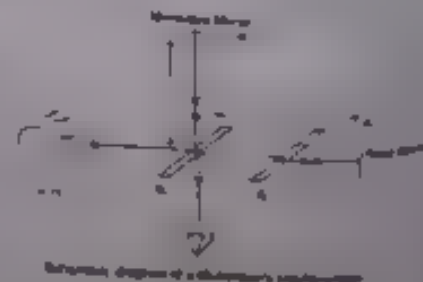
Diffraction

- (i) Diffraction is due to waves coming from different parts of the same wave front
- (ii) Diffraction bright bands are of low intensity
- (iii) The dark fringes are not perfectly dark
- (iv) The diffraction fringes are not equally spaced

Q 3 In a Michelson interferometer a second glass plate is also used, why?

Ans

- ▶ The ray I have to travel through half silvered glass plate two times and it decreases speed of light two times
- ▶ The glass plate  $G_2$  cut from the same piece of glass as  $G_1$  and is equal in thickness to  $G_1$  is introduced in the path of beam I to decrease its speed of light two times
- ▶  $G_2$  therefore equalizes the path length of the beam I and II in glass and is called compensating glass plate or compensator plate
- ▶ The two beams having their different paths are coherent. They produce interference effects when they arrive at observer's eyes



## How you can explain Brewster's law of polarization?

- Ans
- When unpolarized light falls on glass, water etc. the reflected light is in general partially plane polarized but at a certain angle of incidence called polarizing angle the polarization is complete.
  - At polarizing angle the reflected ray and the refracted ray in a transmitted medium are found to be at right angle to each other. The vibrations in the reflected ray are parallel to the surface as shown. Applying Snell's law we have

$$n_1 \sin i_p = n_2 \sin r$$

(1)

Where  $n_1$  and  $n_2$  are the absolute refractive indexes of the medium 1 and 2. Hence from the Fig 9.24 we have

$$i_p + 90^\circ + r = 180^\circ$$

$$r = 90^\circ - i_p$$

Hence, putting this value in Eq. (1)

$$n_1 \sin i_p = n_2 \sin (90^\circ - i_p)$$

$$n_1 \sin i_p = n_2 \cos i_p$$

$$\frac{n_2}{n_1} = \frac{\sin i_p}{\cos i_p}$$

$$\frac{n_2}{n_1} = \tan i_p$$

This is known as Brewster's law

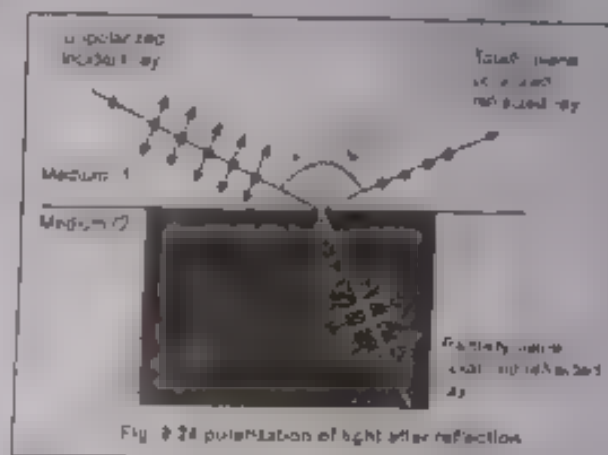


Fig. 9.24 polarization of light after reflection

## What is meant by the path difference with reference to the interference of two wave motion?

### Ans: Interference of Light Waves

The effect produced by the superposition of light waves from two coherent sources passing through same region in same direction is called interference of light.

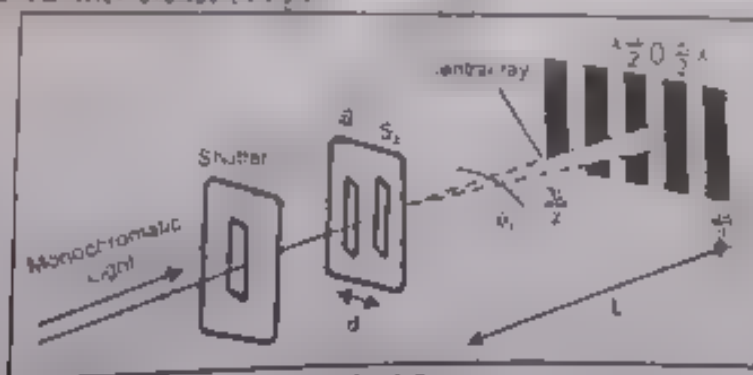


Fig. 9.5

There are two types of interference

- Constructive interference
- Destructive interference

### Constructive Interference

If crest of one wave falls on the crest of the other wave then they support each other. Such a interference is known as constructive interference.

The path difference is the difference between the distances travelled by two waves meeting at a point.

For constructive interference path difference is either zero or integral multiple of wavelength  $\lambda$ .

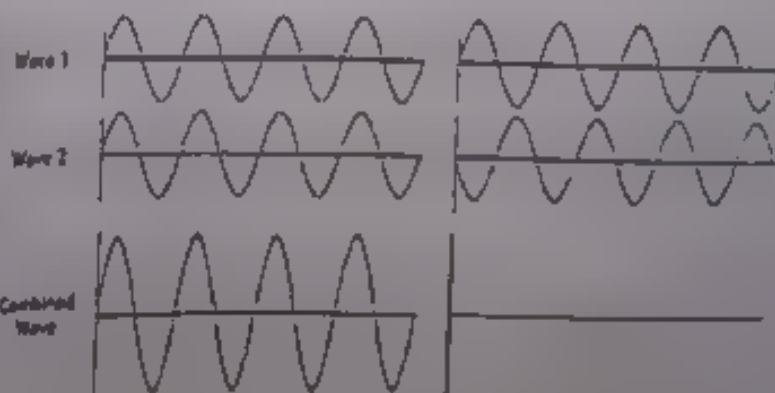
$$\text{Path difference} = d = (0, 2\lambda, 4\lambda, 6\lambda, \dots)$$

$$\text{Path difference} = d = m\lambda$$

Where  $m = 0, 1, 2, 3, \dots$

Two waves in phase  
(Constructive interference)

Two waves out of phase  
(Destructive interference)





Q 10 How would you justify that light waves are transverse?

Ans The phenomenon of polarization of light proves that light waves are transverse waves.

► The light has electric and a magnetic field component which are perpendicular to each other and perpendicular to the direction of propagation.

► The transverse waves can be polarized but longitudinal waves cannot be polarized. Hence the phenomenon of polarization of light proves that light waves are transverse waves.



► As the wave propagates in the z-direction, the electric field is oscillating in the y-direction and the magnetic field is oscillating in the x-direction. Since the x and y axes are perpendicular to each other and to the z-axis, light waves are transverse waves.

## Comprehensive Questions

Give a short response to the following questions.

What is meant by the dual nature of light? Discuss the history about the nature of light in detail.

Ans See Theory

Explain the diffraction of X-rays by crystal and derive an expression for Bragg's law to find the wavelength of light used?

Ans See 3<sup>rd</sup> from book

Describe the experimental arrangement for the production of interference fringes by Young's double slits method and get an expression for the fringes space.

Ans See 3<sup>rd</sup> from book

State and explain Huygen's principle. What is the difference between spherical and plane wavefronts?

Ans See 2<sup>nd</sup> from book

Explain the interference effect produced by thin film.

Ans See 4<sup>th</sup> from book

What is the principle of interference of light? Discuss the necessary condition for interference of light.

Ans See 3<sup>rd</sup> from book

What is diffraction grating? How can the wavelength of a beam of light be measured with it?

Ans See 1<sup>st</sup> from book

- 8 Describe the construction and working of Michelson's interferometer. How one can determine the wavelength of light used by this instrument?

**Ans** See Q 7 from book

- 9 What is meant by plane polarized light? How does this phenomenon decide that light waves are transverse in nature.

**Ans** See Q 12 from book

## Numerical Problems

- 1 In a young double slit experiment the separation of the slits is 1 mm and red light of wave length 620 nm is falling on it. Determine the distance between the central bright band and the fifth bright fringe on the screen which is 3m away from the slit.

**Data** Separation between slits is  $d = 1 \text{ mm}$   
 Wavelength of red light  $= \lambda = 620 \text{ nm} = 620 \times 10^{-9} \text{ m}$   
 For fifth bright fringe  $= m = 5$   
 Distance between slits and screen  $= L = 3 \text{ m}$   
 Distance of 5<sup>th</sup> bright fringe from center of screen  $= Y$

**Solution**

Formula  $Y = \frac{m\lambda L}{d}$

Putting values

$$Y = \frac{5 \times 620 \times 10^{-9} \times 3}{1 \times 10^{-3}}$$

$$Y = 9.3 \times 10^{-3} \text{ m}$$

$$Y = 9.3 \text{ mm}$$

**Answer**

- 2 Two parallel slits are illuminated by light of two wavelengths, one of which is  $5.8 \times 10^{-7} \text{ m}$ . On the screen the fourth dark line of the known wavelength coincides with the fifth bright line of the light of unknown wavelength. Find the unknown wavelength.

**Data** wave length of 1<sup>st</sup> light  $= \lambda_1 = 5.8 \times 10^{-7} \text{ m}$   
 For 4<sup>th</sup> dark fringe  $= m = 3$   
 Wavelength of 2<sup>nd</sup> light  $= \lambda_2 = ?$   
 For 5<sup>th</sup> bright fringe  $= m = 5$

**Solution:**

Position of dark fringe is

$$Y_{\text{dark}} = \left(3 + \frac{1}{2}\right) \frac{\lambda_1 L}{d}$$

$$= \left(\frac{6+1}{2}\right) \frac{\lambda_1 L}{d}$$

$$Y_{\text{dark}} = \frac{7 \times L}{2d}$$

(Putting  $m = 3$  for 4<sup>th</sup> dark line)

Position of bright fringe is

$$Y_{\text{bright}} = \frac{m\lambda_2 L}{d}$$

Putting  $m = 5$  for 5<sup>th</sup> bright fringe.

$$y_{\text{bright}} = \frac{\lambda}{d} \cdot \frac{1}{2}$$

According to given condition 4<sup>th</sup> dark line of light of wavelength  $\lambda$  is seen with 5<sup>th</sup> bright line of 2<sup>nd</sup> light of same wave length

$$y_{\text{dark}} = y_{\text{bright}}$$

Put values from eq. and eq.

$$\frac{7\lambda}{2d} = \frac{5\lambda}{d}$$

$$\frac{7}{2} = 5$$

$$7 = 10$$

$$7 - 10 = -3$$

$$\frac{-3}{1} = -3$$

Putting value of

$$d = 1 \times 10^{-3} \text{ m}$$

Answer

3. When the movable mirror of a Michelson interferometer is moved 0.1 mm. How many dark fringes pass through the reference point, if light of wavelength 580 nm is used?

Data Distance moved by movable mirror  $P = 0.1 \text{ mm} = 0.1 \times 10^{-3} \text{ m}$

Wavelength of light  $= \lambda = 580 \text{ nm} = 580 \times 10^{-9} \text{ m}$

Number of fringes  $= m = ?$

Solution:

Formula

$$P = m \frac{\lambda}{2}$$

$$m = \frac{2P}{\lambda}$$

Putting values

$$m = \frac{2 \times 0.1 \times 10^{-3}}{580 \times 10^{-9}}$$

$$m = 344.827$$

Answer

4. A soap film has a refractive index of 1.40. How thick must the film be, if it appears black, when mercury light of wavelength 546.1 nm falls on it normally?

Data Refractive index  $n = 1.40$

Wavelength of light  $\lambda = 546.1 \text{ nm} = 546.1 \times 10^{-9} \text{ m}$

Thickness of film  $= x = ?$

For 1<sup>st</sup> order  $= n = 1$

Solution:

Formula for destructive interference for thin film is

$$2nx = m\lambda$$

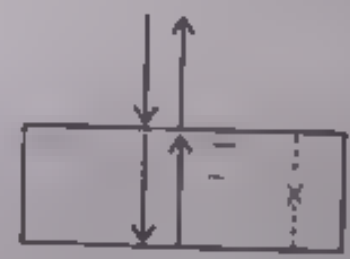
$$x = \frac{m\lambda}{2n}$$

$$x = \frac{546.1 \times 10^{-9}}{2 \times 1.40}$$

$$x = 1.947 \times 10^{-7} \text{ m}$$

$$x = 194.7 \text{ nm}$$

Answer



5. A diffraction grating has 5000 lines per centimeter. At what angle does the second order spectrum of the sodium yellow light of wavelength 589 nm occur?

Data No. of lines  $N = 5000 \text{ lines/cm}$

$$N = 5000 \text{ lines/cm} = \frac{1}{200} \text{ m}$$

$$N = 5000 \times 100 \text{ lines/m}$$

$$N = 500000 \text{ lines/m}$$

$$\text{Grating element} = d = \frac{1}{N}$$

$d = 2.5 \times 10^{-3} \text{ m}$   
 $\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$   
 $n = 1$

**Solution**

For the first order,  $n = 1$

$$d \sin \theta = n \lambda$$

$$2.5 \times 10^{-3} \sin \theta = 1 \times 500 \times 10^{-9}$$

$$\sin \theta = \frac{500 \times 10^{-9}}{2.5 \times 10^{-3}}$$

$$\sin \theta = 0.0002$$

$$\theta = \sin^{-1}(0.0002)$$

$$\theta = 0.011^\circ$$

**Answer**

6. Light is incident normally on a grating which has 250 lines/cm. Find the wavelength of spectral line for which the deviation in second order is  $12^\circ$ .

**Given Data**

$n = 250 \text{ lines/cm}$   
 $\theta = 12^\circ$   
 $n = 2$

**Solution**

$d = \frac{1}{n} = \frac{1}{250} \text{ cm} = 10^{-3} \text{ m}$   
 $d \sin \theta = n \lambda$   
 $10^{-3} \sin 12^\circ = 2 \lambda$   
 $\lambda = \frac{10^{-3} \sin 12^\circ}{2}$

$$\lambda = 1.04 \times 10^{-5} \text{ m}$$

**Answer**

7. In a certain X rays diffraction experiment the first order image is observed at an angle of  $5^\circ$  for a crystal plane spacing of  $2.8 \times 10^{-10} \text{ m}$ . What is the wavelength of X rays used?

**Given Data**

$d = 2.8 \times 10^{-10} \text{ m}$   
 $\theta = 5^\circ$   
 $n = 1$   
 $\lambda = ?$

**Solution**

$d \sin \theta = n \lambda$   
 $2.8 \times 10^{-10} \sin 5^\circ = 1 \times \lambda$   
 $\lambda = 2.45 \times 10^{-11} \text{ m}$

$$\lambda = \frac{2 \times 2.8 \times 10^{-10} \sin 5^\circ}{1}$$

$$\lambda = 0.488 \times 10^{-10} \text{ m}$$

$$\lambda = 0.488 \times 10^{-10} \text{ m}$$

$$\lambda = 0.488 \times 10^{-10} \text{ m}$$

Answer

An X-ray beam of wavelength  $0.48 \times 10^{-10} \text{ m}$  is used to get Bragg reflection from a crystal at an angle of  $20^\circ$  for the first order maximum. What are the possible layer plane spacing which give rise to this maximum?

Given Data

Wavelength of X-rays =  $0.48 \times 10^{-10} \text{ m}$

Angle of diffraction =  $\theta = 20^\circ$

For 1<sup>st</sup> order maximum =  $m = 1$

Crystal plane spacing =  $d = ?$

Solution

$$2d \sin \theta = m\lambda$$

$$d = \frac{m\lambda}{2 \sin \theta}$$

Putting values

$$d = \frac{1 \times 0.48 \times 10^{-10}}{2 \sin 20^\circ}$$

$$d = \frac{0.48}{2 \times 0.342}$$

$$d = 0.70 \times 10^{-10} \text{ m}$$

Answer

The spacing of one set of crystal planes in NaCl (table salt) is  $d = 0.282 \text{ nm}$ . A monochromatic beam of x-rays produces a Bragg maximum when its glancing angle with these planes is  $\theta = 7^\circ$ . Assuming that this is a first order maximum ( $n=1$ ). Find the wavelength of x-ray.

Given Data

Order of maximum =  $n$

Crystal plane spacing =  $d = 0.282 \times 10^{-9} \text{ m}$

Angle =  $7^\circ$

Wave length =  $\lambda = ?$

To find

Solution

$$d \sin \theta = n\lambda$$

$$\lambda = \frac{d \sin \theta}{n} = \frac{0.282 \times 10^{-9} \times \sin 7^\circ}{1}$$

$$\lambda = 6.9 \times 10^{-11} \text{ m}$$

$$\lambda = 0.69 \times 10^{-10} \text{ m}$$

$$\lambda = 0.69 \times 10^{-10} \text{ m}$$

Answer

## Additional Conceptual Short Questions With Answers

How can you increase the fringe width in Young's double slit experiment?

The fringe width is

$$\Delta y = \frac{\lambda D}{d}$$

The fringe width can be increased by increasing

(i) by increasing wavelength  $\lambda$

- (iii) by increasing the distance between the coherent source  $L$  and  
 (iv) by decreasing the distance between the coherent source  $d$

**2** The Young's double slit experiment apparatus is taken from air into water. What will happen to the interference pattern?

**Ans:** The fringe width is

$$\Delta y = \frac{\lambda L}{d}$$

When Young's double slit apparatus is brought from air into water then wavelength  $\lambda$  of light decreases and the fringe width  $\Delta y$  decreases.

**3** What changes would you expect if monochromatic light is replaced by white light in single slit diffraction experiment?

**Ans:** The diffraction pattern is coloured. In higher order spectra due to higher dispersion there will be overlapping colours and a diffused pattern is obtained.

**4** How can we obtain the plane wave fronts (Parallel Rays) of light?

- Ans:** (i) As the wave fronts are the parts of spheres of very large radii. Then at small distances from the source a part of the spherical wave front can be regarded as a plane wave front (parallel rays of light).  
 (ii) If the source of light is at the focus of a convex lens. Then the rays coming out the lens will constitute the plane wave fronts.

**5** If a class room door is open even just a small amount, you can hear sounds coming from the hallway. Yet you cannot see what is going on in the hallway. Why is there this difference?

**Ans:** The space between the slightly open door and the wall is acting as a slit for waves. The sound waves have wavelengths comparable with the slit width, so sound is diffracted effectively by the opening and sound spread out in the whole room. Light wavelengths are much smaller than the slit width, so there is a very little or no diffraction for the light. So they do not spread out in the room and we are not able to see what is going on in the hallway.

**6** Why does a diamond sparkle more than a glass of the same shape and size?

**Ans:** The refractive index of diamond is very high and its critical angle is small (i.e.  $24^\circ$ ). When a beam of light enters it, it is totally reflected number of times inside it and it emerges in random directions causing the sparkling of the diamond. But in case of glass refractive index is less and critical angle greater than diamond, so it does not sparkle like a diamond.

### MCQ's From Past FBISE Papers (FEDERAL BOARD)

- By using the transmitted light, the central spot of Newton's ring appears to be:  
 (a) Bright (b) dark (c) coloured (d) all of these
- Longitudinal waves do not exhibit:  
 (a) Reflection (b) refraction (c) diffraction (d) polarization
- The phase change of  $180^\circ$  is equal to the path difference of  
 (a)  $\lambda$  (b)  $\frac{\lambda}{2}$  (c)  $2\lambda$  (d)  $3\lambda$
- Bending of light around edges of an obstacle is called:  
 (a) Reflection (b) polarization (c) interference (d) diffraction
- If 5000 lines per cm are ruled on diffraction grating, then slit spacing will be:  
 (a)  $5 \times 10^{-4} \text{ m}$  (b) 0.02 m (c)  $2 \times 10^{-4} \text{ m}$  (d)  $2 \times 10^4 \text{ m}$
- Condition of constructive interference for first order image for diffraction grating is ..  
 (a)  $d \sin \theta = \lambda$  (b)  $d \sin \theta = 0$  (c)  $d \sin \theta = 2\lambda$  (d)  $d \sin \theta = 3\lambda$
- Which one of the following wave cannot be polarized?  
 (a) Radio wave (b) X-ray (c) longitudinal wave (d) transverse wave.

8. What happened to the fringe spacing, when Young double slit experiment is performed in water instead of air?  
 a) Fringe spacing increases (b) Fringe spacing decreases (c) Double (d) None of these
9. Soap film in sunlight appears coloured due to  
 a) Scattering of light (b) Interference (c) Dispersion (d) None of these
10. The blue of sky is due to  
 a) Rayleigh scattering (b) Reflection (c) Dispersion (d) None of these
11. Optically active substances are those substances which  
 a) Rotate the plane of polarization (b) Produce double refraction (c) Rotate the plane of polarization of polarized light (d) Convert a plane polarized light into circularly polarized light
12. The refractive index of rarer medium with respect to a denser medium is  
 a) Greater than 1 (b) 1 (c) Greater than 1 (d) None of these
13. A diffraction grating has 5000 lines per cm. Its grating element is  
 a)  $2 \times 10^{-6}$  m (b)  $2 \times 10^{-5}$  m (c)  $2 \times 10^{-4}$  m (d)  $2 \times 10^{-3}$  m
14. The Bragg equation is given as  
 a)  $2d \sin \theta = n\lambda$  (b)  $2d \cos \theta = n\lambda$  (c)  $2d \tan \theta = n\lambda$  (d)  $2d \cot \theta = n\lambda$
15. In Michelson's interferometer a fringe is shifted each time the mirror is displaced by  
 a)  $\frac{\lambda}{2}$  (b)  $\lambda$  (c)  $\frac{\lambda}{4}$  (d)  $\frac{\lambda}{8}$

Answer Key



## SELF ASSESSMENT PAPER

Total Mark 40

Question No 1 Choose the correct answer from the given options

(1 x 8 = 8)

## SECTION - A

- 1 The fringe spacing varies inversely with the
  - A distance between slits and screen
  - B separation between the slits
  - C wavelength of light
  - (D) none of these
- 2 The blue colour of sky is due to
  - A diffraction of light
  - B reflection of light
  - C polarization of light
  - D scattering of light
- 3 Soap film in sunlight appears coloured due to
  - A scattering of light
  - B interference of light
  - (C) refraction of light
  - (D) dispersion of light
- 4 If 5000 lines per cm are ruled on a diffraction grating the slit spacing or grating element will be
  - A  $2 \times 10^{-3}$  m
  - B  $2 \times 10^{-4}$  m
  - C  $2 \times 10^{-5}$  m
  - D  $2 \times 10^{-6}$  m
- 5 Two monochromatic radiations X and Y are incident normally on a diffraction grating. The second order intensity maximum for X coincides with the third order intensity maximum for Y. What is the ratio of wavelength of X to wavelength of Y
  - (A)  $\frac{1}{2}$
  - (B)  $\frac{2}{3}$
  - (C)  $\frac{3}{2}$
  - (D)  $\frac{4}{3}$
- 6 The tip of a needle does not give a sharp image this is due to
  - (A) Diffraction of light
  - (B) Interference of light
  - (C) Polarization of light
  - (D) Refraction of light

Question No 2 Choose short answers of following questions

(3 x 7 = 21)

## SECTION - B

- (i) Why is the fly wheel of an engine made heavy in the rim?
- (ii) A ball is suspended by a string without cleaving. If it is set swinging it makes a sound. Why?
- (iii) Why does the rotating system is known as water drop?
- (iv) Explain why the wheels of cycles make a sound when they rotate.
- (v) Explain, why is there weightlessness in satellites?
- (vi) At what speed is a bank angle of  $45^\circ$  required for aeroplane to turn in a radius of 60 m?
- (vii) How and why is a rainbow produced in the sky? Explain.

Question No 3 Extensive Questions

(13)

## SECTION - C

- (a) Describe the Young's double slit experiment for demonstration of interference. Give an expression for fringe spacing.
- (b) Two parallel slits are illuminated by light of two wavelengths, one of which is  $580 \text{ nm}$ . On the screen the fourth dark line of the known wavelength coincides with the fifth bright line of the other wavelength. Find the unknown wavelength.

\*\*\* The End \*\*\*

# CHAPTER

# 10

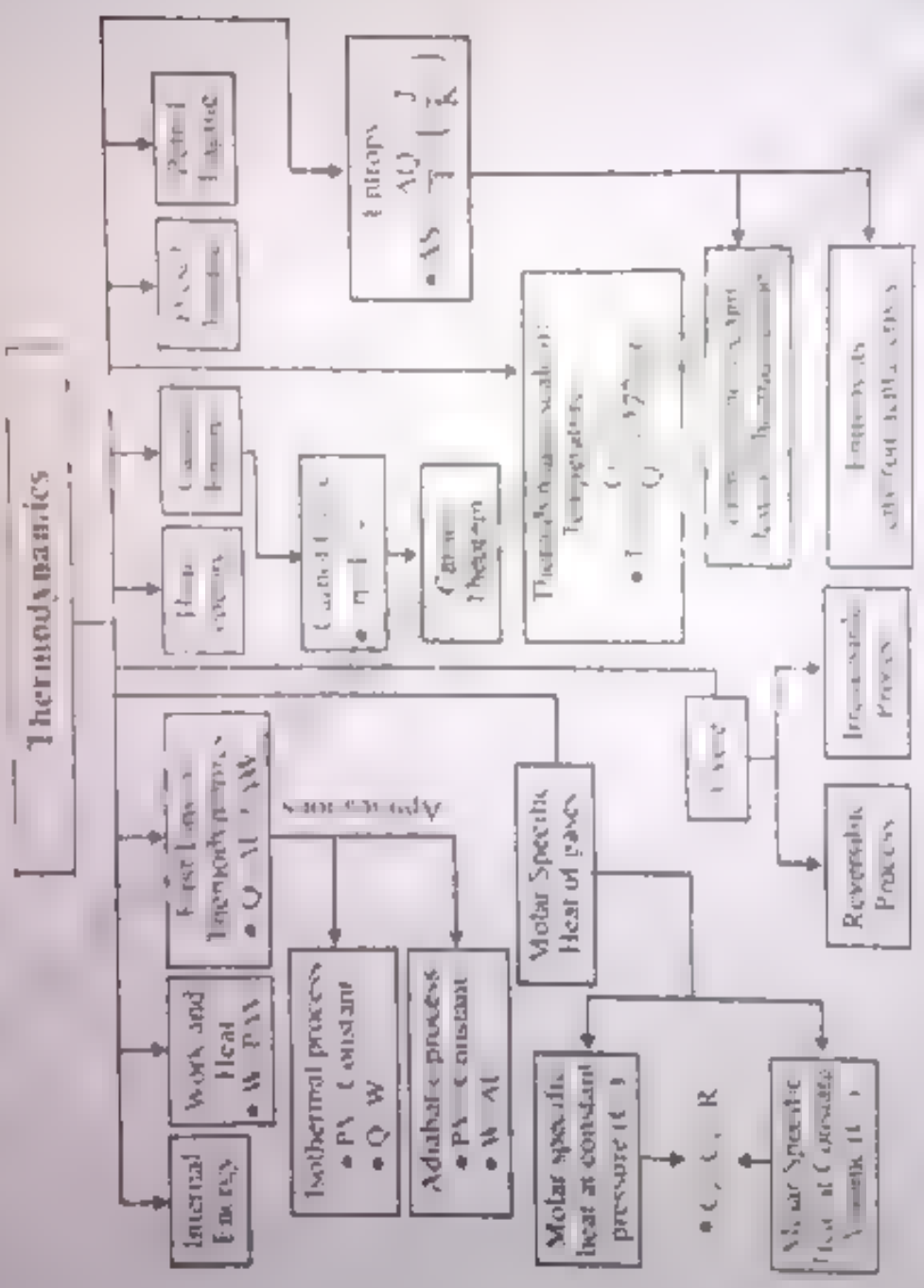
## THERMODYNAMICS

### Learning Objectives

- ◆ Describe that thermal energy is transferred from a region of higher temperature to a region of lower temperature
- ◆ Describe that regions of equal temperatures are in thermal equilibrium
- ◆ Describe that heat flow and work are two forms of energy transfer between systems and a system and being transferred
- ◆ Define thermodynamics and various terms associated with it
- ◆ Relate a small temperature of a body to an increase in its thermal energy
- ◆ Describe the mechanical equivalent of heat concept as it was historically developed to solve problems involving work being done and temperature change
- ◆ Explain that thermal energy is determined by the state of the system and that it can be expressed as the sum of the random distribution of kinetic and potential energies associated with the molecules of the system
- ◆ Calculate work done by a thermodynamic system during a volume change
- ◆ Describe the first law of thermodynamics expressed in terms of the change in internal energy due to heating of the system and work done on the system
- ◆ Explain that first law of thermodynamics expresses the conservation of energy
- ◆ Define the terms specific heat and molar specific heats of a gas
- ◆ Apply first law of thermodynamics to derive  $C_p - C_v = R$
- ◆ State the working principle of heat engine
- ◆ Describe the concept of reversible and irreversible processes
- ◆ State and explain second law of thermodynamics
- ◆ Explain the working principle of Carnot's engine
- ◆ Explain that the efficiency of a Carnot engine is independent of the nature of the working substance and depends on the temperatures of hot and cold reservoirs
- ◆ Describe that refrigerator is a heat engine operating in reverse as that of an ideal heat engine
- ◆ Derive an expression for the coefficient of performance of a refrigerator
- ◆ Describe that change in entropy is positive when heat is added and negative when heat is removed from the system
- ◆ Explain that increase in temperature increases the disorder of the system
- ◆ Explain that increase in entropy means degradation of energy
- ◆ Explain that energy is degraded during all natural processes
- ◆ Identify that systems tend to become less orderly over time

# Chapter No. 10

## CONCEPT MAP



"Thermodynamics is a combination of two words thermo and dynamics. The word thermo is related to heat while dynamics is related to the motion of particles."

Therefore we define thermodynamics as the branch of physics that deals with the transformation of heat into other forms of energy such as mechanical, chemical and electrical energy and vice versa.

► Principally it is based on two laws of thermodynamics i.e. the first and second laws of thermodynamics.

► It is a practical subject that explains the working of heat engines, refrigerators and heat pumps.

### For Your Information:

The first thermodynamic textbook was written in 1859 by William Rankine, who was a physicist and a civil and mechanical engineering professor at the University of Glasgow.

### Role of Thermodynamics

► It plays a central role in technology.

► All the raw energy available for our use is liberated in the form of heat.

### Q1 What is thermal equilibrium?

#### Ans: Thermal Equilibrium

► When two objects at different temperatures are brought into contact with each other, energy is transferred from the hotter to the colder object until the bodies reach thermal equilibrium (that is, they are at the same temperature).

► These observations reveal that heat is energy transferred spontaneously due to a temperature difference. Figure 10.1 shows an example of heat transfer.

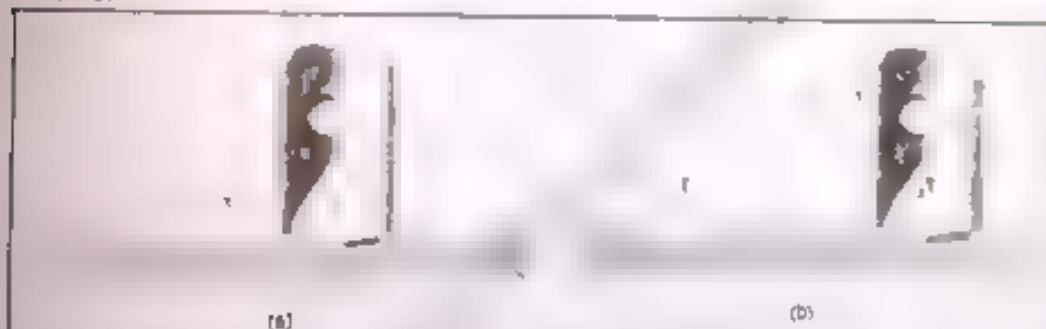


Figure 10.1 (a) Here, the soft drink has a higher temperature than the ice, so they are not in thermal equilibrium. (b) When the soft drink and ice are allowed to interact, heat is transferred from the drink to the ice due to the difference in temperatures until they reach the same temperature  $T$ , achieving equilibrium.

► In fact, since the soft drink and ice are both in contact with the surrounding air and the bench, the ultimate equilibrium temperature will be the same as that of the surroundings.

► In other words, if two objects are in thermal equilibrium, they have the same temperature.

### Point to Ponder

Q. Two bodies, one hot and the other cold, are kept in vacuum. What will happen to the temperature of the hot body after some time?

A. Due to thermal radiation, after some time the cold body will gain heat and the hot body will lose heat. When the temperature of the two bodies become equal, then bodies are said to be in thermodynamic equilibrium.

### FIGURE



Work done by a system

### Q.2 Explain work done in thermodynamics.

#### Ans: WORK

► Work  $W$  is defined as  $\int \vec{F} \cdot d\vec{S}$ .

► In thermodynamics, work is said to be done by a system when the system as a whole expands.

► By means of the boundary, the system exerts a force on the surroundings and thereby displaces the surroundings.

► The sum of all  $\int \vec{F} \cdot d\vec{S}$  over the entire boundary of the system is the work done by a system on its surroundings.

- ▶ Similarly, in the process of contraction of a system, work is done on the system by its surroundings
- ▶ By convention the work done by a system is considered positive and the work done on the system is negative



**Q** The temperature of a normal healthy person is  $37^{\circ}\text{C}$ . What will be the temperature of a dead person?

**A** It depends on how long the person has been dead. Temperature of dead body slowly approaches that of the surrounding place/surrounding air.

**Q 3** Define and explain Internal energy.

**Ans:** Internal Energy

The sum of all the forms of molecular energies (such as kinetic and potential energy) of a substance is called internal energy. (OR)

The sum of the kinetic and potential energy associated with the random motion of the atoms of the substance is the internal energy of the substance

The kinetic energy may be in the form of translational, rotational and vibrational kinetic energy

- ▶ Atoms and molecules are in constant motion about their mean positions
- ▶ Molecules in a liquid wander around among the other molecules. They have frequent collisions with them and thus exchanging energy
- ▶ In gases, the molecules travel about at high speeds and have frequent elastic collisions with the neighbors.
- ▶ When we heat a substance, the random motion and the energy associated with it are increased and energy is transferred into the internal energy of the substance
- ▶ Similarly, work can be performed on a substance in such a way as to increase the random motion of the atoms.
- ▶ Since Temperature  $\propto \langle K.E. \rangle$
- ▶ Therefore internal energy of an ideal gas is directly proportional to its temperature
- ▶ The increase in temperature of the object indicates an increase in the internal energy

**Internal energy is a state function**

- ▶ In thermodynamics, internal energy is function of state. Consequently, it does not depend on the path between its initial and final states of the system



**Q 1** Is the temperature of a normal healthy old man is less than that of a normal healthy young man?

**A 1** When people get very old, blood circulation often becomes poor, resulting in cold hands and feet. The mean body temperatures of old man lower than the reference temperature

**Q 2** A glass full of water contains ice cubes floating in it. What will happen to the water level when ice melts?

**A 2** The water level remains the same when the ice cube melts.

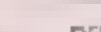
According to the Archimedes principle, the floating substance displaces some liquid, so when the ice melts, there will be no change in the water level as the melted ice will occupy the same volume as it was occupying earlier

**Q 4** What is heat and work?

**Ans:** Heat, Work and internal energy

Heat is form of energy which flows from the hotter body to the colder body till the temperatures of the two bodies become equal

- ▶ To raise the temperature of some water we "heat" the water by placing the water pot on a flame

- 

QUIZ

**Q** Bore with a small drill into a hard board. The drill becomes hot to touch. Why? What happens if hard board is replaced by a soft board?

**A** A large amount of work has to be done in making the bore which produces a large amount of heat and the drill becomes too hot to touch. In case of the soft board less work has to be done in making the bore. Hence heat produced is also small.

### Expt: Equivalence of Heat and Work

- Where  $J$  is called "Mechanical Equivalent" of heat (Joule's constant) and  $n$  is called

- ▶ The ratio of the work done in joules to the heat produced in calories (old unit of heat) is called the mechanical equivalent of heat (J).
- ▶ The mechanical equivalent of heat may also be defined as the amount of mechanical energy or work required to produce a unit quantity of heat.

- ▶ The mechanical equivalent of heat is  $1 \text{ cal} = 4.18 \text{ J}$ .
- ▶ If work and energy then both work ( $W$ ) and heat ( $Q$ ) are measured in terms of energy, then the conservation of energy can be stated as follows: The total energy of a system of heat is conserved.



QUIZ

- Q What is the significance of Joule's experiment for determining the value of  $J$ ?
- A Joule's experiment proved that heat was actually a form of mechanical energy, so it is not a new form of energy. This led to the modern understanding of the conservation of energy.

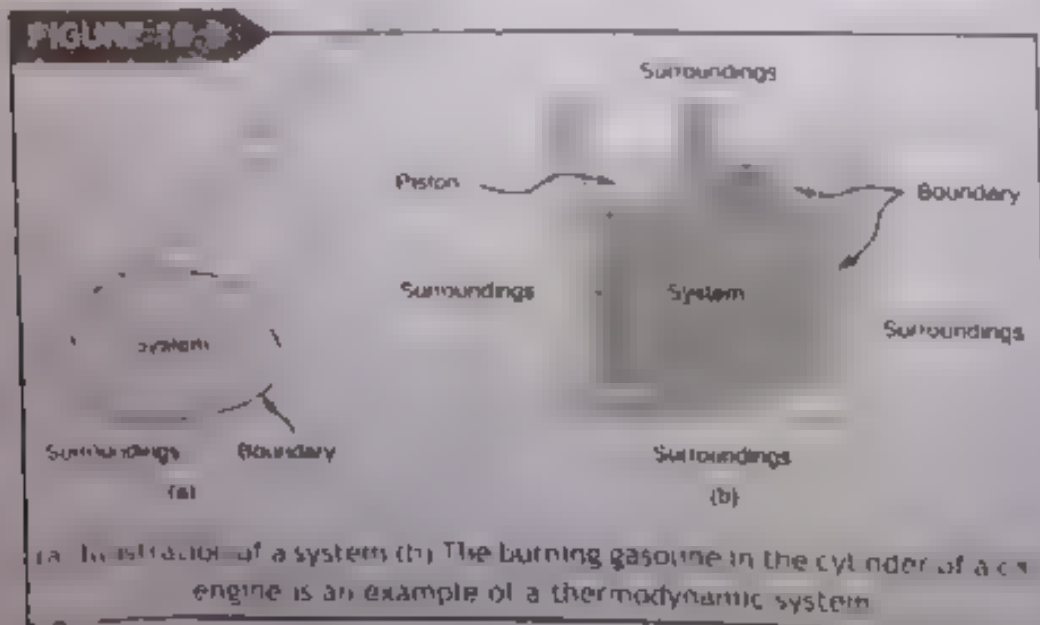
- Q The heat capacity of a normal healthy old man is less than that of a normal healthy young man.
- A The heat capacity of an old man is less than that of a young man because the old man has a smaller mass and therefore a smaller heat capacity. The heat capacity of a young man is about  $3.5 \text{ kJ/K}$ , while that of an old man is about  $2.5 \text{ kJ/K}$ .
- Q A glass full of water contains ice cubes floating in it. What will happen to the water level when the ice melts?
- A The water level remains the same when the ice cube melts.

Explanation: According to the Archimedes principle, the floating substance displaces an equal volume of the liquid in which it is floating. When the ice melts, the volume of water displaced by the ice is equal to the volume of the ice itself. Therefore, there is no change in the water level as the melted ice will occupy the same volume as it was already occupying.

- Q 6 Explain thermodynamic system and its types. Also discuss the thermodynamic state variable.

### Thermodynamic system

- ▶ A thermodynamic system includes anything whose thermodynamic properties are of interest.
- ▶ The surroundings are everything outside the system. The boundary separates the system from the surroundings.



- ▶ The thermodynamic system and its surroundings are interactive with each other. The system and its surroundings are separated by a boundary. The boundary separates the system from the surroundings. The boundary can be real or imaginary. The boundary can be fixed or movable. The boundary can be permeable or impermeable. The boundary can be adiabatic or diathermic. The boundary can be rigid or flexible. The boundary can be smooth or rough. The boundary can be straight or curved. The boundary can be open or closed. The boundary can be finite or infinite. The boundary can be static or dynamic. The boundary can be stable or unstable. The boundary can be linear or non-linear. The boundary can be isotropic or anisotropic. The boundary can be homogeneous or heterogeneous. The boundary can be uniform or non-uniform. The boundary can be continuous or discontinuous. The boundary can be differentiable or non-differentiable. The boundary can be smooth or rough. The boundary can be straight or curved. The boundary can be open or closed. The boundary can be finite or infinite. The boundary can be static or dynamic. The boundary can be stable or unstable. The boundary can be linear or non-linear. The boundary can be isotropic or anisotropic. The boundary can be homogeneous or heterogeneous. The boundary can be uniform or non-uniform. The boundary can be continuous or discontinuous. The boundary can be differentiable or non-differentiable.

Example: Consider a gas contained in a cylinder. The gas is the thermodynamic system. The piston and the cylinder walls are the surroundings of the system. The boundary then consists of the piston and the cylinder walls.

temperature

- Any change in a series of changes in thermodynamic process.
- We can define the system as follows.

### System

### Surroundings of the system

### Boundary of the system

- The system is surrounded by its boundary.
- The boundary separates the system and the surroundings.

### Closed system

The system in which heat energy can flow in to or out of the system but mass cannot enter or leave it, is called closed system.

FIGURE 10.2



(a) This boiling tea kettle is an open thermodynamic system. It transfers heat and mass (steam) to the surroundings. (b) This boiling tea kettle is a closed thermodynamic system. It transfers heat to the surroundings but mass cannot enter or leave it.

### Open system

- In an open system the transfer of heat energy can take place from the system to the surroundings or vice versa. Plants and animals are examples of open systems because they exchange matter and energy with their surroundings.

### Isolated system

- In an isolated system, both mass and energy cannot be exchanged with the surroundings.

### Thermodynamic state and thermodynamic state variables

- The particular condition with which a system is said to be in a thermodynamic state is called its thermodynamic state.
- The variables or functions which describe the physical state of the system are called state functions of the system.

### For You Information

Q. An empty polythene bag burns or melts on a flame of a stove. However we can make a few cups of tea by placing water inside polythene bag on a flame of a stove.

A. The polythene bag is higher than water i.e.  $100^{\circ}\text{C}$ . When water is inside the polythene bag, then by heating the polythene bag, it transfers heat to water and polythene bag does not keep enough heat.

### Point to Ponder

Q. An ink dot on a white porcelain dish appears dark. When the dish is raised to a very large temperature the dot appears brighter than the surroundings. Why?

A. When temperature of dish is raised to very high then it begins to emit radiation of different wavelengths. Due to this reason the ink dot starts to appear brighter than its surroundings.

- ▶ If the system is thermally insulated, it has no heat exchange with the surroundings. In this case, the heat  $Q$  is used as enough to do work  $W$  and the change in internal energy  $\Delta U$  is given by  $\Delta U = W$ .
- ▶ The process depends on the quantities  $P$ ,  $V$ ,  $T$ ,  $Q$ ,  $W$ ,  $\Delta U$  and  $I$  are considered. The process is called  $P$ - $V$ ,  $P$ - $T$ ,  $V$ - $T$ ,  $Q$ - $W$ ,  $\Delta U$ - $I$  and so on.
- ▶ Efficiency of the process is given by  $\eta = \frac{W}{Q}$ .

### Q 7 Write a note on Reversible and Irreversible Processes

#### NOTE

A process is called reversible if it can be retraced in the reverse order without producing any change in the surroundings.

#### Reversible Process

A reversible process is one which can be retraced in the reverse order without producing any change in the surroundings.

#### Explanation

- ▶ A reversible process is one which can be retraced in the reverse order without producing any change in the surroundings.
- ▶ The process is called reversible if it can be retraced in the reverse order without producing any change in the surroundings.
- ▶ A reversible process is one which can be retraced in the reverse order without producing any change in the surroundings.
- ▶ A reversible process is one which can be retraced in the reverse order without producing any change in the surroundings.

#### Cycle

- ▶ A cycle is a process which starts from a state and returns to the same state.

#### Examples of Reversible Process

The process of expansion of a gas in a cylinder with a piston is reversible if the piston moves slowly and the process is retraced in the reverse order without producing any change in the surroundings. The process can be changed to irreversible by increasing the speed of the piston.

#### Irreversible Process

An irreversible process is one which cannot be retraced in exactly reverse order without producing any change in the surroundings.

#### Explanation

- ▶ All changes which occur suddenly or with friction or dissipation of energy through conduction, convection and radiation are irreversible.

#### Examples

- ▶ Expansion of a gas in a cylinder with a piston is irreversible if the piston moves suddenly.
- ▶ Work done by a gas in a cylinder with a piston is irreversible if the piston moves suddenly.

### Q 8 Explain the first law of Thermodynamics and its consequences?

#### NOTE

#### First Law of Thermodynamics

When a quantity of heat  $Q$  is added to a system, this energy appears as an increase in the internal energy  $\Delta U$  of the system plus the work done  $W$  by the system on the surroundings.

#### Mathematically

$$\Delta U = Q + W \quad (1)$$

This is equivalent to first law of thermodynamics.

#### Think to Ponder

Q Why do heels crack in winter? What effect does the application of lubricants have on the heels?

A A person with sensitive skin on the bottom of the feet and heels becomes dry due to the lack of moisture. This dryness leaves cracks called fissures on your heels. The application of lubricants decreases the dryness of skin.

### Explanation

- ▶  $\Delta Q$  is taken as positive when heat enters the system
- ▶  $\Delta Q$  is taken as negative when heat leaves the system
- ▶  $\Delta T$  is taken as positive when temperature of the system rises
- ▶  $\Delta T$  is taken as negative when temperature of the system decreases
- ▶ By convention the work done by a system is taken as positive
- ▶ The work done on the system is taken as negative
- ▶ First law of thermodynamics obeys the law of conservation of energy

When heat is added to a system there is an increase in the internal energy, temperature and an increase in pressure or change in the state of the system. If the system is allowed to expand then  $\Delta W$  is the work done on the environment.

change in internal energy  $\Delta U = U_B - U_A$  ... (1)

Where  $U_A$  is initial internal energy of system

$U_B$  is final internal energy of system

From equation (1)

$$\Delta U = \Delta Q - \Delta W \quad (3)$$

Putting values from equation (2) in (3)

$$U_B - U_A = \Delta Q - \Delta W \quad (4)$$

If system under goes cyclic process then

$$U_B = U_A$$

Putting  $U_B = U_A$  in equation (4)

$$U_A - U_A = \Delta Q - \Delta W$$

$$0 = \Delta Q - \Delta W$$

$$\Delta Q = \Delta W$$



QUIZ

**Q** Two blocks of ice when pressed together combine to form single piece Explain how this happen?

- A**
- ▶ When two blocks are pressed together the heat is generated due to pressure and of friction which melts the outer layers and friction decreases.
  - ▶ When pressure is decreased and due to the low temperature of both the ice blocks the water just refreezes and blocks combine to form single piece. It is called Regelation.
  - ▶ Regelation is the phenomenon of melting under pressure and freezing again when the pressure is reduced.

**Q 9** Discuss applications of first law of thermodynamics

**Ans:** Applications of First law of thermodynamics

(a) Isochoric Process:

The thermodynamic process in which the volume of the system remains constant is called isochoric process.

- ▶ We consider the gas contained in a cylinder having a conducting base, non-conducting walls and a fixed piston at the end as shown in the fig (10.4a)
- ▶ Let heat ( $\Delta Q$ ) is supplied to the system at constant volume
- ▶ The pressure of the gas increases from  $P_1$  to  $P_2$
- ▶ Temperature increases from  $(T_1)$  to  $(T_2)$

Since the system neither expands nor contracts, work is neither done by the system nor on the system

$$\Delta W = 0$$

- ▶ According to first law of thermodynamics

Putting

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta W = 0$$

$$\Delta Q = \Delta U > 0$$

$$\Delta U = \Delta T$$

- The above equation shows that in an isochoric process the entire amount of heat supplied to the gas is converted into internal energy of the gas. The pressure and temperature of the gas will increase.
- On the contrary, removal of heat from a system under isochoric condition will cause an equivalent decrease in the temperature of the system which will lead down to the pressure will fall.
- The graph of pressure versus volume for isochoric process which is a straight line parallel to the pressure axis is shown in Fig (10.4 b).



Fig 10.4

## (b) Isobaric Process

An isobaric process is a thermodynamic process in which the pressure remains constant.

- The expansion of a system is often used to convert heat into work.
- The expansion of a gas is often used to convert heat into work.
- A gas expands under constant pressure in a cylinder with a piston and non-conducting walls and the pressure is maintained constant by a weight on the piston.
- A gas expands under constant pressure in a cylinder with a piston and non-conducting walls and the pressure is maintained constant by a weight on the piston.
- When a gas expands under constant pressure, it does work. The gas expands and moves the piston.
- A gas expands under constant pressure from  $V_1$  to  $V_2$  ( $V_2 > V_1$ ).
- A gas expands under constant pressure from  $T_1$  to  $T_2$  ( $T_2 > T_1$ ).
- A gas expands under constant pressure from  $U_1$  to  $U_2$  ( $U_2 > U_1$ ).

$$\Delta W = P \Delta V$$

$$\Delta W = P \Delta V$$

(1)

Where  $P$  is the constant pressure and

$\Delta V$  is the change in volume of the gas during expansion.

From equation (1)

$$\Delta W = P \Delta V$$

(2)

$$P = \frac{F}{A} \quad \Delta V = A \Delta x \quad \text{Hence from (2)}$$

$$\Delta W = P A \Delta x = P \Delta V$$

- Where  $A$  is the area of the piston,  $\Delta x$  is the displacement and  $\Delta V$  is the increase in volume of the gas. Hence the work done by the gas which expands at constant pressure is

$$\Delta W = P \Delta V \quad (3)$$

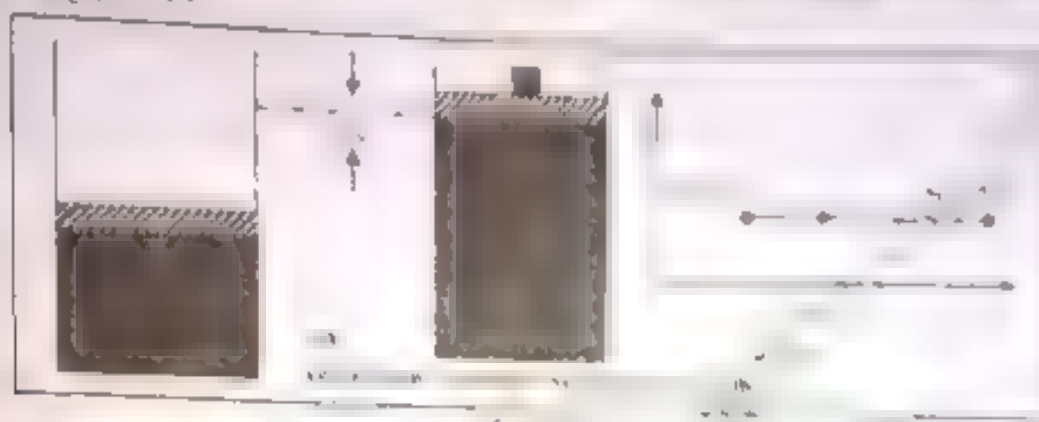
According to first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W \quad (4)$$

Putting value of  $\Delta W$  from equation (3) in above equation

$$\Delta Q = \Delta U + P \Delta V$$

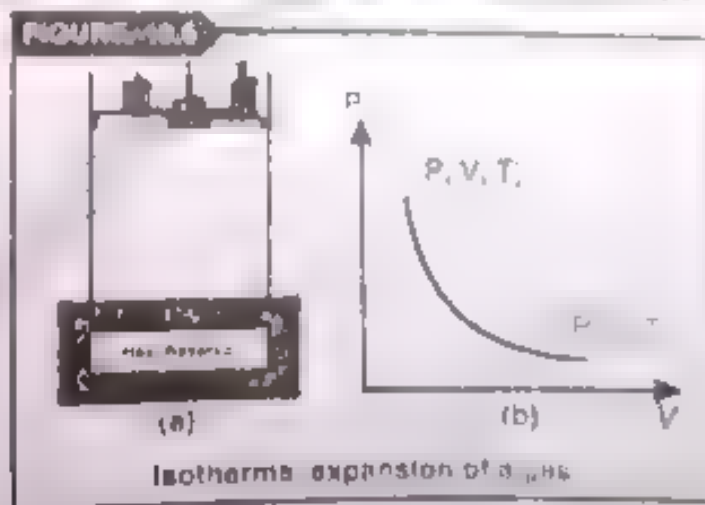
- The work performed by the expanding or contracting gas comes from one or both sources: heat supplied to the gas and the internal energy of the gas.
- The graph of isobaric process is called as *isobaric process* as shown in the Fig (10.4b)



#### (c) Isotherms Process

A process in which the temperature of the system remains constant is called an isothermal process.

- Let us consider the ideal gas contained in a cylinder with a movable piston at the end of a cylinder with a non-conducting wall and with a movable piston at the end of the cylinder.
  - The base of the cylinder is placed on a heat reservoir at temperature  $T$ .
- A reservoir is a body of large heat capacity such that the temperature of the gas at any time is equal to the temperature of the reservoir.



- Pressure of the system (on piston) is decreased from  $P_1$  to  $P_2$  ( $P_2 < P_1$ )
- Volume increases from  $V_1$  to  $V_2$  ( $V_2 > V_1$ )
- Due to this expansion the temperature of the gas tends to decrease. To keep the temperature constant, heat is supplied to the gas so that the temperature of the system remains constant equal to the temperature of the reservoir.
- The whole process from an initial state ( $P_1, V_1, T$ ) to a final state ( $P_2, V_2, T$ ) is called an isothermal process. The continuous curve which is called an isotherm as shown in the Fig (10.5b).

- ▶ During isothermal expansion some work ( $\Delta W$ ) is done by the gas in pushing up the piston in the cylinder.
- ▶ Since the temperature remains constant, there is no change in the internal energy of the gas that is,  $\Delta U = 0$ .

According to the first law of thermodynamics,  $\Delta Q = \Delta U + \Delta W$

$$\text{Putting } \Delta U = 0$$

$$\Delta Q = 0 + \Delta W$$

$$\Delta Q = \Delta W$$

- ▶ This shows that if the gas expands and does external work, an equivalent amount of heat must be supplied to the gas to keep its temperature constant.
- ▶ Conversely, if the piston does work is negative and heat is added to the gas, the gas expands.
- ▶ For an isothermal process, the equation  $P_1 V_1 = P_2 V_2$  holds.

$$P_1 V_1 = P_2 V_2$$

#### (d) Adiabatic Process

A process in which no heat enters or leaves the system is called an adiabatic process.  
(OR)  
The process in which heat energy of the system remains constant is called an adiabatic process.

Explanation

- ▶ In an adiabatic process, the cylinder and piston are insulated so that no heat can enter or leave the system.
- ▶ Since no heat is exchanged, the internal energy of the gas changes only due to work done.

- ▶ Pressure decreases

- ▶ Temperature decreases

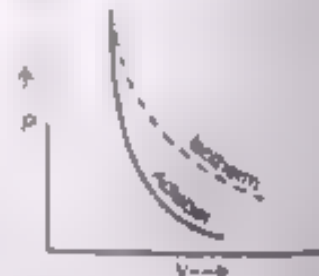
According to first law of thermodynamics

$$\Delta U = \Delta W$$

$$\Delta U = \Delta Q + \Delta W$$

$$\Delta U = 0 + \Delta W$$

$$\Delta U = \Delta W$$



**Adiabatic Expansion:** Above equation shows that work is done by the system at the expense of internal energy of the gas and hence the temperature of the gas falls.

- ▶ Therefore,  $\Delta Q = 0$  and  $\Delta U = \Delta W$  and  $\Delta U < 0$  and  $\Delta W < 0$ .

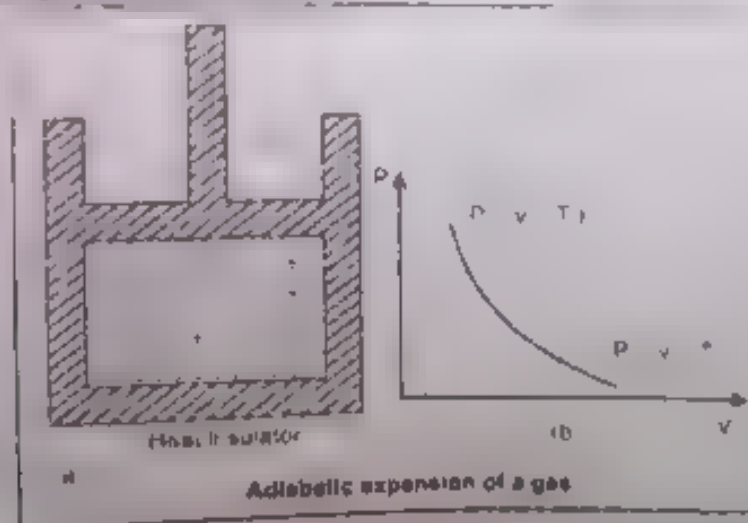


Fig 10.7

**Adiabatic compression.**

- ▶ Pressure increases
- ▶ Volume decreases
- ▶ Work is done on the gas & increases the temperature one if it is compressed.

$$W = \Delta U$$

**Condition for adiabatic change**

- ▶ Adiabatic change occurs when the gas expands or compresses rapidly so that it does not have time to exchange heat with the surroundings.
- For adiabatic process  $PV^\gamma = \text{constant}$
- Where  $\gamma = \frac{C_p}{C_v}$  is the ratio of specific heats.
- $n$  is constant  $\gamma = \frac{f+2}{f}$

**Adiabatic**

An adiabat is steeper than an isotherm

**Examples of Adiabatic Process**

1. The rapid escape of air from a burst tyre
2. The rapid expansion and compression of air through which a sound wave travels
3. Conduction in the atmosphere



**QUIZ**

- Q. The rate of evaporation of water from a pond is higher in the morning than in the evening. Why?
- A. The air in the morning is warmer than in the evening, and the air is a better conductor of heat than water.
- Q. The rate of evaporation of water from a pond is higher in the morning than in the evening. Why?
- A. The air in the morning is warmer than in the evening, and the air is a better conductor of heat than water.

Which remains constant in adiabatic process?

- (A) Volume (B) Pressure (C) Entropy (D) Temperature

Which one is false for an adiabatic process?

- (A)  $Q = 0$  (B)  $T = \text{constant}$  (C)  $Q = W$  (D) None of these

In case of adiabatic expansion process, the law of thermodynamics is written as

- (A)  $\Delta U = 0$  (B)  $\Delta U = W$  (C)  $\Delta U = -W$  (D)  $\Delta U = Q$

According to first law of thermodynamics the quantity which is conserved is

- (A) Energy (B) Mass (C) Force (D) None of these

A good example of first law of thermodynamics is

- (A) Simple pendulum (B) Adiabatic process (C) Isothermal process (D) Isochoric process

Boyle's law is applicable to

- (A) isobaric process (B) isochoric process (C) isothermal process (D) adiabatic process

Which is called the internal energy of an ideal gas?

- (A) Kinetic energy (B) Translational kinetic energy (C) Rotational kinetic energy (D) None of these

An isotherm is drawn as



Cloud formation in atmosphere is an example of

- (A) Isothermal process (B) Adiabatic process (C) Isobaric process (D) Isochoric process

The measure of the degree of hotness or coldness of a substance is called

- (A) Temperature (B) Heat (C) Thermal energy (D) None of these

The formula connecting the pressure and volume of a gas undergoing a process is

$$P \propto V^{-\gamma} \quad \text{A} \quad P \propto V^{-1} \quad \text{B} \quad P \propto V^{-2} \quad \text{C} \quad P \propto V^{-3} \quad \text{D} \quad P \propto V^{-4}$$

12. At which of the following temperatures a body has maximum thermal energy?

$$\text{A} \quad 0^\circ \text{C} \quad \text{B} \quad 273^\circ \text{C} \quad \text{C} \quad 100^\circ \text{C} \quad \text{D} \quad 373^\circ \text{C}$$

13. In a thermodynamic system, internal energy decreases by  $100 \text{ J}$  and  $100 \text{ J}$  of work is done on the system. Heat added to the system is

$$\text{A} \quad 0 \text{ J} \quad \text{B} \quad 100 \text{ J} \quad \text{C} \quad 200 \text{ J} \quad \text{D} \quad 300 \text{ J}$$

14. A thermodynamic system has

$$\text{A} \quad \text{internal energy} \quad \text{B} \quad \text{potential energy} \quad \text{C} \quad \text{kinetic energy} \quad \text{D} \quad \text{thermal energy}$$

15. An ideal gas has potential energy associated with it. It is

$$\text{A} \quad \text{zero} \quad \text{B} \quad \text{very small} \quad \text{C} \quad \text{very large} \quad \text{D} \quad \text{infinite}$$

16. For an ideal gas system, the internal energy is directly proportional to

$$\text{A} \quad \text{volume} \quad \text{B} \quad \text{pressure} \quad \text{C} \quad \text{temperature} \quad \text{D} \quad \text{density}$$

17. A gas is compressed in a cylinder. The work done on the gas is

$$\text{A} \quad \text{zero} \quad \text{B} \quad \text{positive} \quad \text{C} \quad \text{negative} \quad \text{D} \quad \text{infinite}$$

18. A gas is compressed in a cylinder. The work done on the gas is

|   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |     |
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| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|

An ideal gas is compressed in a cylinder. The work done on the gas is

The pressure of the gas is constant. The initial volume of the gas is

The final volume of the gas is

The initial temperature of the gas is

The final temperature of the gas is

19. The work done on the gas is

Work done by the gas is

Change in volume

$$= 1.01 \times 10^3 \text{ Pa}$$

$$= 7.50 \times 10^3$$

$$= 10 \times 10^3 - 2.00 \times 10^3$$

$$= 2.00 \times 10^3 = 3.00 \times 10^3$$

20. The

Work done

is

the

work

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that the work done on the gas is

**For Your Information**

However, thermodynamics also applies to living systems, such as our own bodies. This is the basis of the biological thermodynamics (Figure 10.10).

$$\Delta U = Q - W + \text{food energy}$$

$$\Delta U = \text{stored food energy}$$



(a)



(b)

Figure 10.10: Thermodynamics applies to living systems. (a) A human figure. The body receives energy  $Q$  from food and loses energy  $W$  as heat. The energy balance is  $\Delta U = Q - W + \text{food energy}$ . (b) A plant figure. The plant receives energy  $Q$  from sunlight and loses energy  $W$  as heat. The energy balance is  $\Delta U = Q - W + \text{food energy}$ . The food energy is stored in the plant as chemical energy, which is used for photosynthesis.

The energy balance for a human figure is  $\Delta U = Q - W + \text{food energy}$ . The energy balance for a plant figure is  $\Delta U = Q - W + \text{food energy}$ . The food energy is stored in the plant as chemical energy, which is used for photosynthesis. The energy balance for a human figure is  $\Delta U = Q - W + \text{food energy}$ . The energy balance for a plant figure is  $\Delta U = Q - W + \text{food energy}$ . The food energy is stored in the plant as chemical energy, which is used for photosynthesis.

Q 10 (a) Define the following terms:

- molar specific heat
- molar specific heat at constant volume ( $C_v$ )
- molar specific heat at constant pressure ( $C_p$ )

(b) Prove that  $C_p - C_v = R$

**ANSWER** Specific heat

The amount of heat energy required to raise the temperature of one kilogram of a substance by  $1^\circ\text{C}$  is called specific heat.

- One kilogram of different substances contains different number of molecules. Sometimes it is preferred to consider a quantity called mole. One mole of any substance contains same number of molecules.

**Molar Specific Heat of a Gas**

Molar specific heat of the substance is defined as the heat energy required to raise the temperature of one mole of a substance through  $1\text{K}$ .

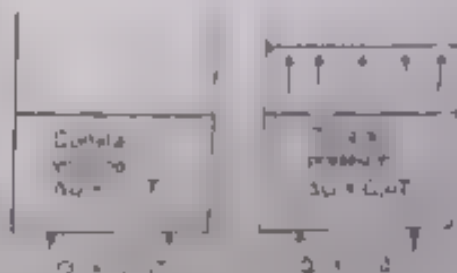
$$\Delta Q = nC\Delta T$$

$$\Delta Q = C_M n\Delta T$$

- Where  $C_M$  is molar specific heat and  $n$  is number of moles

$$C_M = \frac{\Delta Q}{n\Delta T}$$

- S-I unit of molar specific heat is  $\text{J mol}^{-1} \text{K}^{-1}$



## Note

- In case of solids and liquids the change of volume and hence work done against external pressure during a change of temperature is negligible.
- For gases, after variation in pressure as well as in volume with change in temperature. Hence to study the effect of heating the gases, either pressure or volume is kept constant.

## (1) Molar specific heat at constant volume

The amount of heat energy required to raise the temperature of one mole of the gas through  $1\text{K}$  at constant volume is called molar specific heat at constant volume.

$$\Delta Q_v = C_v n \Delta T$$

- Where  $C_v$  is molar specific heat at constant volume and its SI unit is  $\text{mol}^{-1} \text{K}^{-1}$ .

## (2) Molar specific heat at constant pressure

The amount of heat energy required to raise the temperature of one mole of the gas through  $1\text{K}$  at constant pressure is called molar specific heat at constant pressure.

$$\Delta Q_p = C_p n \Delta T$$

- Where  $C_p$  is molar specific heat at constant pressure and its SI unit is  $\text{mol}^{-1} \text{K}^{-1}$ .

(3) Derivation of  $C_p - C_v = R$ 

## At constant volume

If  $n$  moles of an ideal gas are heated at constant volume so that its temperature rises by  $\Delta T$  then the heat transferred  $\Delta Q_v$  is given by

$$\Delta Q_v = n C_v \Delta T$$

Applying first law of thermodynamics,

$$\Delta Q_v = \Delta U + \Delta W$$

Putting value of  $\Delta Q_v$  from equation (1)

$$n C_v \Delta T = \Delta U + \Delta W$$

Since volume remains constant i.e.  $\Delta V = 0$ , so work done by the system is zero. Thus the above equation becomes

$$n C_v \Delta T = \Delta U \quad \text{[ } \Delta W = P \Delta V = P(0) = 0 \text{ ]}$$

Hence

$$n C_v \Delta T = \Delta U$$

$$C_v = \frac{\Delta U}{n \Delta T} \quad \text{----- (2)}$$

## At constant pressure

- If  $n$  moles of an ideal gas are heated at constant pressure so that its temperature rises by  $\Delta T$  then the heat transferred  $\Delta Q_p$  is given by

$$\Delta Q_p = n C_p \Delta T \quad \text{----- (3)}$$

- The internal energy increases by the same amount as at constant volume for the same rise in temperature  $\Delta T$ .

Thus  $\Delta U = n C_v \Delta T$  ----- (4)

- Since the gas expands to keep the pressure constant, so the work done by the gas is

$$\Delta W = P \Delta V \quad \text{----- (5)}$$

$$P V = n R T$$

$$P \Delta V = n R \Delta T$$

Putting value of  $P \Delta V = n R \Delta T$  in equation (5)

FIGURE 10.9

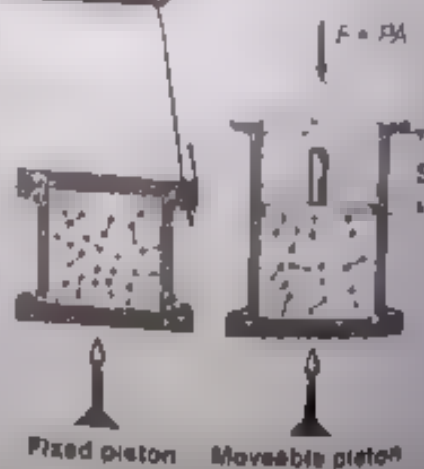


Specific heat capacity at constant volume

FIGURE 10.10



FIGURE 10.11



According to first law, thermal energy is

$$dQ = dU + dW$$

Put the values from conditions 2, 3, 4 and 6 in equation 1.

$$nC_p \Delta T = nC_v \Delta T + nR \Delta T$$

$$nC_p \Delta T = n \Delta T (C_v + R)$$

$$C_p = C_v + R$$

$$C_p - C_v = R$$

It is called Mayer's relation.

### Point to ponder

Q A football is inflated in a warm room. It is used out of door on a cold day. What happened to ball? Why?

Ans The volume of the ball will decrease.  
According to ideal gas law,  $PV = nRT$

Where  $V$  is the volume,  $n$  is the number of moles of gas,  $R$  is the universal gas constant and  $T$  is the temperature.  
The temperature falls on a cold day, so the pressure will decrease.

### MCQs

1. The difference between  $C_p$  and  $C_v$  is equal to

A.  $R$  B.  $2R$  C.  $3R$  D.  $4R$

2.  $C_v$  is the molar heat capacity at constant volume and  $C_p$  is the molar heat capacity at constant pressure.

A.  $C_p > C_v$  B.  $C_p < C_v$  C.  $C_p = C_v$  D.  $C_p$  and  $C_v$  are not comparable

3. The ratio of  $C_p$  to  $C_v$  for a diatomic gas is equal to

A.  $1.4$  B.  $1.67$  C.  $1.33$  D.  $1.25$

4. For a diatomic gas  $C_p = \frac{5R}{2}$  then  $\gamma$  for this gas is

A.  $\frac{5}{3}$  B.  $\frac{4}{3}$  C.  $\frac{3}{2}$  D.  $\frac{5}{2}$

5. What is the example of irreversible process?

A. Explosion B. Evaporation C. Slowly compressing a gas D. All of these

6. Which one is an example of reversible process?

A. Work done against friction B. Heat produced by current C. Melting of ice D. None of these

7. In reversible process the entropy of system

A. Remains constant B. Decreases C. Increases D. None of these

### Answers Key

|      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|
| 1. B | 2. B | 3. C | 4. C | 5. A | 6. C | 7. A |
|------|------|------|------|------|------|------|

Q.11 Write a note on Heat Engine?

Ans Heat Engine

Heat engine is a device which converts heat energy into mechanical work.

### Introduction

- The earliest heat engine was the steam engine. It was developed on the fact that when water is boiled in a vessel covered with a lid, the steam inside tries to push the lid off, showing the ability to do work. This observation helped to develop a steam engine.

### Do You Know?



The steam engine is a thermodynamic system.

**Construction**

- **Hot reservoir or Heat source**  
A system that is at a high temperature  $T_1$  and can supply heat  $Q_1$  to the working substance (HTR)
- **Cold reservoir or Heat sink**  
A system that is at a low temperature  $T_2$  and can absorb heat  $Q_2$  from the working substance (LTR)
- **Working substance**  
A substance that is used as working substance in heat engine. The working substance is taken through a cyclic process.

**Working**

- ▶ A heat engine is made cyclic to provide a continuous supply of work
- ▶ Working substance takes heat  $Q_1$  from heat source, converts some of it into work  $W$  by expansion and rejects the rest of the heat  $Q_2$  to the cold reservoir or sink.

After a complete cycle the change in internal energy is zero.

Heat supplied =  $Q_1$

Heat rejected =  $Q_2$

Work done =  $W$

According to first law of thermodynamics

$Q_1 = W + Q_2$

or  $W = Q_1 - Q_2$

OR  $W = Q_1 - Q_2$

OR  $W = Q_1 - Q_2$

**Efficiency of heat engine**

- ▶ The efficiency of a heat engine is the ratio of work done by it to the heat supplied (input). It is denoted by  $\eta$ .

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\text{OR } \eta = \frac{Q_1 - Q_2}{Q_1} = \frac{Q_1}{Q_1} - \frac{Q_2}{Q_1}$$

- ▶ If  $Q_2 = 0$ , i.e., heat were exhausted by engine so that all the heat  $Q_1$  absorbed were converted into work  $W$ , then  $\eta = 1$  or  $100\%$ .

- ▶ Practically it is not possible to construct 100% efficient engine because every heat engine reject some heat to the sink. Therefore  $Q_2$  can never be zero.
- ▶ It is impossible to construct a heat engine which converts all the heat absorbed from a hot reservoir into work without rejecting any heat to sink.

**FIGURE 10.12**



QUIZ

- Q What is function of spark plug in a petrol engine?  
 A The function of spark plug is to ignite the air/fuel mixture. It delivers electric current from an ignition system to the combustion chamber of a spark ignition engine to ignite the compressed air/fuel mixture by an electric spark. Electrical energy is transmitted to the spark plug, jumping the gap in the plug's firing end if the voltage supplied to the plug is high enough. This electrical spark ignites the gasoline/air mixture in the combustion chamber.
- Q There is no spark plug in diesel engine, then how does fuel burn in it?  
 A In diesel engine, no spark plugs are not needed. The air from the air sphere is sucked into the cylinder of the engine and then the air is compressed to high pressure where eventually leads to the increase of temperature when the diesel is supplied at the compression stroke, the temperature develops more enough to ignite the diesel. This high temperature makes the fuel to burn and then expansion of gases takes place. As a result, the power stroke is obtained.

Assignment 10.2

During one cycle, an engine extracts  $2.00 \times 10^3$  J of energy from a hot reservoir and transfers  $1.50 \times 10^3$  J to a cold reservoir. (a) Find the thermal efficiency of the engine. (b) How much work does this engine do in one cycle? (c) What average power does the engine generate if it goes through four cycles in 2.50 s?

- Given Data Heat supplied to the system =  $Q_1 = 2.00 \times 10^3$  J  
 Heat energy rejected from the system =  $Q_2 = 1.50 \times 10^3$  J  
 Time for four cycles = 2.50 s  
 Hence time for one cycle is  $t = 2.50 / 4 = 0.625$  s  
 Thermal efficiency of the engine =  $\eta = ?$   
 Work done by the system in each cycle =  $\Delta W = ?$   
 Average power delivered =  $P = ?$

- Solution Using the equation  

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{2.00 \times 10^3 - 1.50 \times 10^3}{2.00 \times 10^3} = \frac{0.50 \times 10^3}{2.00 \times 10^3} = 0.25$$
  
 Power delivered is  $P = \frac{W}{t} = \frac{0.50 \times 10^3}{0.625} = 8.00 \times 10^2$  watt

Q.12 State and explain Second Law of Thermodynamics

Ans: Second Law of Thermodynamics

Lord Kelvin's statement for working of heat engine  
 "It is impossible to construct a heat engine operating in cycle which absorbs heat from a hot reservoir and converts it completely into work without rejecting any heat to sink"  
 OR

- ▶ The second law of thermodynamics is a generalization of the law of conservation of energy.
  - ▶ The second law of thermodynamics tells us how heat energy can be converted into useful work.
  - ▶ A statement of second law of thermodynamics, two bodies at different temperatures, the conversion of heat energy into work.
  - ▶ Heat cannot be completely converted into useful work during a complete cycle.
  - ▶ Working substance absorbs heat  $Q_1$  from heat source, converts some heat energy into work  $W$  in expansion and rejects the rest of the heat  $Q_2$  to cold reservoir or sink.
  - ▶ Therefore, efficiency less than input and efficiency can never be 100%.
  - ▶ Petrol engine converts roughly 25% and Diesel engine 35% to 40% available heat energy into work.
- Rudolf Clausius Statement  
 It is impossible to cause heat to flow from a cold body to a hot body without expenditure of work.

- ▶ The process of the refrigerator is a good example of the second law.
- ▶ A refrigerator engine has to be expanded to transfer heat from inside the refrigerator to outside it, because heat cannot flow spontaneously from cold to hot.

**Q 13 What is Carnot's Engine? Explain its working and calculate its efficiency. Also state Carnot's theorem.**

### **Carnot's Engine**

A Carnot engine is a hypothetical engine that operates on the Carnot cycle. Sadi Carnot in 1824 proposed this cycle as a model for an ideal heat engine operating in an ideal gas. The Carnot cycle consists of four processes: two isothermal and two adiabatic processes.

FOR YOUR INFORMATION



#### **Principle**

- ▶ A Carnot engine works on the same principle as that of cyclic heat engine.
- ▶ It takes heat from hot body, convert a part of it into work and reject the remaining part to cold body.

#### **Working**

##### **1. Isothermal expansion**

- ▶ Pressure decreases from  $P_1$  to  $P_2$ .
- ▶ Volume increases from  $V_1$  to  $V_2$ .
- ▶ The gas is allowed to expand isothermally at temperature  $T$  (high) absorbing heat  $Q_1$  from the hot reservoir.
- ▶ The process is represented by curve AB.

##### **2. Adiabatic expansion**

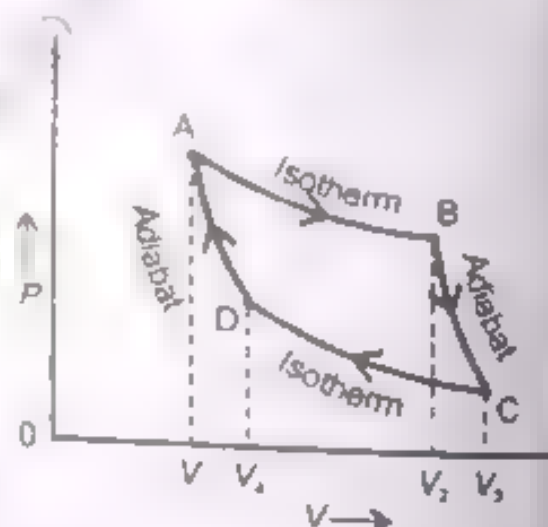
- ▶ Pressure decreases from  $P_2$  to  $P_3$ .
- ▶ Volume increases from  $V_2$  to  $V_3$ .
- ▶ The gas is then allowed to expand adiabatically until its temperature drops to  $T_2$  (low). The process is represented by curve BC.

##### **3. Isothermal compression**

- ▶ Pressure increases from  $P_3$  to  $P_4$ .
- ▶ Volume decreases from  $V_3$  to  $V_4$ .
- ▶ The gas is then compressed isothermally at constant temperature  $T_2$  rejecting heat  $Q_2$  to the cold reservoir. The process is represented by curve CD.

##### **4. Adiabatic compression**

- ▶ Pressure increases from  $P_4$  to  $P_1$ .
- ▶ Volume decreases from  $V_4$  to  $V_1$ .
- ▶ The gas is then compressed adiabatically to restore its initial state at temperature  $T_1$ . The process is represented by curve DA.
- ▶ The net work done in one cycle is equal to the area enclosed by the path ABCDA of the P-V diagram.



#### **Expression for Efficiency**

- ▶ As the engine returns to the initial state, there is no change in its internal energy.
- ▶ The net work done in one cycle equals to the area enclosed by the path ABCDA of the P-V diagram.
- ▶ The net work done in one cycle is equal to the net heat  $Q$  absorbed in the cycle.

$$W = Q_1 - Q_2$$

From 1<sup>st</sup> law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W \quad (1)$$

Putting value of  $\Delta Q$  and  $\Delta U$  in equation (1), we get

$$Q_1 - Q_2 = 0 + \Delta W$$

OR  $\Delta W = Q_1 - Q_2$

If the efficiency of heat engine is defined as

$$\eta = \frac{\text{output work}}{\text{input energy}}$$

$$\eta = \frac{\Delta W}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} \quad (2)$$

The energy transfer in isothermal expansion or compression turns out to be proportional to Kelvin temperature

i.e.  $Q_1$  and  $Q_2$  are proportional to Kelvin temperature  $T_1$  and  $T_2$  respectively.

Hence  $\frac{T_1}{T_2} = \frac{Q_1}{Q_2}$

Thus eq. (2) becomes

"

hence efficiency is always taken in percentage

Percentage efficiency  $\eta = \frac{T_1 - T_2}{T_1} \times 100$

#### For Your Information

The first successful petrol engine was invented by Nikolaus Otto in the year 1866 and the Diesel engine was invented by Rudolph Diesel in 1892.

#### Dependence of Efficiency

- Efficiency of a Carnot Engine depends on the temperature of hot & cold reservoir
- It is independent of the nature of the working substance
- The larger the temperature difference of the two reservoirs, the greater is the efficiency
- In most practical cases the cold reservoir is near room temperature. So the efficiency can be increased by raising the temperature of hot reservoir

#### Can efficiency of heat engine be 100%?

- It can never be one or 100% unless cold reservoir is at absolute zero temperature. Such reservoirs are not available & hence maximum efficiency is always less than one

#### Carnot's Theorem

##### Statement

- No real heat engine can be more efficient than a Carnot engine operating between the same two temperatures

##### Extended statement

- All Carnot's engines operating between the same two temperatures have the same efficiency irrespective of the nature of working substance

##### Note

- All real heat engines are less efficient than Carnot engine due to friction & heat losses

**Q 14** Explain the working of refrigerator and discuss coefficient of performance of refrigerator

### 10.1 Refrigerator

- ▶ The device is used to transfer body temperature below that of the surrounding environment.
- ▶ Hence, heat must be made to flow from a body at low temperature to the surrounding at high temperature.
- ▶ The device in which no work is done, performs cycle is called as perfect refrigerator.
- ▶ No work is done as it is not a thermodynamic process.

### Working

- ▶ A refrigerator works in such a way that some amount of heat ( $Q_2$ ) is removed from a cold reservoir at temperature ( $T_2$ ).
- ▶ A work  $W$  is done by the compressor. The refrigerator then rejects the heat  $Q_1$  to the hot reservoir.
- ▶ The heat  $Q_1$  is rejected to the hot reservoir at temperature ( $T_1$ ) and the heat  $Q_2$  is removed from the cold reservoir at temperature ( $T_2$ ).

$$Q_1 = W + Q_2$$

$$W = Q_1 - Q_2$$

- ▶ The coefficient of performance (COP) for a refrigerator is defined as the ratio of the amount of heat removed from the cold reservoir ( $Q_2$ ) to the work done ( $W$ ) on the refrigerator.

### Coefficient of performance or energy ratio of refrigerator

- ▶ The coefficient of performance is defined as the ratio of the amount of heat removed from the cold reservoir to the work done on the refrigerator.
- ▶ The coefficient of performance for cooling or cooling energy ratio is

$$COP = \frac{Q_2}{W}$$

$$COP = \frac{Q_2}{Q_1 - Q_2}$$

$$COP = \frac{Q_2}{Q_1 - Q_2}$$

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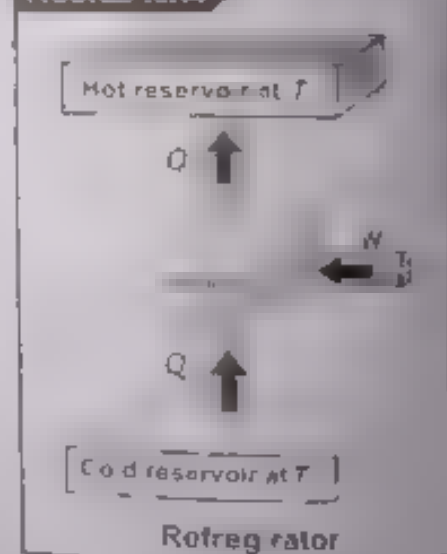
$$COP = \frac{Q_2}{Q_1 - Q_2}$$

$$COP = \frac{Q_2}{Q_1 - Q_2}$$

$$COP = \frac{Q_2}{Q_1 - Q_2}$$

$$COP = \frac{Q_2}{Q_1 - Q_2}$$

FIGURE 10.14



- ▶ Coefficient of performance for heating or heating energy ratio is given by

$$COP_{heating} = \frac{Q_1}{W}$$

$$COP_{heating} = \frac{Q_1}{Q_1 - Q_2}$$

$$COP_{heating} = \frac{Q_1}{Q_1 - Q_2}$$

$$COP_{heating} = \frac{Q_1}{Q_1 - Q_2}$$

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$$COP_{heating} = \frac{Q_1}{Q_1 - Q_2}$$

- ▶ No cycle device has ever been built that will extract heat ( $Q_2$ ) from a cold reservoir and reject it entirely to a hot reservoir without the expenditure of energy.

- ▶ This statement is referred to as the Clausius statement of the second law of thermodynamics.

## Point to Ponder

1. Why is the freezer in the upper part of refrigerator?
- Ans. It follows phenomenon of convection in which warm air (less denser, rises up and cool air (denser air) moves downward. Freezer is the source for the refrigerant's coolness. When so cold air being dense moves down to other part of fridge and the warm air rises up to get cooled in the freezer.
2. Why do we not keep bananas in the refrigerator?
- Ans. Bananas grow in hot climates so they are unused to the cold. If they are kept at a cold temperature, the enzymes that enable them to grow are inhibited. And as those enzymes become inactive, the bananas become more efficiently. Some cause cell damage while others browning enzymes cause the fruit to ripen.
3. Why does the refrigerator switch itself OFF intermittently with some noise?
- Ans. ► The refrigerator is trying to maintain temperature. It does that by following the setting of its thermostat. ON when called or OFF when satisfied.  
► The noise is due to compressing ON and turning OFF the compressor. Also noise is typically caused when the refrigerator enters the defrosting cycle and the compressor isn't off. The compressor is under load at that point and will come to an abrupt stop, this causes the motor to vibrate momentarily as it powers down.

## Q 15 Define and explain the term entropy

## Ans: Entropy

Entropy is a thermodynamic property that measures the randomness, of a system.

- Entropy is state variable of thermodynamically system.
  - It was introduced by Rudolf Clausius in 1856.
  - Entropy is a thermodynamic property that measures the randomness, of a system.
- (Change in entropy is denoted by  $\Delta S$ )
- If  $\Delta Q$  is the heat added to the system, at temperature  $T$ , then change in entropy (state variable) of the system is,

$$\Delta S = \frac{\Delta Q}{T}$$

- Just as we can have a negative and positive energy, we can have a negative entropy, which is more important than its absolute value.

## Sign Convention

- The change in entropy is positive (means that entropy increases) when heat is added to a system.
- Change in entropy is negative means that entropy decreases when heat is taken out of a system.

Unit: The SI unit for change of entropy or entropy is  $\text{Joule/K}$  or  $\text{J/K}$ .

- Let  $T_1$  and  $T_2$  are the temperature of hot reservoir and cold reservoir respectively.

- $Q$  amount amount of heat flows through a conduction rod from  $T_1$  to  $T_2$  and  $T_1 > T_2$  then

Decrease of entropy of H.R  $= \frac{Q}{T_1}$

Increase of entropy of C.R  $= \frac{Q}{T_2}$

## Do You Know?

Approximate efficiencies of various devices

| Device                              | Efficiency % |
|-------------------------------------|--------------|
| Electric generator                  | 70-90        |
| Electric motor                      | 50-91        |
| Dry cell battery                    | 90           |
| Domestic gas furnace                | 70-85        |
| Storage battery                     | 72           |
| Hydrogen-oxygen fuel cell           | 60           |
| Liquid fuel rocket                  | 47           |
| Steam turbine                       | 35-46        |
| Fossil-fuel power plant             | 30-40        |
| Nuclear power plant                 | 30-35        |
| Nuclear reactor                     | 30           |
| Automobile gas turbine engine       | 30           |
| Solid state laser                   | 30           |
| Internal combustion gasoline engine | 25-30        |
| Gasoline automobile engine          | 20           |
| Fluorescent lamp                    | 20           |
| Silicon solar cell                  | 12-16        |
| Steam locomotive                    | 8            |
| Incandescent lamp                   | 5            |
| Waste steam engine                  | 1            |

Net change in entropy  $= \frac{Q}{T} - \frac{Q}{T}$  positive

- ▶ As  $T_1 > T_2$  so the sign of net change of entropy is +ve or we can say that net entropy of the system is increased.
- ▶ This proves that there is **net increase of entropy** due to a natural process (i.e. flow of heat from higher to lower temperature). This is also called another statement of second law of thermodynamics.

### Second Law of Thermodynamics in terms of Entropy

*"If a system undergoes a natural process, it will go in the direction that causes the entropy of system plus the environment to increase"*

- ▶ It is observed that a natural process tends to proceed towards a state of greater disorder.
- ▶ For example, an irreversible heat flows from a hot body to a cold body, it increases the disorder. So we can say that the entropy is increased.
- ▶ Addition of heat increases the disorder, hence the entropy is also increased.
- ▶ The free expansion of a gas increases its disorder because molecules have greater randomness if they are expanded than before.
- ▶ The process in which entropy remains constant is a reversible process.
- ▶ For a irreversible process entropy of system increases.

### Entropy as Unavailability of Mechanical Work (i.e., Degradation of energy)

- ▶ Let us consider two water tank of different temperature. The molecules in higher temperature water have more kinetic energy. This energy can be used as source and sink to do the work. When the temperature between two tanks becomes equal, no work can be obtained.
- ▶ If the temperature difference between two tanks is small, then the work obtained is small. As the temperature difference increases, the work obtained increases. So, we can say that **increase in entropy means the degradation of energy**.
- ▶ The degradation of energy is called entropy. The degradation of energy is continuous, degrading for doing useful work.

### Heat death of Universe

- ▶ When the entropy of the universe will reach at maximum value, every thing will be at same temperature. At this time, chemical and physical processes will have ceased and there will be no way to convert the energy into useful work. At this stage, all efforts are failed and death of universe.

### MCQs

- The efficiency of Carnot engine depends upon
  - A. Sink temperature only
  - B. Source temperature only
  - C. Both source and sink temperature
  - D. The working substance
- Which of the following is the efficiency of petrol engine?
  - A. 25-30%
  - B. 45-50%
  - C. 34.5%
  - D. 48-60%
- The efficiency of Carnot engine depends upon is
  - A. Nature of the working substance
  - B. Size of engine
  - C. Construction of engine
  - D. Temperature of hot and cold reservoirs
- If the temp of the sink is decreases, the efficiency of Carnot engine
  - A. Decrease
  - B. Increase
  - C. Remains the same
  - D. First increases then decreases
- Absolute zero corresponds to
  - A.  $-273^\circ\text{C}$
  - B.  $360^\circ\text{F}$
  - C.  $0^\circ\text{F}$
  - D.  $480^\circ\text{F}$
- A heat engine operates between the temperature  $1000\text{ K}$  and  $400\text{ K}$ . Its efficiency is
  - A. 100%
  - B. 70%
  - C. 60%
  - D. 50%
- Which of the following is the expression for the change in entropy of a system?
  - A.  $\Delta S = \frac{Q}{T}$
  - B.  $\Delta S = \frac{1}{T}$
  - C.  $\Delta S = \frac{Q}{T}$
  - D.  $\Delta S = \frac{1}{T}$

1. Which of the following is the latent heat of fusion of ice?  
 (A)  $3.36 \times 10^5 \text{ J/kg}$  (B)  $3.36 \times 10^6 \text{ J/kg}$  (C)  $3.36 \times 10^7 \text{ J/kg}$  (D)  $3.36 \times 10^8 \text{ J/kg}$
2. The Carnot cycle can be shown by which graph?  
 (A) P-T graph (B) V-T graph (C) P-V graph (D) P-V-T graph
3. No entropy change takes place in an  
 (A) isothermal process (B) isobaric process (C) Adiabatic process (D) isochoric process
4. Which of the following is the SI unit of entropy?  
 (A) J/K (B) J/K (C) JK (D) J/K
5. Which of the following are the Dimensions of entropy?  
 (A)  $\text{ML}^2\text{T}^{-2}\text{K}^{-1}$  (B)  $\text{ML}^2\text{T}^{-2}\text{K}^{-1}$  (C)  $(\text{ML}^2\text{T}^{-2}\text{K}^{-1})$  (D)  $(\text{ML}^2\text{T}^{-2})$
6. The entropy of the universe with passage of time is  
 (A) increasing (B) Decreasing (C) Remains constant (D) increases & decreases
7. Which of the following is the Triple point of water?  
 (A) 273 K (B) 273.15 °C (C) 273.15 K (D) 373.15 K
8. Net change in entropy of a system after one complete Carnot cycle is:  
 (A) Positive (B) Negative (C) Sometimes positive and sometimes negative (D) Sometimes positive and sometimes negative
9. Which of the following is the temperature scale is independent of nature of substance?  
 (A) Thermodynamic scale (B) Centigrade scale (C) Fahrenheit scale (D) Regnault scale
10. What would be the approximate efficiency of a Carnot engine operating with boiling water as one reservoir and a freezing mixture of ice and water as the other reservoir?  
 (A) 20% (B) 20.8% (C) 20.4% (D) 12%
11. Change in entropy of reversible process is  
 (A) Positive (B) Negative (C) 0 (D) Maximum
12. The increase in entropy means the increase in  
 (A) Disorder (B) Unavailability of energy (C) Randomness (D) All of these
13. In reversible process, entropy of a system  
 (A) Remains constant (B) Decreases gradually (C) Increases (D) Increases and decreases

Answers Key

|      |      |      |      |      |      |      |      |     |      |      |      |
|------|------|------|------|------|------|------|------|-----|------|------|------|
| 1 C  | 2 A  | 3 D  | 4 B  | 5 A  | 6 C  | 7 C  | 8 A  | 9 C | 10 C | 11 A | 12 C |
| 13 A | 14 C | 15 C | 16 A | 17 B | 18 C | 19 D | 20 A |     |      |      |      |

FOR YOUR INFORMATION

Diesel fuel evaporates more slowly because it is heavier. It contains more carbon atoms in longer chains than gasoline does. Gasoline is typically  $\text{C}_9\text{H}_{20}$  while diesel fuel is typically  $\text{C}_{14}\text{H}_{30}$ . It takes less energy to create diesel fuel, which is why it used to be cheaper than gasoline. Diesel fuel has a higher energy density than gasoline. On average, 1 gallon (3.8 L) of diesel fuel contains approximately  $1.4 \times 10^6$  joules (147,000 BTU), while 1 gallon of gasoline contains  $1.32 \times 10^6$  joules (125,000 BTU). This combined with the improved efficiency of diesel engines explains why diesel engines get better mileage than equivalent gasoline engines.

Assignment 10.3:

Find the change in entropy of  $3.00 \times 10^2 \text{ g}$  of lead when it melts at  $327^\circ \text{C}$ . Lead has a latent heat of fusion of  $2.45 \times 10^4 \text{ J/kg}$ . (b) Suppose the same amount of energy is used to melt part of a piece of silver, which is already at its melting point of  $961^\circ \text{C}$ . Find the change in the entropy of the silver.

Given Data: Mass of lead  $m = 3.00 \times 10^2 \text{ g} = 0.3 \text{ kg}$   
 Constant temperature at melting point for silver  $T = 327^\circ \text{C} = (327 + 273) \text{ K} = 600 \text{ K}$   
 Constant temperature at melting point for silver  $T = 961^\circ \text{C} = (961 + 273) \text{ K} = 1234 \text{ K}$   
 Latent heat of fusion of Lead  $2.45 \times 10^4 \text{ J/kg}$   
 Change in entropy  $\Delta S_{\text{lead}}$   
 Change in entropy  $\Delta S_{\text{silver}}$

**Solution** Heat absorbed by the lead:  $\Delta Q = mH_f = 0.3 \text{ kg} \times 2.45 \times 10^5 \text{ J kg}^{-1}$   
 Change in entropy of lead:  $\Delta S_{\text{lead}} = \frac{\Delta Q}{T} = \frac{73500 \text{ J}}{273 \text{ K}} = 270 \text{ JK}^{-1}$   
 Change in entropy of silver for same amount of heat is  
 $\Delta S_{\text{silver}} = \frac{\Delta Q}{T} = \frac{73500 \text{ J}}{323 \text{ K}} = 228 \text{ JK}^{-1}$

## FORMULAE

|                                  |                                            |                                            |                              |
|----------------------------------|--------------------------------------------|--------------------------------------------|------------------------------|
| Work done                        | $W = P \Delta V$                           |                                            |                              |
| First law of thermodynamics      | $\Delta Q = \Delta U + W$                  | $\Delta Q = (U_2 - U_1) + W$               | $\Delta U = \Delta Q - W$    |
| Isothermal process               | $\Delta Q = W$                             |                                            |                              |
| Adiabatic process                | $W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$ | $W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$ |                              |
| Relation between $C_p$ and $C_v$ | $C_p - C_v = R$                            |                                            |                              |
| Efficiency of heat engine        | $\eta = \frac{W}{Q_1}$                     | $\eta = \frac{W}{Q_1}$                     | $\eta = 1 - \frac{Q_2}{Q_1}$ |
| Efficiency of Carnot's engine    | $\eta = 1 - \frac{T_2}{T_1}$               | $\eta = 1 - \frac{T_2}{T_1}$               |                              |
| Entropy                          | $\Delta S$                                 |                                            |                              |



## Key Points

- ❖ **Thermodynamics.** The branch of physics which deals with the laws of transformation of heat into other forms of energy and vice versa is called thermodynamics.
- ❖ **Internal energy:** The sum of the kinetic and potential energies associated with the random motion of the atoms of the substance is called the internal energy of the substance.
- ❖ **First law of thermodynamics.** This law states that if an amount of heat energy  $\Delta Q$  is supplied to a system a part of it may increase its internal energy by an amount  $\Delta U$  while the remaining part may be used up as the external work  $\Delta W$  by the system.  $\Delta Q = \Delta U + \Delta W$
- ❖ **Molar specific Heat.** The quantity of heat required to raise the temperature of one mole of the substance (gas) by  $1^\circ\text{C}$  or  $1\text{ K}$  is called molar specific heat or molar specific heat capacity of the substance.
- ❖ **Reversible Process:** A process is said to be reversible if it can be retraced exactly in reverse order without producing any change in the surroundings.
- ❖ **Irreversible Process:** A process which cannot be retraced in the backward direction by reversing the controlling factors is said to be irreversible.
- ❖ **Heat engine:** A heat engine is a device for converting heat energy into mechanical work.
- ❖ **Reversible heat engine.** The engine in which the process can be retraced at any stage of its operation by reversing the boundary conditions is called a reversible heat engine.
- ❖ **Heat Source.** A body of infinite heat capacity which is capable of absorbing or rejecting an unlimited quantity of heat without any change in its temperature is called heat source or heat reservoir.
- ❖ **Principle of increase of entropy.** Any process taking place within a thermally isolated system the entropy of the system either increases or remains constant.
- ❖ **Degradation of energy.** The conversion of heat energy from high ordered state to a less ordered state is called degradation of energy.



## Solved Examples

## Example 18.3:

A heat engine operates with 65.0 kcal of heat supplied and exhausts 40.0 kcal of heat. How much work did the engine do?

Given:

Heat input  $Q_H = 65.0 \text{ kcal}$ Heat rejected  $Q_L = 40.0 \text{ kcal}$ Mechanical equivalent of heat  $1 \text{ kcal} = 4,184 \text{ J}$ 

Required:

Work done =  $W$ 

Solution:

The relationship between these quantities is found in equation

$$W = J(Q_H - Q_L)$$

Putting values

$$= 4,184 \text{ (J/kcal)} (65.0 \text{ kcal} - 40.0 \text{ kcal})$$

$$= 4,184 \text{ (J/kcal)} (25.0 \text{ kcal})$$

$$= 4,184 \times 25.0 \text{ (J/kcal kcal)}$$

$$= 104,600 \text{ J} = 105 \text{ kJ}$$

## Example 18.4:

In a certain process, 400J of heat energy is supplied to a system and at the same time 150J of work is done by the system. What is the increase in internal energy of the system?

Given:

Heat energy supplied to the system  $Q_H = 400 \text{ J}$ Work done by the system,  $\Delta W = 150 \text{ J}$ 

Required:

Increase in internal energy of the system  $\Delta U = ?$ 

Solution:

Using the first law of thermodynamics

$$\Delta U = Q_H - \Delta W$$

$$= 400 \text{ J} - 150 \text{ J}$$

$$= 250 \text{ J}$$

$$\Delta U = 250 \text{ J}$$

## Example 18.5:

What is the change in internal energy of 200g of nitrogen as it is heated from  $10^\circ\text{C}$  to  $30^\circ\text{C}$  at constant volume? [For nitrogen gas  $C_v = 20.815 \text{ J mole}^{-1} \text{ K}^{-1}$ ]

Given:

Mass of nitrogen gas  $m = 200 \text{ g}$ Initial temperature of nitrogen gas  $T_1 = 10^\circ\text{C}$ Final temperature of nitrogen gas  $T_2 = 30^\circ\text{C}$ Change in temperature  $\Delta T = T_2 - T_1 = 30^\circ\text{C} - 10^\circ\text{C}$   
 $= 20^\circ\text{C} = 20 \text{ K}$ 

Required:

Change in internal energy  $\Delta U = ?$ 

Solution:

Molecular mass of nitrogen gas  $M = 28 \text{ g per mole}$ 

$$\text{Number of moles, } n = \frac{m}{M} = \frac{200 \text{ g}}{28 \text{ g per mole}} = 7.143 \text{ mole}$$

The heat added is converted entirely into the internal energy of the nitrogen gas

$$\Delta U = n C_v \Delta T$$

$$\Delta U = 7.143 \text{ mole} \times 20.815 \text{ J mole}^{-1} \text{ K}^{-1} \times 20 \text{ K}$$

$$\Delta U = 2973.6 \text{ J}$$

## Example 18.6:

A reversible engine works between two temperatures whose difference is  $100^\circ\text{C}$ . If it absorbs 745 J of heat from the source and rejects 545 J to the sink, calculate the temperature of the source and the sink.

Solution:

Temperature of the source =  $T_1 = ?$ Temperature of the sink =  $T_2 = ?$

Difference between temperatures,

$$T_1 - T_2 = 100^\circ\text{C} = 100\text{K}$$

$$\text{Heat absorbed, } Q_1 = 746\text{ J}$$

$$\text{Heat rejected, } Q_2 = 546\text{ J}$$

The efficiency of a reversible heat engine is given by the formula

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

Substitute the given values in the above equation, we get

$$\eta = \frac{746\text{ J} - 546\text{ J}}{746\text{ J}} = \frac{100\text{ K}}{T_1}$$

$$\Rightarrow \frac{200\text{ J}}{746\text{ J}} = \frac{100\text{ K}}{T_1}$$

$$\Rightarrow T_1 = \frac{746\text{ K}}{2} = 373\text{ K} = (373 - 273)^\circ\text{C}$$

$$T_1 = 100^\circ\text{C}$$

$$\text{Since } T_1 - T_2 = 100^\circ\text{C}$$

$$\Rightarrow T_2 = T_1 - 100^\circ\text{C} = 100^\circ\text{C} - 100^\circ\text{C} = 0^\circ\text{C}$$

$$T_2 = 0^\circ\text{C}$$

### Example 10.6

A Carnot heat engine has a maximum efficiency of 55%. It takes certain amount of heat from a source, converts a part of it into work and rejects the remaining heat towards the heat sink. If the temperature of the heat source is 600 K then find the temperature of the heat sink.

**Solution**

Temperature of the heat source,  $T_1 = 600\text{ K}$

Maximum efficiency of Carnot heat engine

$$\eta = 55\% = 0.55$$

Temperature of the heat sink,  $T_2 = ?$

Using the equation

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{600\text{ K}} = 1 - 0.55$$

$$T_2 = 600\text{ K} \times 0.45 = 270\text{ K}$$

### Example 10.7

A refrigerator has a coefficient of performance 8. If the temperature in the freezer is  $-23^\circ\text{C}$ . What is the temperature at which it rejects heat?

**Solution**

The coefficient of performance,  $\epsilon = 8$

Temperature of the heat sink,  $T_2 = -23^\circ\text{C} = -23 + 273 = 250\text{ K}$

Temperature of the heat source,  $T_1 = ?$

Using the equation

$$\epsilon = \frac{T_2}{T_1 - T_2}$$

$$8 = \frac{250\text{ K}}{T_1 - 250\text{ K}}$$

$$T_1 - 250 = \frac{250}{8}$$

$$T = 250 = 3 \times 25$$

$$T = 281.25 \text{ K} = 281.25 - 273 = 8.2^\circ\text{C}$$

**Example 10.7:**

What is the change in entropy of 30 g of water at  $0^\circ\text{C}$  as it is changed into ice at  $0^\circ\text{C}$ ? Take the latent heat of fusion of ice =  $336000 \text{ J kg}^{-1}$

**Solution:**

Mass of water  $m = 30 \text{ g} = 0.03 \text{ kg}$

Constant temperature at fusion point  $T = 0^\circ\text{C} = 273 \text{ K}$

Latent heat of fusion of ice  $L_f = 336000 \text{ J kg}^{-1}$

Heat removed from water  $= \Delta Q = mL_f = 0.03 \times 336000 \text{ J} = 10080 \text{ J}$

Change in entropy:  $\Delta S = \frac{\Delta Q}{T} = \frac{10080}{273 \text{ K}} = 36.92 \text{ J/K}$

## Text Book Exercises

**Q 1** Select the correct answer of the following questions

- 1) Assume we can change the equilibrium state of a system via two different process. Assume that the initial and the final state are the same. Which of the quantities  $\Delta U$ ,  $\Delta Q$ ,  $\Delta W$  and  $\Delta T$  must be the same for the two process?
  - (a) Only  $\Delta Q$  and  $\Delta W$
  - (b) Only  $\Delta U$  and  $\Delta T$
  - (c) Only  $\Delta Q$  and  $\Delta T$
  - (d) Only  $\Delta U$  and  $\Delta W$
- 2) In any process the maximum amount of mechanical energy that can be converted to heat
  - (a) can be converted
  - (b) Depends upon the amount of heat
  - (c) Depends upon the intake and exhaust temperature
  - (d) Depends upon whether kinetic or potential energy is involved
- 3) In an isothermal change, internal energy
  - (a) Decreases
  - (b) Increases
  - (c) Remains the same
  - (d) Depends upon the process
- 4) A thermos bottle containing hot coffee is vigorously shaken. Consider coffee as the system, then its temperature:
  - (a) Increases
  - (b) Decreases below  $0^\circ\text{C}$
  - (c) Remains the same
  - (d) Decreases
- 5) Maximum work can be obtained in the process called
  - (a) Cyclic
  - (b) Isothermal
  - (c) Adiabatic
  - (d) Isobaric
- 6) A heat engine takes in 800 J of heat at 1000 K and exhausts 600 J of heat at 400 K. What is the actual efficiency of this engine?
  - (a) 25%
  - (b) 40%
  - (c) 50%
  - (d) 75%
- 7) If the temperature of the heat source is increased, the efficiency of a Carnot's engine
  - (a) increases
  - (b) Decreases
  - (c) Remains constant
  - (d) First increases and then becomes constant
- 8) Triple point of water is
  - (a)  $273.16^\circ\text{C}$
  - (b)  $273.16 \text{ K}$
  - (c)  $273.16^\circ\text{F}$
  - (d)  $273.16 \text{ K}$
- 9) A real gas can be approximated to an ideal gas at
  - (a) Low density
  - (b) High pressure
  - (c) High density
  - (d) Low temperature
- 10) If the volume of the gas is to be increased by 4 times, then:
  - (a) Temperature and pressure must be double
  - (b) At constant P the temperature must be increased by four times
  - (c) At constant T the pressure must be increased by four times
  - (d) It cannot be increased

- (11) In which of the system listed below is the entropy decreasing?  
 a. A gas is cooled  
 b. A gas is heated  
 c. A gas is compressed  
 d. A gas is expanded
- (12) If the temperature of source and sink of a Carnot engine have efficiency  $\eta$  are each decreased by 100K then the efficiency  $\eta$

| No. | Options | Answer                              | Explanation                                                                                                                                                    |
|-----|---------|-------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1   |         | Depends upon the amount of friction |                                                                                                                                                                |
| 2   |         | Remains same                        |                                                                                                                                                                |
| 3   |         |                                     |                                                                                                                                                                |
| 4   |         |                                     |                                                                                                                                                                |
| 5   |         |                                     |                                                                                                                                                                |
| 6   |         |                                     |                                                                                                                                                                |
| 7   |         |                                     |                                                                                                                                                                |
| 8   |         |                                     |                                                                                                                                                                |
| 9   |         |                                     |                                                                                                                                                                |
| 10  |         |                                     |                                                                                                                                                                |
| 11  |         |                                     |                                                                                                                                                                |
| 12  |         |                                     | Let $T = 400K$<br>Since<br>Now<br>When temperatures are decreased by 100 K, then,<br>$\eta = 1 - \frac{100}{400}$<br>$\eta = 1 - 0.25 = 0.75$<br>$\eta = 0.75$ |

### Short Answers of the Exercise

Q 2 Write short answers of the following questions

Q 1 Why is the earth not in thermal equilibrium with the sun?

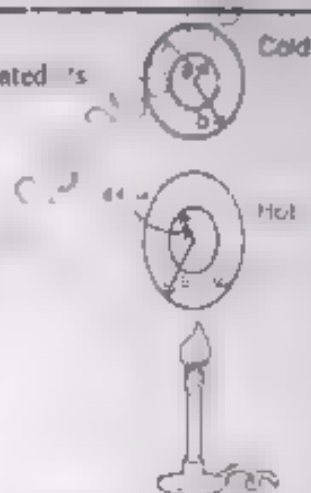
Ans The two bodies are said to be in thermal equilibrium if they are at the same temperature. The distance between Earth to the Sun is about  $1.496 \times 10^8$  km.

- ▶ The earth is not in thermal equilibrium with sun because while earth is being warmed by absorbing radiant energy, it is also losing heat in various ways e.g. re-emission of radiation,
- ▶ The Earth and the Sun are not in contact with each other. Usually things reach thermal equilibrium if the particles can directly interact with each other.
- ▶ But the only way for energy to move from the Sun to the Earth is via E.M radiation, because the space between the Earth and the Sun is mostly a vacuum.
- ▶ A so very small percentage of sun energy can reach earth.

**Q.2** When a block with a hole in it is heated, why does not the material around the hole expand into the hole and make it small?

**Ans:**

- ▶ If you heat the ring it expands, making the circumference bigger. When a block is heated its temperature increases.
- ▶ The block will expand on heating and its all dimensions increase.
- ▶ Therefore diameter (size) of hole also increases.



**Q.3** A thermometer is placed in direct sunlight. Will it read the temperature of the air, or of the sun or of something else?

**Ans:**

- ▶ When thermometer is placed in direct sunlight it will read the temperature of its surroundings or temperature of air surrounding the glass bulb.
- ▶ When the temperature of the thermometric substance becomes equal to the temperature of surrounding then thermal equilibrium is reached.
- ▶ A normal thermometer cannot be used to measure temperature of sun.
- ▶ Temperature of sun can be estimated by studying its spectrum.

**Q.4** The pressure in a gas cylinder containing hydrogen will leak more quickly than if it is containing oxygen. Why?

**Ans:** According to Graham's law of diffusion, the rate of diffusion of a certain gas is inversely proportional to square

root of its molecular mass.  $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$

- ▶ The smaller the molecular mass of a gas the greater will be the rate of diffusion and vice versa.
- ▶ As the hydrogen is lighter than oxygen i.e. its molecular mass and density is less than that of oxygen therefore rate of diffusion of hydrogen gas is greater than oxygen.
- ▶ That is why the pressure in a gas cylinder containing hydrogen will leak more quickly than the gas cylinder containing oxygen.

**Q.5** What happens to the temperature of a room in which an air conditioner is left running on a table in the middle of the room?

**Ans:** The temperature of the room will not decrease but it increases slightly.

Reason

- ▶ Air conditioner absorbs heat from the room as well as rejects heat in the same room at the same rate.
- ▶ During compression of gas some heat is produced due to friction in the compressor. This heat is also expelled in the same room.
- ▶ Therefore the temperature of the room will increase slightly.

**Q 6** Why does the pressure of the air in automobile tyre increases if the automobile is driven for a while?

**Ans** When the automobile is driven for a while then heat produce due to friction between road and tyre i.e. the work done against friction appears as heat energy.

- This heat is absorbed by the gas molecules.
- It increases the temperature and average kinetic energy of gas molecules.

As pressure  $P \propto KE$

- When  $\langle KE \rangle$  of gas molecules increases then number of collision of gas molecules with walls of tyre increases per unit time, therefore pressure of gas inside tyre also increases.

**Q 7** On removing the valve, the air escaping from a cycle tube cool. Why?

**Ans** The air inside cycle tube is at high pressure and high temperature.

- When the valve is removed the air rushes out from the tube.
- The outside atmospheric pressure is less than the air pressure inside the tube.
- Therefore adiabatic expansion produces which causes cooling.
- As a result the escaping air falls down and hence becomes cool.

**Q 8** Write the limitations of first law of thermodynamics.

**Ans**

- First law of thermodynamics does not provide a clear idea about the direction of absorption or evolution of heat.
- It does not tells that under what conditions and in which direction heat and work takes place.
- Does not tell about the complete system. The energy conservation which can never be achieved if entropy is achieved, conservation has failed.

**Q 9** Is it possible, according to the second law of thermodynamics, to construct a heat engine that is free from thermal pollution?

**Ans** No, it is not possible to construct a heat engine that is free from thermal pollution.

**Reason:**

- According to the second thermodynamics and Kelvin's statement, It is impossible to construct a heat engine operating in a cycle which absorbs heat from a hot reservoir and converts it completely into work without rejecting any heat to a sink.
- Every heat engine expels heat energy to the surrounding and produce thermal pollution.
- It is possible then it will be violation of second law of thermodynamics.

**Q 10** Can specific heat of a gas be zero or infinity? Can specific heat be negative?

**Ans** i) No, specific heat can be zero in adiabatic process.

As we know that

$$\Delta Q = nC\Delta T$$

Where  $C$  is molar specific heat

$$C = \frac{\Delta Q}{n\Delta T} \quad (1)$$

- In adiabatic process no heat enters or leaves the system i.e.  $\Delta Q = 0$

Putting  $\Delta Q = 0$  in equation (1)

$$C = \frac{0}{n\Delta T} \text{ (or) } C = 0$$

- ii) Molar specific heat can be infinity in isothermal process

- In isothermal process the temperature of the system remains constant

And  $\Delta T = 0$

Putting  $\Delta T = 0$  in equation (1)

$$C = \frac{\Delta Q}{n\Delta T} \quad (\text{OR}) \quad C = \text{Infinity}$$

- iii) Molar specific heat is always positive. It can never be negative.

Q.11 An inventor claims to have developed a heat engine, working between  $27^\circ\text{C}$  and  $227^\circ\text{C}$  having an efficiency of 45%. Is the claim valid? Why?

Ans: Temperature of source  $T_1 = 227^\circ\text{C} = 227 + 273 = 500\text{ K}$   
 Temperature of sink  $T_2 = 27^\circ\text{C} = 27 + 273 = 300\text{ K}$

$$\eta\% = 0.4 \times 100 = 40\%$$

### Comprehensive Revision

Q3 Give answers to the following questions.

1 Explain, briefly, the following terms used in thermodynamics: System, Surroundings, Boundary and State variables.

Ans: See Q 5 from book

2 Distinguish among the three forms of energy: work, heat and internal energy.

Ans: See Q 2 and Q 3 from book

3 State and explain the first law of thermodynamics.

Ans: See Q 6 from book

4 In the light of the first law of thermodynamics describe the processes:

- (a) Isochoric process (b) Isobaric process  
 (c) Isothermal process and (d) Adiabatic process

Ans: See Q 7 from book

5 Define the molar heat capacities  $C_p$  and  $C_v$  for a gas. Show that  $C_p - C_v = R$ .

Ans: See Q 8 from book

6 Explain with examples reversible and irreversible processes.

Ans: See Q 9 from book

7 What is meant by a heat engine? What is its main purpose? How is its efficiency defined?

Ans: See Q 10 from book

8 State the second law of thermodynamics in its alternative forms. Discuss the implications of the first and second laws about heat and work energies.

Ans: See Q 11 from book

9 What were the basic questions that led Carnot to invent Carnot engine?

Ans: See Q 12 from book

10 What is meant by Carnot cycle and by Carnot engine?

Ans: See Q 12 from book

- 11 State Carnot Theorem about the characteristics of a Carnot engine

**Ans** See Q 12 from book

- 12 What do you mean by a refrigerator? How does it function? Derive an expression for the Coefficient of performance of a refrigerator

**Ans** See Q 13 from book

- 13 Explain the concept of entropy. Mention its major properties. How is the second law of thermodynamics expressed in terms of entropy?

**Ans** See Q 14 from book

## Numerical Problems

- 1 Water at  $20^\circ\text{C}$  falls from a height of 854 m. If the whole energy is used in increasing the temperature, find out the final temperature. Specific heat of water is  $4200 \text{ J K}^{-1} \text{ kg}^{-1}$

**Data:** Initial temperature of water =  $T = 20^\circ\text{C}$ ,  $T = 293 \text{ K}$

Final temperature of water =  $T_2 = ?$

Height from which water falls =  $h = 854 \text{ m}$

Specific heat of water =  $c = 4200 \text{ J kg}^{-1} \text{ K}^{-1}$

**Solution**

When water falls on ground then its P.E.

Converts into heat energy

Loss of P.E. = gain of heat energy

$$mgh = mc\Delta T$$

$$gh = c\Delta T$$

$$\Delta T = \frac{gh}{c}$$

$$T_2 - T = \frac{gh}{c}$$

$$T_2 = T + \frac{gh}{c}$$

$$T_2 = T + \frac{gh}{c}$$

$$T_2 = 293 + \frac{9.8 \times 854}{4200} \quad \text{Putting values}$$

$$T_2 = 293 + 2 = 295 \text{ K}$$

$$T_2 = 295 - 273 = 22^\circ\text{C}$$

$$T_2 = 22^\circ\text{C}$$

**Alternate method:**

Final velocity of water just before hitting the ground is  $v = \sqrt{2gh}$

$$v = \sqrt{2 \times 9.8 \times 854}$$

$$v = 29.4 \text{ m/s}$$

As P.E. of water converts into heat energy on hitting the ground

Loss of P.E. of water = Heat energy

$$mgh = mc\Delta T$$

$$\Delta T = \frac{gh}{c}$$

$$T_2 - T = \frac{gh}{c}$$

$$T_2 = T + \frac{gh}{c}$$

$$2gh = v^2 - u^2$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 854}$$

$$v = 29.4 \text{ m/s}$$

$$T_1 = \frac{P_1 V_1}{nR}$$

$$T_2 = \frac{P_2 V_2}{nR}$$

$$T_1 = \frac{100 \times 10^3 \times 29.4}{1 \times 8.314}$$

$$T_1 = 353.15 \text{ K}$$

$$T_2 = \frac{10 \times 10^3 \times 29.4}{1 \times 8.314}$$

$$T_2 = 35.315 \text{ K}$$

2. 25200 J of heat is supplied to the system while the system does 6000 J of work. Calculate the change in internal energy of the system.

Data: Heat supplied  $Q = 25200 \text{ J}$   
 Work done by system  $W = 6000 \text{ J}$   
 Change in internal energy  $\Delta U = ?$

Solution.

According to 1st law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta U = \Delta Q - \Delta W$$

$$\Delta U = 25200 - 6000 = 19200 \text{ J}$$

3. A sample of ideal gas is uniformly heated at constant pressure. If the amount of 180 J of heat is supplied to the gas, calculate the Change in internal energy of the gas and Work done by the gas. Take  $\gamma = 1.41$

Given Data: Heat supplied at constant pressure  $\Delta Q_p = 180 \text{ J}$

a. Change in internal energy  $= \Delta U = ?$

b. Work done by system  $= \Delta W = ?$

Solution (a)  $\gamma = \frac{C_p}{C_v}$

$$C_p = \gamma C_v$$

$$C_p = \gamma n \Delta T \quad (\text{Putting } \Delta Q_p = 180 \text{ J})$$

$$180 = \gamma n \Delta T \quad (C_p = \gamma C_v)$$

$$180 = \gamma \Delta U$$

$$\Delta U = \frac{180}{\gamma} \quad (\text{Putting } C_v n \Delta T = \Delta U)$$

$$\Delta U = \frac{180}{1.41}$$

$$\Delta U = 127.66 \text{ J}$$

$$\Delta W = \Delta Q - \Delta U$$

$$\Delta W = 180 - 127.66$$

$$\Delta W = 52.34 \text{ J}$$

(b)  $\Delta W = \Delta Q - \Delta U$   
 $\Delta W = 180 - 127.66$   
 $\Delta W = 52.34 \text{ J}$

4. Find the efficiency of a Carnot's heat engine working between the steam and ice points?

Data: High temperature  $T_1 = 100^\circ \text{C} = 100 + 273 = 373 \text{ K}$

Low temperature  $= T_2 = 0^\circ \text{C} = 0 + 273 = 273 \text{ K}$

Efficiency  $= \eta = ?$

Solution.

$$\eta = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{273}{373}$$

$$= 0.2681$$

$$T_c = 26 \text{ K}$$

$$T_h = 296 \text{ K}$$

5. A Carnot heat engine absorbs 2000 J of heat from the source of heat engine at  $227^\circ\text{C}$  and rejects 1200 J of heat during each cycle to sink. Calculate efficiency of engine, temperature of sink and amount of work done during each cycle.

**Data**

Heat absorbed  $Q_1 = 2000 \text{ J}$

Heat rejected  $Q_2 = 1200 \text{ J}$

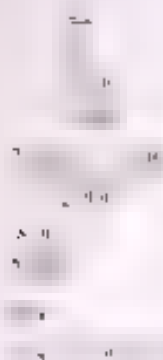
Temperature of source  $T_1 = 227 + 273 = 500 \text{ K}$

Temperature of sink  $T_2 = ?$

Work done  $W = ?$

**Solution**

(a)



(b)

$$\eta = \frac{W}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = \frac{2000 - 1200}{2000}$$

$$\eta = \frac{800}{2000}$$

$$\eta = 0.4$$

(c)

$$W = Q_1 - Q_2$$

$$W = 2000 - 1200$$

$$W = 800 \text{ J}$$

6. What is the least amount of work that must be performed to freeze one gram of water at  $0^\circ\text{C}$  by means of a refrigerator? Take the temperature of the surrounding as  $37^\circ\text{C}$ . How much heat is passed on to the surrounding during this process?

**Given Data**

Mass of water  $m = 1 \text{ g}$

Temperature  $T_1 = 0^\circ\text{C} = 273 \text{ K}$

Temperature  $T_2 = 37^\circ\text{C} = 32 + 273 = 310 \text{ K}$

Latent heat of fusion  $L_f = 336 \text{ J/g}$

Amount of heat removed  $Q = mL_f$

$$Q = 1 \times 336 = 336 \text{ J}$$

Heat rejected to surrounding  $Q_2 = ?$

Work done  $W = ?$

**Solution**

Coefficient of performance of refrigerator is

$$F = \frac{1}{\frac{1}{F_1} + \frac{1}{F_2}}$$

$$F = \frac{1}{\frac{1}{10} + \frac{1}{20}}$$

$$F = \frac{1}{\frac{2+1}{20}}$$

$$F = \frac{20}{3} \text{ N}$$

Ans

$$F = \frac{20}{3}$$

$$\Delta W = Q$$

$$\Delta W = \frac{336}{1.5 \times 10^8}$$

$$\Delta W = 45.54 \text{ J}$$

$$\Delta W = Q - Q_2$$

$$Q_2 = \Delta W + Q_1$$

$$= 45.54 + 336$$

$$Q_2 = 381.54 \text{ J}$$

- 7 Calculate the change in entropy when 10 kg of water is heated from 90 °C to 100 °C? (Specific heat of water is 4180 J mol<sup>-1</sup> K<sup>-1</sup>)

Given Data: mass = m = 10 kg

$$\text{Initial temperature} = T_1 = 90^\circ\text{C} = 90 + 273 = 363 \text{ K}$$

$$\text{Final temperature} = T_2 = 100^\circ\text{C} = 100 + 273 = 373 \text{ K}$$

$$\text{Change in temperature} = \Delta T = 373 - 363 = 10 \text{ K}$$

$$\text{Specific heat} = C = 4180 \text{ J mol}^{-1} \text{ K}^{-1}$$

Solution

$$\text{Average temperature} = \frac{T_1 + T_2}{2}$$

$$= \frac{363 + 373}{2}$$

$$T = 368 \text{ K}$$

Heat supplied to water-

$$\Delta Q = C \cdot m \cdot \Delta T$$

$$4180 \times 10 \times 10$$

$$[ \Delta Q = 418000 \text{ J} ]$$

Change in entropy

$$\Delta S = \frac{\Delta Q}{T}$$

$$= \frac{418000}{368}$$

$$\Delta S = 1135.8 \text{ K}$$

8. A system absorbs 1176 J of heat and at the same time does 352.8 J of external work. Find the change in internal energy of the system? Find the change in internal energy in the system when it absorbs 1050 J of heat while 84 J of work is done? What will be the change in internal energy of the gas if 210 J of heat is removed at constant volume?

Given Data.

(a) Heat supplied =  $\Delta Q = 1176 \text{ J}$

Work done =  $\Delta W = 352.8 \text{ J}$

Change in internal energy =  $\Delta U = ?$

- (b) Heat supplied =  $\Delta Q = 1050 \text{ J}$   
 Work done =  $\Delta W = 84 \text{ J}$   
 $\Delta U = \quad ?$
- (c) Heat removed =  $\Delta Q_1 = -210 \text{ J}$   
 At constant volume =  $\Delta W = 0$   
 $\Delta U = \quad ?$

**Solution:**

- (a) According to 1<sup>st</sup> law of thermodynamics

$$\begin{aligned}\Delta Q &= \Delta U + \Delta W \\ \Delta U &= \Delta Q - \Delta W \\ &= 1176 - 352.8\end{aligned}$$

$$\Delta U = 823.2 \text{ J}$$

- (b) Formula

$$\begin{aligned}\Delta Q &= \Delta U + \Delta W \\ \Delta U &= \Delta Q - \Delta W \\ &= 1050 - 84\end{aligned}$$

$$\Delta U = 966 \text{ J}$$

- (c) Formula

$$\begin{aligned}\Delta Q &= \Delta U + \Delta W \\ \Delta U &= \Delta Q - \Delta W \\ &= -210 - 0\end{aligned}$$

$$\Delta U = -210 \text{ J}$$

9. An ideal gas at  $20.0^\circ\text{C}$  and a pressure of  $1.50 \times 10^5 \text{ Pa}$  is in a container having a volume of  $1.00 \text{ L}$ .
- (a) Determine the number of moles of gas in the container.
- (b) The gas pushes against the piston, expanding to twice the original volume, while pressure falls to atmospheric pressure. Find the final temperature.

**Data**

Temperature of ideal gas =  $T = 20^\circ\text{C} = 273 + 20 = 293 \text{ K}$

Pressure =  $P_1 = 1.50 \times 10^5 \text{ Pa}$

Initial Volume =  $V = 1.00 \text{ L} = \frac{1.00}{1000} \text{ m}^3 = 10^{-3} \text{ m}^3$

Universal gas constant =  $R = 8.315 \text{ J/mol K}$

Final Volume =  $V_2 = 2V$

- (a) Number of moles of gas =  $n = ?$

- (b) Final temperature =  $T_2 = ?$  When  $P_2 = 1 \text{ atm} = 1.01325 \times 10^5$   $V_2 = 2V_1$

**Solution:**

- (a) From ideal gas equation,

$$\begin{aligned}P_1 V_1 &= nRT \\ 1.50 \times 10^5 \times 10^{-3} &= n \times 8.315 \times 293 \\ 50 \times 10^2 &= n(2436.295)\end{aligned}$$

$$\begin{aligned}n &= \frac{1.50 \times 10^2}{2436.295} \\ n &= 6.16 \times 10^{-2} \text{ moles}\end{aligned}$$

- (b) From ideal gas equation

$$\begin{aligned}P_2 V_2 &= nRT_2 \\ P_2 \times 2V_1 &= nRT_2 \\ 1.01 \times 10^5 \times 2 \times 10^{-3} &= 6.16 \times 10^{-2} \times 8.315 \times T_2 \\ T_2 &= \frac{2.02 \times 10^3}{6.16 \times 10^{-2} \times 8.315} = 395 \text{ K}\end{aligned}$$

10. A block of ice at 273 K is put in thermal contact with container of steam at 373 K, converting 26 g of ice to water at 273 K while condensing some of the steam to water at 373 K.
- Find the change in entropy of the ice
  - Find the change in entropy of the steam.
  - Find the change in entropy of the universe

**Given Data:** Temperature of ice  $T_{ice} = 273\text{ K}$   
 Temperature of steam  $= T_{steam} = 373\text{ K}$   
 Mass of ice block  $m = 26\text{ g} = \frac{26}{1000}\text{ kg} = 0.026\text{ kg}$   
 Latent heat of fusion of ice is  $L_f = 336000\text{ J/kg}$   
 Change in the entropy of ice  $= \Delta S_{ice}$   
 Change in the entropy of steam  $= \Delta S_{steam}$   
 Change in the entropy of ice  $= \Delta S_{universe} = ?$

**Solution.**

The heat absorbed from the steam is

$$\Delta Q = mL_f = 0.026\text{ kg} \times 336000\text{ J/kg} = 8400\text{ J}$$

The change in the entropy of ice  $\Delta S_{ice}$

$$\Delta S_{ice} = \frac{\Delta Q}{T_{ice}} = \frac{8400}{273} = 30.76\text{ J/K} \approx 30.8\text{ J/K}$$

Change in the entropy of ice  $\Delta S_{ice}$

$$\Delta S_{steam} = -\frac{\Delta Q}{T_{steam}} = -\frac{8400}{373} = -22.52\text{ J/K} \approx -22.5\text{ J/K}$$

Negative sign shows a decrease in the entropy of the steam

Change in the entropy of ice  $\Delta S_{universe}$

$$\Delta S_{universe} = \Delta S_{ice} + \Delta S_{steam} = 30.8\text{ J/K} + (-22.5\text{ J/K})$$

$$\Delta S_{universe} = 8.3\text{ J/K} \approx 8.3\text{ J/K}$$



## Additional Conceptual Short Questions With Answers

1. Why the food is cooked quicker in pressure cooker?

**Ans:** In pressure cooker, due to increase in pressure the boiling point of the food rises so the food is cooked quicker.

2. A person is painting the ceiling, and drop of paint from the brush falls onto an operating incandescent light bulb. The bulb breaks. Why?

**Ans:** An incandescent light bulb has glass that receives light from the bulb filament. The glass heats up by radiation of filament and by convection through gas filled in glass. Thus, glass becomes very hot.

When drop of paint falls onto the glass, that portion of glass suddenly becomes cold, and the contraction of this region creates thermal stresses that causes the glass to break.

3. Write down values of molar specific heat at constant volume and at constant pressure and find their ratio constant  $\gamma$  for mono, di and polyatomic gases?

**Ans:**

|                             | $C_V$          | $C_P$          | $\gamma = \frac{C_P}{C_V}$ |
|-----------------------------|----------------|----------------|----------------------------|
| Mono-atomic gas (Ideal gas) | $\frac{3}{2}R$ | $\frac{5}{2}R$ | $\frac{5}{3} = 1.67$       |
| Diatomic gas                | $\frac{5}{2}R$ | $\frac{7}{2}R$ | $\frac{7}{5} = 1.4$        |
| Polyatomic gas              | $3R/2$         | $9R/2$         | 1.29                       |

4. For a mono-atomic gas if  $\gamma = \left(\frac{5}{3}\right)$ , then find value of  $C_p$  and  $C_v$ ?

**Ans.** We know that

$$C_p - C_v = R$$

Dividing above eq. by  $C_v$

$$\frac{C_p}{C_v} - \frac{C_v}{C_v} = \frac{R}{C_v}$$

$$\left(\frac{C_p}{C_v} - 1\right) = \frac{R}{C_v}$$

putting  $\frac{C_p}{C_v} = \gamma$

$$(\gamma - 1) = \frac{R}{C_v}$$

$$\Rightarrow C_v = \frac{R}{\gamma - 1}$$

$$\Rightarrow C_v = \frac{R}{\frac{5}{3} - 1}$$

$$\Rightarrow C_v = \frac{R}{\frac{5-3}{3}} = \frac{R}{\frac{2}{3}}$$

$$C_v = \frac{3}{2}R$$

Similarly

$$C_p = \gamma C_v$$

$$C_p = \gamma C_v$$

$$C_p = \frac{5}{3} \times \frac{3}{2} R$$

$$C_p = \frac{5}{2} R$$

5. If one mole of a monoatomic gas  $\gamma = \frac{5}{3}$  is mixed with one mole of a diatomic gas  $\gamma = \frac{7}{5}$  then what is value of  $\gamma$  for the mixture?

**Ans.** We know that

$$(C_v)_{\text{mono}} = \frac{3}{2} R$$

$$\text{and } (C_v)_{\text{di}} = \frac{5}{2} R$$

$$\begin{aligned} \text{So } (C_v)_{\text{mixture}} &= \frac{\left(\frac{3}{2} R + \frac{5}{2} R\right)}{2} = \left(\frac{3+5}{2 \times 2}\right) R \\ &= \frac{8}{4} R \end{aligned}$$

$$C_1 + C_2 = 2R$$

From the relation

$$C_1 + C_2 = 2R$$

$$C_1 + 2R = 2R$$

$$C_1 = 2R - R$$

$$C_1 = R$$

Since  $\gamma = \frac{C_p}{C_v} = \frac{3R}{2R} = \frac{3}{2} \Rightarrow \gamma_{\text{mix}} = \frac{3}{2}$

6 If  $PV^\gamma = \text{constant}$ , prove that  $TV^{\gamma-1} = \text{constant}$

Ans

$$PV^\gamma = \text{Constant} \quad (1)$$

Also

$$PV = nRT$$

For one mole  $n = 1$

$$PV = RT$$

$$P = \frac{RT}{V} \quad (2)$$

Putting value from eq. (2) in (1)

$$\frac{RT}{V} V^\gamma = \text{Constant}$$

$$RT V^{\gamma-1} = \text{Constant}$$

$$TV^{\gamma-1} = \frac{\text{Constant}}{R}$$

As R is also a constant

$$TV^{\gamma-1} = \text{Constant}$$

7 Explain why adiabatic is steeper than an isotherm?

Ans

If an isotherm and an adiabatic are drawn in the same graph, it can be seen that adiabatic is steeper than the isotherm. This steepness is due to the reason that in an adiabatic expansion, the system does work at the cost of its own internal energy. While in an isothermal expansion, energy is supplied by the heat reservoir.

► Adiabatic process is faster than isothermal process.

Due to these reasons, adiabatic is more steeper than isotherm.

(Alternate Method)

For an adiabatic process

$$PV^\gamma = k$$

$$P = k/V^\gamma$$

$$\frac{d}{dV} (P) = \frac{d}{dV} (k/V^\gamma)$$

$$\frac{dP}{dV} = -\gamma \frac{k}{V^{\gamma+1}}$$

$$\frac{dP}{dV} = -\gamma k (1/V^{\gamma+1}) \left( \frac{dV}{dV} \right)$$

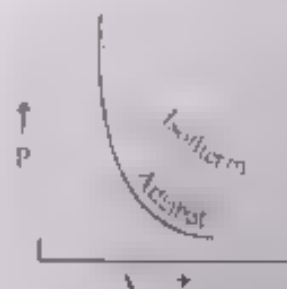
$$\frac{dP}{dV} = -\gamma k (1/V^{\gamma+1})$$

$$\frac{dP}{dV} = -\frac{\gamma}{V} (k/V^\gamma)$$

putting  $k/V^\gamma = P$

$$\frac{dP}{dV} = -\gamma \left( \frac{P}{V} \right)$$

$$\text{slope of adiabatic process} = \frac{dP}{dV} = -\gamma \left( \frac{P}{V} \right) \quad (1)$$



For isothermal process

$$PV = k$$

$$P = \frac{k}{V}$$

(1) Differentiating

$$\frac{d}{dV} (PV) = \frac{d}{dV} k$$

$$\frac{dP}{dV} = k \frac{d}{dV} \left( \frac{1}{V} \right)$$

$$\frac{dP}{dV} = k \left( -\frac{1}{V^2} \right) \frac{dV}{dV}$$

$$\frac{dP}{dV} = -\frac{k}{V^2} \left( \frac{1}{V} \right) \quad \text{putting } \frac{k}{V} = P$$

$$\frac{dP}{dV} = -\frac{P}{V}$$

Slope of an isothermal process

$$\frac{dP}{dV} = -\frac{P}{V}$$

As we know that  $V \propto T$ 

From equation (1) and (2) it is clear that the slope of an isothermal process is negative and its magnitude is more steep than unity.

8. Specific heat of a gas at constant pressure is greater than specific heat at constant volume. Why?

**Ans:** Reason

When gas is heated at constant pressure then

(i) a part of heat is used to do work on piston(ii) rest of heat is used to increase the temperature through 1K

When gas is heated at constant volume then all the heat absorbed is used to increase the temperature through 1K. As piston is fixed, therefore no work is done by system.

That is why the molar specific heat at constant pressure is greater than molar specific heat at constant volume.

Also  $C_p = C_v + R$ 

$$(C_p > C_v)$$

9. Does entropy of a system increases or decreases due to friction?

**Ans:** The entropy of a system increases due to friction.

Reason

Due to friction, some mechanical energy is converted into heat which increases the entropy of system.

$$\Delta S = \frac{\Delta Q}{T}$$

When  $\Delta Q$  increase then  $\Delta S$  also increases.

10. Give an example of a process in which no heat is transferred to or from the system but the temperature of the system changes.

**Ans:** In adiabatic process no heat enters or leaves the system take place but temperature of the system changes.

During adiabatic expansion temperature falls

During adiabatic compression temperature increases

Examples

- Rapid escape of air from a burst tyre
- Rapid expansion and compression of air through which sound wave is passing
- Cloud formation in the atmosphere

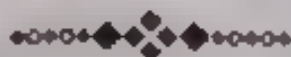


# MCQ's From Past FBISE Papers (FEDERAL BOARD)

1. For a mono atomic gas  $C_v = 3R/2$ , therefore gamma  $\gamma$  for this gas is  
(a) 7/5 (b) 5/3 (c) 4/3 (d) 15/4
2. In thermodynamic process the equation  $\Delta Q = \Delta U$  represents process.  
(a) Isothermal (b) isobaric (c) Adiabatic (d) Isochoric
3. An isothermal process is represented by:  
(a) Charles' law (b) Gay-Lussac law (c) Ideal gas law (d) Boyle's law
4. In an isothermal process, internal energy of the system  
(a) Increases (b) Decreases then increase (c) Decreases (d) Remains constant
5. If the temperature of the source increases, the efficiency of a Carnot engine.  
(a) Decreases (b) increases then decreases (c) increases (d) Remains same
6. The efficiency of a Carnot engine working between higher and lower temperatures  $T_1$  &  $T_2$  respectively is given by  
(a)  $\eta = \frac{T_1 - T_2}{T_1}$  (b)  $\eta = \frac{T_1}{T_1 - T_2}$  (c)  $\eta = \frac{T_1 \cdot T_2}{T_1 - T_2}$  (d)  $\eta = \frac{T_2}{T_1 - T_2}$
7. The expression for pressure exerted by the gas on container on any side is  
(a)  $P = \frac{1}{2} \rho \langle v^2 \rangle$  (b)  $P = \frac{2}{3} \rho \langle v^2 \rangle$  (c)  $P = \frac{1}{3} \rho \langle v^2 \rangle$  (d)  $P = \frac{1}{2} \rho \langle v^2 \rangle$
8. Maximum efficiency of Carnot engine is always  
(a) greater than one (b) less than one (c) equal to one (d) none of these
9. In reversible process, the entropy  
(a) remains constant (b) increases (c) decreases (d) initially increases then decreases
10. The triple point of water is  
(a) 0K (b) 100K (c) 373.16K (d) 273.16K
11. What would be the efficiency of a Carnot engine operating with boiling water as one reservoir and a freezing mixture of ice and water as the other reservoir? (FBISE-2017)  
(a) 27% (b) 67% (c) 12% (d) 100%
12. In a reversible cycle, the entropy of the system (FBISE-2017)  
(a) First increases and then decreases (b) Increases (c) Decreases (d) Does not change
13. A frictionless heat engine can be 100% efficient only if its exhaust temperature is. (FBISE (ON) 2017)  
(a) Zero kelvin (b) Equal to its input temperature (c) Kelvin temperature (d) Zero  $^{\circ}\text{C}$  temperature
14. The temperature at which a system undergoes a reversible isothermal process without transfer of heat is called as. (FBISE(ON) 2017)  
(a) Reversible temperature (b) Critical temperature (c) Kelvin temperature (d) Absolute zero temperature
15. The entropy of universe always (FBISE-2018)  
(a) Increases and decreases simultaneously (b) Remains constant (c) Increases (d) Decreases
16. The efficiency of diesel engine is about: (FBISE(ON) 2018)  
(a) 30% to 35% (b) 25% to 30% (c) 45% to 50% (d) 35% to 40%

## Answers Key

|    |   |    |   |    |   |    |   |    |   |
|----|---|----|---|----|---|----|---|----|---|
| 1  | b | 2  | d | 3  | d | 4  | d | 5  | c |
| 6  | b | 7  | c | 8  | b | 9  | a | 10 | d |
| 11 | a | 12 | d | 13 | a | 14 | d | 15 | c |
| 16 | d |    |   |    |   |    |   |    |   |



## SELF - ASSESSMENT PAPER

Total Mark 40

Question No 1 Choose the correct answer from the given options

(1 x 8 = 8)

## SECTION - A

1. For a mono-atomic gas  $C_v = \frac{3R}{2}$  then  $\gamma$  is:
 

|                   |                   |                    |                    |
|-------------------|-------------------|--------------------|--------------------|
| (A) $\frac{3}{5}$ | (B) $\frac{5}{3}$ | (C) $\frac{4}{15}$ | (D) $\frac{15}{4}$ |
|-------------------|-------------------|--------------------|--------------------|
2. Which of following is correct for 1 calorie = \_\_\_\_\_ J
 

|           |          |          |           |
|-----------|----------|----------|-----------|
| (A) 420 J | (B) 10 J | (C) 40 J | (D) 4.2 J |
|-----------|----------|----------|-----------|
3. Efficiency of Carnot engine can be:
 

|          |             |              |          |
|----------|-------------|--------------|----------|
| (A) 100% | (B) Maximum | (C) Infinite | (D) Zero |
|----------|-------------|--------------|----------|
4. Temperature would be same on  $^{\circ}\text{F}$  and  $^{\circ}\text{C}$  at \_\_\_\_\_
 

|                   |                  |          |          |
|-------------------|------------------|----------|----------|
| (A) $-40^{\circ}$ | (B) $40^{\circ}$ | (C) $30$ | (D) $30$ |
|-------------------|------------------|----------|----------|
5. In an isothermal change internal energy
 

|               |               |                  |                      |
|---------------|---------------|------------------|----------------------|
| (A) Decreases | (B) Increases | (C) Becomes zero | (D) Remains constant |
|---------------|---------------|------------------|----------------------|
6. A real gas can be approximated to an ideal gas at \_\_\_\_\_
 

|                 |                   |                  |                     |
|-----------------|-------------------|------------------|---------------------|
| (A) Low density | (B) High pressure | (C) High density | (D) Low temperature |
|-----------------|-------------------|------------------|---------------------|

(FBISE-2019)

Question No 2 Give short answers of following

(3 x 7 = 21)

## SECTION - B

- (i) Why does the pressure of gas in a car tyre increase when it is driven through some distance?
- (ii) Prove that area of P-V graph is equal to work done by the system.
- (iii) Can specific heat of a gas be zero or infinity? Can specific heat be negative?
- (iv) An inventor claims to have developed a heat engine working between  $27^{\circ}\text{C}$  and  $227^{\circ}\text{C}$  having an efficiency of 45%. Is the claim valid? Why?
- (v) What happens to the temperature of a room in which an air conditioner is left running on a table in the middle of the room?
- (vi) Differentiate between reversible and irreversible process.
- (vii) During one cycle, an engine extracts  $2.00 \times 10^3 \text{ J}$  of energy from a hot reservoir and transfers  $1.50 \times 10^3 \text{ J}$  to a cold reservoir. (a) Find the thermal efficiency of the engine. (b) How much work does the engine do in one cycle? (c) What average power does the engine generate if it goes through four cycles in 2.50 s?

Question No.3 Extensive Questions.

(13)

## SECTION - C

- (a) What is Carnot's Engine? Explain its working and calculate its efficiency. Also state Carnot's theorem.
- (b) A Carnot heat engine absorbs 2000 J of heat from the source of heat engine at  $227^{\circ}\text{C}$  and rejects 1200 J of heat during each cycle to sink. Calculate efficiency of engine, temperature of sink and amount of work done during each cycle.
- (c) What is mechanical equivalent of heat?

The End

THE END

## FEDERAL PHYSICS HSSC - I

Time: 25 Minutes

## SECTION-A

Marks: 17

Note: Section-A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Choose the correct answer i.e. A/B/C/D by filling the relevant bubble for each question on the OMR Answer Sheet according to the instructions given there. Each part carries one mark.

- In a cricket match 500 spectators are counted one by one. How many significant figures will be there in final result?  
A 3                      B 1                      C 2                      D 0
- The SI unit of solid angle is:  
A degree                      B radian                      C steradian                      D revolution
- If  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ ,  $\vec{A}$  the scalar product is:  
A additive                      B associative                      C commutative                      D multiplicative
- The rectangular components of a force of 5N are:  
A 3 and 4N                      B 2.5 and 2.5N                      C 1 and 2N                      D 2 and 3N
- Distance covered by free falling body in 2 second is:  
A 4.9 m                      B 19.6 m                      C 39.2 m                      D 44.1 m
- If momentum is increased by 20% then K.E. increases by:  
A 44%                      B 55%                      C 66%                      D 77%
- The consumption of energy of 60 watt bulb is:  
A 30 J                      B 8 J                      C 42 J                      D 16 J
- SI unit of moment of inertia is:  
A Kg/m                      B Kg/m<sup>2</sup>                      C Kg m<sup>2</sup>                      D Kg m
- Who gave the inverse square law for gravity?  
A. Einstein                      B. Galileo                      C. Newton                      D. Plank
- Pressure will be low where the speed of the fluid is:  
A. Zero                      B. High                      C. Low                      D. Constant
- The displacement of particle having amplitude 'a' in SHM in one time period is:  
A. zero                      B. a                      C. 2a                      D. 4a
- The distance covered by a body in one complete linear vibration is 20 cm. What is the amplitude of body?  
A. 10 cm                      B. 5 cm                      C. 15 cm                      D. 7 cm
- Which waves are used in Sonography?  
A. microwaves                      B. X-rays                      C. ultrasonic waves                      D. Material waves
- Which is not optically active?  
A. Sugar                      B. Tartaric acid                      C. Water                      D. Milk
- The triple point of water is:  
A. 273 K                      B. 0 K                      C. 273.16 K                      D. 37 K
- A heat engine absorbs 50J of energy and give 45J of work. Its efficiency will be:  
A 60%                      B 70%                      C 80%                      D 90%
- Velocity of the efflux is measured by the relation:  
A.  $\sqrt{gh}$                       B.  $\sqrt{2gh}$                       C.  $\sqrt{\frac{1}{2}gh}$                       D.  $\sqrt{\frac{4}{3}gh}$

# FEDERAL PHYSICS HSSC – I

Time allowed: 2 3/4 Minutes

Total Marks: Section B, C and D: 68

Note: Section B, C and D comprise pages 1-2 and questions therein are to be answered in the separately provided Answer Book. Use supplementary answer sheet i.e., sheet B if required. Write your answers neatly and legibly.

## SECTION - B (Marks 21) (Chapters 1 to 5)

NOTE: Please write answer in no more than FIVE / SIX lines.

Q 2 Answer any SEVEN parts. All parts carry equal marks.

(7 × 3 = 21)

- Show that one radian =  $57.3^\circ$
- How are uncertainties measured in final result of power factor and volume?
- Prove that if the vector A & B are parallel to each other, then  $A \cdot B = AB$
- If  $(A + B) \cdot 0$ , then either of the two vectors is a null vector or vector A and B are perpendicular to each other. Prove it.
- Why is it helpful to wear a helmet while riding?
- Does a moving object have impulse? Explain.
- Name the various non-conventional sources of energy and write down their advantages.
- Why are energy savers used instead of normal bulbs?
- Prove that  $r_c = \left[ \frac{GM_E}{4\pi^2} \right]^{1/3}$  of geostationary satellite.
- Why does the coasting rotating system slows down as water drains from the bucket?

## SECTION - C (Marks 21) (Chapters 6 to 10)

Q 3 Answer any SEVEN parts. All parts carry equal marks.

(7 × 3 = 21)

- What is an aerofoil? Explain its working in aerodynamic lift.
- When water falls from a tap, why does the cross-sectional area of the liquid decrease as it comes down?
- What is meant by damped Oscillations? Give an example.
- In relation to SHM, explain the equation  $a = -\omega^2 x$ . Also show  $v = \omega \sqrt{A^2 - x^2}$
- Write down the applications of Doppler Effect.
- How can you generate transverse waves and how can you generate longitudinal waves?
- What do you know about polarizing spectrum?
- Why is ordinary light not polarized?
- Two blocks of ice are pressed together in order to form a single piece. What happens?
- What is the function of spark plug in a petrol engine?

## SECTION - D (Marks 26)

Q 3 Answer any TWO questions. All parts carry equal marks.

(2 × 13 = 26)

- Derive a relation for the time period of a simple pendulum using dimensional analysis. The variables possible factors on which the time period may depend are:
    - length of pendulum
    - mass of the bob
    - angle  $\theta$  which the thread makes with the vertical
    - acceleration due to gravity
  - Consider a ladder weighing 20N resting against a smooth wall such that it makes an angle of  $60^\circ$  with the horizontal. Find the reaction on the ladder due to the wall and ground.
- Q 5
- What is Bernoulli's equation? Show that how it is based on law of conservation of energy and name three applications of Bernoulli's equation.
  - Find the amplitude, frequency and time period of an object oscillating at the end of a spring. The equation for its position at any instant  $t$  is given by  $x = 0.25 \cos \left( \frac{\pi}{8} t \right)$ . Find the displacement of the object after 2 seconds.
- Q 6
- State Carnot theorem and the characteristics of a Carnot engine. Explain Carnot cycle and Carnot engine.
  - Find the polarizing angle for a glass of refractive index of 1.55.



## Federal Board HSSC-I Examination

## Physics Model Question Paper

## SECTION - A

Version Number    

Time allowed, 25 Minutes

Marks 17

NOTE: Section-A is compulsory. All parts of this section are to be answered on the separately provided OSMT Answer Sheet which should be completed in the first 25 minutes and handed over to the Center Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

- Q.1 Choose the correct answer A / B / C / D by filling the relevant bubble for each question on the OSMT Answer sheet according to the instructions given there. Each part carries one mark.
- What is the ratio  $1\mu\text{m}/1\text{Gm}$ ?  
(A)  $10^{-3}$  (B)  $10^{-4}$  (C)  $10^{-13}$  (D)  $10^{-15}$
  - For which angle the equation  $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}|$  is correct?  
(A)  $30^\circ$  (B)  $45^\circ$  (C)  $60^\circ$  (D)  $90^\circ$
  - What is the angle between  $\vec{A}$  and  $\vec{B}$  for which  $\vec{A} + \vec{B} = \vec{A} \cdot \vec{B}$ ?  
(A)  $30^\circ$  (B)  $45^\circ$  (C)  $60^\circ$  (D)  $90^\circ$
  - What does NOT change when force is applied on a body?  
(A) Mass (B) Velocity (C) Position (D) Acceleration
  - A projectile is thrown so that it travels a maximum range of 100m. How high will it rise?  
(A) 500 m (B) 50 m (C) 400 m (D) 100 m
  - If momentum is increased by 20% then K.E. increases by  
(A) 44% (B) 55% (C) 66% (D) 77%
  - The atmosphere is held to the earth by:  
(A) winds (B) gravity (C) clouds (D) rotation of earth
  - The angular speed in rad/hrs for daily rotation of our earth is.  
(A)  $2\pi$  (B)  $4\pi$  (C)  $\frac{\pi}{6}$  (D)  $\frac{\pi}{12}$
  - Artificial satellite moves around:  
(A) Moon (B) Sun (C) Stars (D) Earth
  - The pressure will be low where the speed of the fluid is:  
(A) Zero (B) High (C) Low (D) Constant
  - The periods of the pendulum at Karachi ( $T_k$ ) and at Murree ( $T_m$ ) are related as  
(A)  $T_k > T_m$  (B)  $T_k < T_m$  (C)  $T_k = T_m$  (D)  $2T_k = 3T_m$
  - Which one of the following factors has no effect on the speed of sound in a gas?  
(A) Humidity (B) Pressure (C) Temperature (D) Density
  - Which one of the following properties is NOT exhibited by the longitudinal waves?  
(A) Reflection (B) Interference (C) Diffraction (D) Polarization
  - The tip of a needle does NOT give a sharp image. It is due to  
(A) Polarization (B) Interference (C) Diffraction (D) Refraction
  - The Principle of Michelson Interferometer is based on the division of:  
(A) Wave front (B) Amplitude (C) Frequency (D) Speed of light
  - In an isothermal change, internal energy of a system:  
(A) Decreases (B) Increases (C) Becomes Zero (D) Remains Constant
  - Triple point of water is:  
(A)  $273.16^\circ\text{F}$  (B)  $372.16\text{ K}$  (C)  $273.16^\circ\text{C}$  (D)  $273.16\text{ K}$

# PHYSICS HSSC-I

Time allowed: 2 1/2 Hours

Total Mark: Section B, C and D: 68

Note: Section B, C and D comprise pages 1-2 and questions therein are to be answered on the separately provided Answer Book. Use supplementary answer sheet (i.e., sheet B if required). Write your answers neatly and legibly.

## SECTION – B (Marks 21)

(Chapters 1 to 5)

Q 2 Answer any SEVEN parts. All parts carry equal marks. (7 × 3 = 21)

- State the law of conservation of energy. Give an example to illustrate it.
- Define the term 'work'. State its SI unit and derive its dimensional formula.
- Define the term 'power'. State its SI unit and derive its dimensional formula.
- Define the term 'efficiency'. State its SI unit and derive its dimensional formula.
- Define the term 'moment of a force'. State its SI unit and derive its dimensional formula.
- Define the term 'torque'. State its SI unit and derive its dimensional formula.
- Define the term 'angular momentum'. State its SI unit and derive its dimensional formula.
- Define the term 'angular velocity'. State its SI unit and derive its dimensional formula.
- Define the term 'angular acceleration'. State its SI unit and derive its dimensional formula.
- Define the term 'centripetal force'. State its SI unit and derive its dimensional formula.
- Define the term 'centrifugal force'. State its SI unit and derive its dimensional formula.
- Define the term 'centrifugal force'. State its SI unit and derive its dimensional formula.
- Define the term 'centrifugal force'. State its SI unit and derive its dimensional formula.
- Define the term 'centrifugal force'. State its SI unit and derive its dimensional formula.
- Define the term 'centrifugal force'. State its SI unit and derive its dimensional formula.

## SECTION – C (Marks 21)

(Chapters 6 to 10)

Q 3 Answer any SEVEN parts. All parts carry equal marks. (7 × 3 = 21)

- Why sports car has oblong shape design?
- Describe the working of an engine carburetor.
- Define the term 'simple pendulum'. State its SI unit and derive its dimensional formula.
- What will be the zero of a simple pendulum?
- What are the conditions for constructive and destructive interference?
- How does the position of nodes and anti-nodes in a vibrating string?
- What are the conditions for radar waves? Explain briefly.
- What are the conditions for a process to be reversible?
- What are the conditions for a process to be irreversible?
- What are the conditions for a process to be irreversible?
- What are the conditions for a process to be irreversible?
- What are the conditions for a process to be irreversible?
- What are the conditions for a process to be irreversible?
- What are the conditions for a process to be irreversible?
- What are the conditions for a process to be irreversible?

## SECTION – D (Marks 26)

Q 4 Answer any FIVE questions. All questions carry equal marks. (5 × 5 = 25)

- Prove that Absolute PE =  $\frac{GMm}{R}$ .
- A man whose mass is 70 kg walks up to the third floor of a building which is 2 m above the ground in 20 sec. Find his power in watts and hp.
- Derive equation for kinetic and potential energy of a body executing S.H.M. for a mass-spring system.
- What should be the length of a simple pendulum whose time period is one second? What is its frequency?
- Describe any experimental arrangement for the production of interference fringes by Young's double slit experiment and get an expression for the fringe spacing.
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**PHYSICS HSSC-I (New Course)****OBJECTIVE KEY**

| Serial # | V # 3081 | V # 3082 | V # 3083 | V # 3084 |
|----------|----------|----------|----------|----------|
| 1)       | C        | EC       | A        | A        |
| 2)       | D        | D        | EC       | B        |
| 3)       | B        | A        | A        | EC       |
| 4)       | B        | C        | B        | B        |
| 5)       | C        | C        | D        | C        |
| 6)       | EC       | D        | D        | A        |
| 7)       | A        | C        | A        | A        |
| 8)       | C        | B        | EC       | B        |
| 9)       | C        | C        | C        | EC       |
| 10)      | D        | D        | A        | D        |
| 11)      | B        | A        | A        | B        |
| 12)      | C        | C        | B        | B        |
| 13)      | A        | D        | D        | C        |
| 14)      | B        | B        | A        | A        |
| 15)      | B        | C        | C        | B        |
| 16)      | C        | C        | D        | D        |
| 17)      | EC       | D        | D        | A        |

# SOLUTION PHYSICS HSSC-I (NEW COURSE) SECTION B

(Q)2

(i) **Given Data**Einstein equation  $E = mc^2$ 

To Prove

Einstein equation  $E = mc^2$  is dimensionally consistent

Calculations.

As  $E = mc^2$ Where  $E$  is the energy in joules

Dimensions of L.H.S of equation

$$= [E] = [ML^2T^{-2}] \quad (1)$$

Dimensions of R.H.S of equation

$$= mc^2 = [M][L^2T^{-2}]$$

$$= [ML^2T^{-2}] \quad (2)$$

Thus

$$L.H.S = R.H.S$$

(03)

(ii)

A microwave oven we use electromagnetic waves called microwaves to heat food. Micro means small because these waves are smaller in wavelength than other radio waves. It does not show that its wave-length is a micrometer.

Wave length of microwaves used in microwave oven is 12 cm and frequency is 2450 MHz.

(03)

(iii)

**Steps for addition of vectors**

- Find the x and y-components of all given vectors.
- Add x components of all the vectors to find the x component of Resultant vector.
- Add y components of all the vectors to find the y component of Resultant vector.
- Find the magnitude of resultant vector  $\vec{R}$  by using

$$R = \sqrt{R_x^2 + R_y^2}$$

- Find the direction of resultant vector  $\vec{R}$  by using

$$\theta = \tan^{-1} \left( \frac{R_y}{R_x} \right)$$

(03)

(v)

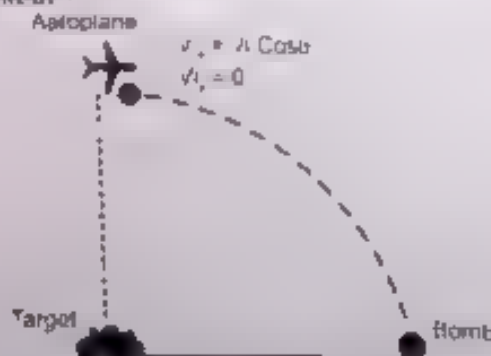
Cranes are made heavy at its bottom which lowers its C.G. and they are constructed such that their line of action of forces lies within its base area (their base area is made large. That's why the cranes mostly do not topple over after lifting load.

(03)

(vi)

The bomb has the same velocity as that of the aero-plane when it is dropped. If the bomb is dropped when the aero plane is vertically above the target. It will strike a point ahead of the target due to constant horizontal velocity component and inertia. The bomb misses the target. The bomb moves like a projectile as shown in fig. It will not hit the target but it will hit at T.

(03)



(Figure not complete)

(vii)

**Relation**

$$\vec{J} = \vec{F} \cdot \Delta t$$

As force is the time rate of change of momentum So

$$\int \frac{d}{dt} (mv) = \frac{d}{dt} (mv)$$

$$F = \frac{d}{dt} (mv) = \frac{d}{dt} (mv)$$

$$\text{Putting } F = \frac{d}{dt} (mv) = \frac{d}{dt} (mv)$$

$$\int F dt = \int \frac{d}{dt} (mv) dt = \int \frac{d}{dt} (mv) dt$$

So the impulse is equal to the change in momentum of the body

(iii) **Work Done by a Variable Force**

In many cases the force is not constant but it varies in magnitude or direction or in both e.g.

- Force of gravity acting on a rocket moving away from earth
- Force exerted by spring increases by the amount of stretch

Let us consider the path of particle in xy plane from point a to b as shown in figure.

Divide the path into n short intervals of displacement  $\Delta d_1, \Delta d_2, \dots, \Delta d_n$

The forces acting during these intervals are  $F_1, F_2, \dots, F_n$  respectively

The force is considered to be approximately constant to each interval of displacement

So work done for the first interval is,

$$\Delta W_1 = \vec{F}_1 \cdot \Delta \vec{d}_1 = F_1 \cos \theta_1 \Delta d_1$$

Similarly,

$$\Delta W = F_2 \cos \theta_2 \Delta d_2$$

and up to nth interval

$$\Delta W_n = F_n \cos \theta_n \Delta d_n$$

Now the total work done in moving the body from point a to b is

$$W = (\Delta W_1 + \Delta W_2 + \dots + \Delta W_n)$$

$$\text{OR } W = (F_1 \cos \theta_1 \Delta d_1 + F_2 \cos \theta_2 \Delta d_2 + \dots + F_n \cos \theta_n \Delta d_n)$$

$$\text{OR } W = \sum F \cos \theta \Delta d$$

OR Graphical Method

(03)

(iv) **Correct any three differences**

**Solar Energy**

Solar energy is the radiant light and heat from the Sun. Solar radiation along with secondary solar resources such as wind and wave power, hydroelectricity and biomass account for most of the available renewable energy on Earth.

There are two methods to use solar energy depending on the way they capture, convert and distribute sunlight.

**Active solar techniques** include the use of photovoltaic panels and solar thermal collectors (with electrical or mechanical equipment) to convert sunlight into useful outputs.

**Passive solar techniques** include orienting a building to the Sun, selecting materials with favorable thermal mass or light dispersing properties, and designing spaces that naturally circulate air.

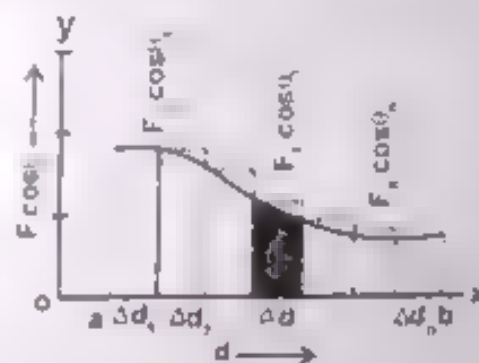
**Wind Energy**

The non-uniform distribution of heat due to solar energy in different regions causes the movement of hot and cold air over the earth's surface. Winds blow from area of high pressure to areas of low pressure. At sea the winds are even stronger than on the land surface.

In Pakistan places like Chhar, Gogit, coastal areas and high mountain valleys are suitable for the exploitation of wind power. (03)

(v) **Relation between Angular and Linear velocities**

Consider a particle that is moving in a circle of radius 'r' with center at O. Let particle moves from point 'A' to



point "B" in a circle such that it

If we take " $\theta$ " in radians, then  $S = r\theta$  (1)

Similarly, in linear motion, when a body moves with uniform velocity  $\vec{v}$ , in time ' $t$ ', its linear displacement will be:

$$S = vt \quad (2)$$

Comparing the above equations, we can derive

$$vt = r\theta$$

$$v = r(\theta/t)$$

Putting

$$\frac{\theta}{t} = \omega$$

$$v = r\omega$$

In vector form, we can write  $\vec{v} = \vec{\omega} \times \vec{r}$

(03)

#### (x) Expression for Orbital Velocity

Consider a satellite of mass  $m$ , moving with orbital velocity  $v$  around the earth of mass  $M$ . If  $r$  is the radius of the orbit then centripetal force  $F$  can be expressed as

$$F_c = \frac{m v^2}{r} \quad (1)$$

This force is provided by gravitational force of attraction between earth and satellite and is given by

$$F = G \frac{Mm}{r^2} \quad (2)$$

Equating equation (1) and (2), we get

$$\frac{m v^2}{r} = G \frac{Mm}{r^2}$$

Or

$$v^2 = \frac{GM}{r}$$

Or

$$v = \sqrt{\frac{GM}{r}}$$

(03)

#### SECTION (C)

Q n 3

#### (i) Lift of an aero-plane (Aerofall) (Figure is not compulsory)

(01 + 02)

The lift of an aero-plane is due to the effect, where speed of fluid is high, its pressure will be low.

**Explanation**

The design of wing deflects the air in such a way that

Stream lines are closer together above the wing than lower side. Air moves faster at the upper side of the wing than the lower side. Pressure is lower at the top of the wing. Hence, the wing experiences a net upward force.



#### (ii) Each time your heart beats, it generates a high pressure pulse of blood into the arteries. Counting these pulses gives you an accurate measurement of a person's heart rate in Beats per Minute.

When heart pumps the blood in the vessels, the pressure of blood in the vessels varies accordingly. The increase in pressure inflates the vessels and decrease in pressure squeezes the vessels. These changes in the pressure can be felt in the form of pulsations in the pulse.

Checking your pulse is a simple way to get information about your health.

(03)

#### (iii) Free Oscillations

(03)

A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force.

For example

A simple pendulum vibrates freely with its natural frequency that depends only upon the length of the pendulum.

**Forced Oscillations**

A body is said to be executing forced vibrations if it oscillates with the interference of an external force.

For example

If the mass of vibrating pendulum is struck repeatedly, then forced vibrations are produced.

The vibrations of factory floor caused by the running of heavy machinery is another example.

(iv) **Applications of Resonance (any two applications)**

a. Radio and Resonance

b. Magnetic Resonance Image (M.R.I)

(03)

(v) If  $E$  be the elastic modulus and  $\rho$  be the density of the medium then speed of sound waves is

$$v = \sqrt{\frac{E}{\rho}}$$

Although the density of solids is greater than that of gases but the modulus of elasticity of solids is much greater than that of gases ( $E_{\text{solids}} \gg E_{\text{gases}}$ ).

Therefore  $\left[\frac{E}{\rho}\right]_{\text{solids}} > \left[\frac{E}{\rho}\right]_{\text{gases}}$

Also molecules are closer in solids than in the gases, so they respond more quickly to a disturbance. That is why speed of sound is greater in solids than the gases.

(03)

(vi) **Any THREE differences.**

(03)

| Progressive waves |                                                                                                                                                                                                                 | Stationary waves |                                                                                                                                                                                                                                                     |
|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1.                | The disturbance produced in the medium travels onward, it being handed over from one particle to the next. Each particle executes the same type of vibration as the preceding one, though not at the same time. | 1.               | There is no onward motion of the disturbance as no particle transfers its motion to the next. Each particle has its own characteristic vibration.                                                                                                   |
| 2.                | The amplitude of each particle is the same but the phase changes continuously,                                                                                                                                  | 2.               | The amplitudes of the different particles are different, ranging from zero at the nodes to maximum at the antinodes. All the particles in a given segment vibrate in phase but in opposite phase relative to the particles in the adjacent segment. |
| 3.                | No particle is permanently at rest. Different particles attain the state of momentary rest at different instants,                                                                                               | 3.               | The particles at the nodes are permanently at rest but other particles attain their position of momentary rest simultaneously.                                                                                                                      |
| 4.                | All the particles attain the same maximum velocity when they pass through their mean positions.                                                                                                                 | 4.               | All the particles attain their own maximum velocity at the same time when they pass through their mean positions.                                                                                                                                   |

(vii) The ray I have to travel through half silvered glass plate two times and it decreases speed of light two times. The glass plate  $G_2$  cut from the same piece of glass as  $G_1$  and is equal in thickness to  $G_1$  is introduced in the path of beam II to decrease its speed of light two times.  $G_2$  therefore equalizes the path length of the beam I and II in glass and is called compensating glass plate or compensator plate. (03)

(viii) **The Dual nature of light**

- Sometimes it behaves like a particle (called a photon), which explains how light travels in straight lines
- Sometimes it behaves like a wave, which explains how light bends (or diffracts) around an object.
- Scientists accept the evidence that supports this dual nature of light (even though it intuitively doesn't make sense to us!)

(03)

(ix) **Reversible Process**

- A reversible process is one which can be retraced in exactly reverse order, without producing any change in the surroundings.

- In the reverse process, the working substance passes through the same stages as in the direct process, but thermal and mechanical effects at each stage are exactly reversed.
- **Example of Reversible Process (any one)**  
The process of liquefaction and the evaporation of a substance performed slowly are reversible processes.

**Irreversible Process**

- An irreversible process is one which can not be retraced in exactly reverse order, without producing any change in the surroundings.
- All changes which occur suddenly or which involve friction or dissipation of energy through conduction, convection and radiation are irreversible.

**Example (any one):**

- Explosion is an example of highly irreversible process (03)
- (x) a) First law of thermodynamics does not provide a clear idea about the direction of absorption or evolution of heat.
- b) The informations provided by the first law of thermodynamics are not enough to predict the spontaneity or feasibility of a process.
- c) It does not tell about the entropy of system. If all energy is conserved, entropy can never be achieved; if entropy is achieved, conservation has failed. (03)

**SECTION (D)****Q # 4**

- a. Definition and figure of projectile motion (01 + 01)  
Derive: Maximum height + time of flight + range of projectile (02 + 02 + 02)
- b. Numerical: (0.5 + 1.5 + 1.5 + 1.5)
- Given Data:** Mass of ball =  $m = 100\text{gm} = 0.1\text{kg}$   
Speed of ball =  $v = 25\text{ m/sec}$ .
- To Find:** (a) Height to which the ball would reach =  $h = ?$  (ans : 31.9 m)  
(b) If height =  $h' = 25\text{m}$ , then  
(i) work done = ? (6.7 J) (ii) Force of friction =  $F = ?$  (0.3 N)

**Q # 5**

- a. Derivation of Kinetic and Potential Energies in terms of S.H.M and also draw graph (03 + 03 + 01)
- b. Numerical: (03 + 03)
- Solution:**  
Frequency of sound =  $f = 500\text{ Hz}$   
Speed of sound =  $v = 340\text{ m/s}$   
Speed of car =  $u = 20\text{ m/s}$

- (a) The apparent frequency when the car approaches =  $f' = ?$  :  $f' = \left( \frac{v}{v - u} \right) f$  (Ans: 531 Hz)
- (b) The apparent frequency when receding the car =  $f'' = ?$  :  $f'' = \left( \frac{v}{v + u} \right) f$  (Ans: 472 Hz)

**Q # 6**

- (a) Explain the diffraction of X-Rays by crystal and derivation Bragg's Law and also finds the wavelength of Light. (03 + 04 + 01)
- (b) Numerical (01 + 01 + 03)

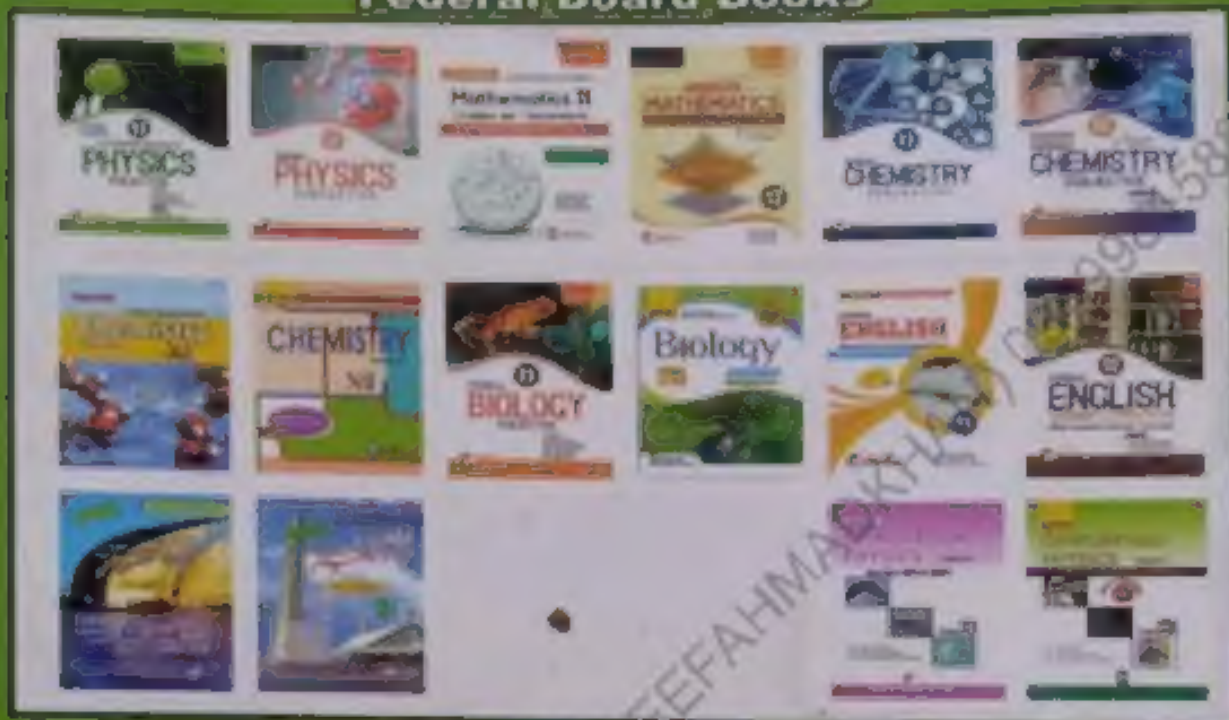
Data ( $E = 2$ ,  $T_1 = -23^\circ\text{C} = 250\text{ K}$ ) + formula ( $E = \frac{T_2}{T_1 - T_2}$ ) + calculation and result

$$(T_1 = 281.25\text{ K} = 281.25 - 273 = 8.2^\circ\text{C})$$

**THE END**

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